UNSTEADY AND TRANSIENT STATE

The process of heat transfer by conduction where the temperature varies with time and with space coordinates, is called 'unsteady or transient'. All transient state systems may be broadly classified into two categories:

(a) Non-periodic Heat Flow System - the temperature at any point within the system changes as a non-linear function of time.

(b) Periodic Heat Flow System - the temperature within the system undergoes periodic changes which may be regular or irregular but definitely cyclic.

There are numerous problems where changes in conditions result in transient temperature distributions and they are quite significant. Such conditions are encountered in - manufacture of ceramics, bricks, glass and heat flow to boiler tubes, metal forming, heat treatment, etc.

7.2. Biot and Fourier Modulus-Definition and Significance

Let us consider an initially heated long cylinder (L >> R) placed in a moving stream of fluid at $T_\infty < T_s$, as shown In Fig. 3.1(a). The convective heat transfer coefficient at the surface is h, where,

$Q = hA\,(T_s - T_\infty)$

This energy must be conducted to the surface, and therefore,

$Q = -kA(dT / dr)_{r = R}$

or, $h(T_s - T_\infty) = -k(dT/dr)_{r=R} \approx -k(T_c-T_s)/R$

where T_c is the temperature at the axis of the cylinder

By rearranging, $(T_s - T_c) / (T_s - T_\infty)\ h/Rk$ (3.1)

The term, hR/k, IS called the 'BlOT MODULUS'. It is a dimensionless number and is the ratio of internal heat flow resistance to external heat flow resistance and plays a fundamental role in transient conduction problems involving surface convection effects. I t provides a measure 0 f the temperature drop in the solid relative to the temperature difference between the surface and the fluid.

For Bi << 1, it is reasonable to assume a uniform temperature distribution across a solid at any time during a transient process.

Founer Modulus - It is also a dimensionless number and is defind as

$$Fo = \alpha t/L^2 \quad (3.2)$$

where L is the characteristic length of the body, a is the thermal diffusivity, and t is the time

The Fourier modulus measures the magnitude of the rate of conduction relative to the change in temperature, i.e., the unsteady effect. If Fo << 1, the change in temperature will be experienced by a region very close to the surface.

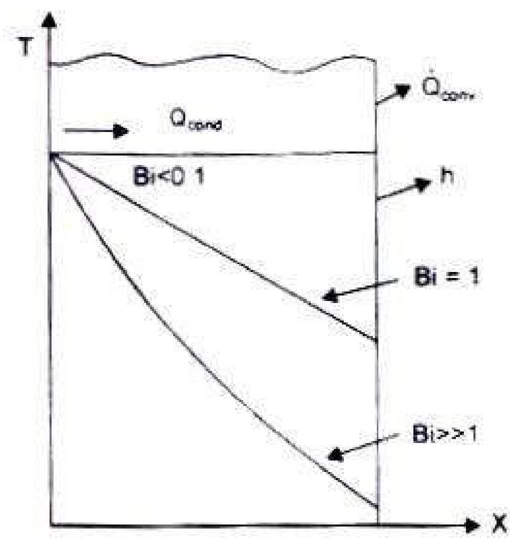


Fig. 1.18 Effect of Biot Modulus on steady state temperature distribution in a plane wall with surface convection.

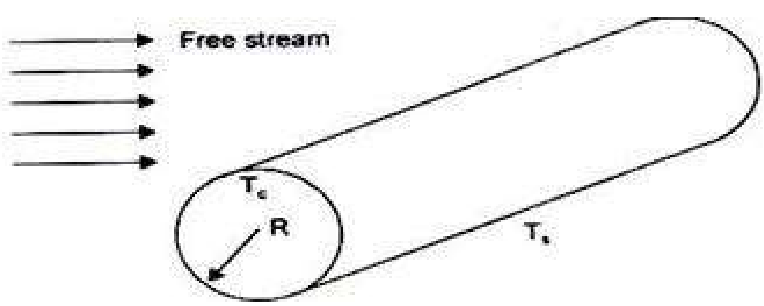


Fig. 1.18 (a) Nomenclature for Biot Modulus

7.3. Lumped Capacity System-Necessary Physical Assumptions

We know that a temperature gradient must exist in a material if heat energy is to be conducted into or out of the body. When $Bi < 0.1$, it is assumed that the internal thermal resistance of the body is very small in comparison with the external resistance and the transfer of heat energy is primarily controlled by the convective heat transfer at the surface. That is, the temperature within the body is approximately uniform. This idealised assumption is possible, if

(a) the physical size of the body is very small,

(b) the thermal conductivity of the material is very large, and

(c) the convective heat transfer coefficient at the surface is very small and there is a large temperature difference across the fluid layer at the interface.

7.4. An Expression for Evaluating the Temperature Variation in a Solid Using Lumped Capacity Analysis

Let us consider a small metallic object which has been suddenly immersed in a fluid during a heat treatment operation. By applying the first law of

Heat flowing out of the body = Decrease in the internal thermal energy of

during a time dt   the body during that time dt

or, $$hA_s(T - T_\infty)dt = - pCVdT$$

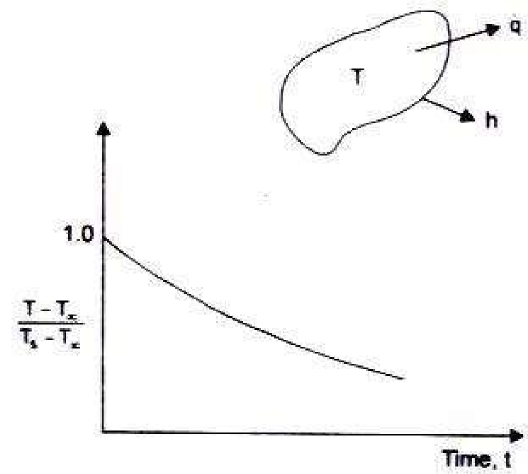
where A_s is the surface area of the body, V is the volume of the body and C is the specific heat capacity.

or, $(hA/\rho CV)dt = - dT/(T - T_\infty)$

with the initial condition being: at $t = 0$, $T = T_s$

The solution is : $(T - T_\infty)/(T_s - T_\infty) = \exp(-hA/\rho CV)t$ (3.3)

Fig. 3.2 depicts the cooling of a body (temperature distribution ⊔ time) using lumped thermal capacity system. The temperature history is seen to be an exponential decay.



We can express

$Bi \times Fo = (hL/k)\times(\alpha t/L^2) = (hL/k)(k/\rho C)(t/L^2) = (hA/\rho CV)t$,

where V / A is the characteristic length L.

And, the solution describing the temperature variation of the object with respect to time is given by

$$(T - T_\infty)/(T_s - T_\infty) = \exp(-Bi\cdot Fo) \qquad (3.4)$$

Example 1.19 Steel balls 10 mm in diameter (k = 48 W/mK), (C = 600 J/kgK) are cooled in air at temperature 35°C from an initial temperature of 750°C. Calculate the time required for the temperature to drop to 150°C when h = 25 W/m2K and density p = 7800 kg/m3.

Solution: Characteristic length, $L = VIA = 4/3\,\pi r^3/4\pi r^2 = r/3 = 5\times 10^{-3}/3$m

$Bi = hL/k = 25 \times 5 \times 10\text{-}^{3}/(3 \times 48) = 8.68 \times 10^{-4} << 0.1$,

Since the internal resistance is negligible, we make use of lumped capacity analysis: Eq. (3.4),

$(T - T_\infty)/(T_s - T_\infty) = \exp(-Bi\ Fo)$; (150 35) / (750 35) = 0.16084

∴ $Bi \times Fo = 1827$; $Fo = 1.827/(8.68 \times 10^{-4})\ 2.1\times 10^3$

or, $\alpha t/L^2 = k/(\rho CL^2)t = 2100$ and t = 568 = 0.158 hour

We can also compute the change in the internal energy of the object as:

$$U_0 - U_t = -\int_0^1 \rho CV dT = \int_0^1 \rho CV\left(T_s - T_\infty\right)\left(-hA/\rho CV\right)\exp t\left(-hAt/\rho CV\right)dt$$

$$= -\rho CV\left(T_s - T_\infty\right)\left[\exp\left(-hAt/\rho CV\right) - 1\right] \quad (3.5)$$

$= -7800 \times 600 \times (4/3)\ \pi (5 \times 10^{-3})^3\ (750\text{-}35)\ (0.16084 - 1)$

$= 1.47 \times 10^3$ J = 1.47 kJ.

If we allow the time 't' to go to infinity, we would have a situation that corresponds to steady state in the new environment. The change in internal energy will be $U_0 - U_\infty = [\rho CV(T_s - T_\infty)\exp(-\infty) - 1] = [\rho CV(T_s - T_\infty]$.

We can also compute the instantaneous heal transfer rate at any time.

or. $Q = -\rho VCdT/dt = -\rho VCd/dt[T_\infty + (T_s - T_\infty)\exp(-hAt/\rho CV)]$

$= hA(T_s - T_\infty)[\exp(-hAt/\rho CV))$ and for t = 60s,

$Q = 25 \times 4 \times 3.142\ (5 \times 10^{-3})^2(750\ 35)\ [\exp(-25 \times 3 \times 60/5 \times 10^{-3} \times 7800 \times 600)]$

= 4.63 W.

Example 1.20 A cylindrical steel ingot (diameter 10 cm. length 30 cm, k = 40 W mK. ρ = 7600 kg/m^3, C = 600 J/kgK) is to be heated in a furnace from 50°C to 850°C. The temperature inside the furnace is 1300°C and the surface heat transfer coefficient is 100 W/m^{2}K. Calculate the time required.

Solution: Characteristic length. $L = V/A = \pi r^2 L/2\pi r(r + L) = rL/2(r + L)$

$= 5 \times 10^{-2} \times 30 \times 10^{-2}/2\ (2\ (5 + 30) \times 10^{-2})$

$= 2.143 \times 10^{-2}$ m.

$Bi = hL/k = 100 \times 2.143 \times 10^{-2}/40 = 0.0536 << 0.1$

$Fo = \alpha t/L^2 = (k/\rho C) \times (t/L^2)$

$= 40 \times t/(7600 \times 600 \times [2.143 \times 1\ 0^{-2})^2] = 191 \times 10^{-2}\ t$

and $(T - T_\infty)/(T_s - T_\infty) = \exp(-Bi\ Fo)$

or, (850 - 1300) /(50 - 1300) = 0.36 = exp (- Bi Fo)

∴ Bi Fo= 102

and Fo = 19.06 and t = 19.06/(1.91×10^{-2}) = 16.63 min

(The length of the ingot is 30 cm and it must be removed from the furnace after a period of 16.63 min. therefore, the speed of the ingot would be 0.3/16.63 = 1.8×10^{-2} m/min.)

Example 1.21 A block of aluminium (2cm × 3cm × 4cm, k = 180 W/mK, $\alpha = 10^{-4} m^2/s$) inllially at 300°C is cooled in air at 30°C. Calculate the temperature of the block after 3 min. Take h = 50W/ m^2K.

Solution: Characteristic length, L= [2 × 3 × 4 /2(2 × 3 + 2 × 4 + 3 × 4)] × 10^{-2}

= 4.6×10^{-3}m

Bi = hL/k = 50 × 4.6 × 10^{-3}/180 = 1.278 × 10^{-3} << 0.1

Fo = $\alpha t/L^2 = 10^{-4} \times 180 / (4.6 \times 10^{-3})^2 = 850$

exp(-Bi Fo) = exp(-1.278 × 10^{-3}×'850) = 0.337

$(T - T_\infty)$ $(T_s - T_\infty)$ - (T - 30)/(300 - 30) = 0.337

∴ T= 121.1°C.

Example 1.22 A copper wire 1 mm in diameter initially at 150°C is suddenly dipped into water at 35°C. Calculate the time required to cool to a temperature of 90°C if h = 100 W/ m^2K. What would be the time required if h = 40 W/m^2K. (for copper; k = 370 W/mK, ρ = 8800 kg/m^3. C = 381 J/kgK.

Solution: The characteristic length for a long cylindrical object can be approximated as r/2. As such,

Bi = hL/k = 100 × 0.5 × 10^{-3}/ (2 × 370) = 6.76 × 10^{-5}<< 0.1

Fo = $\alpha t/L^2 = (k/\rho C) \times (t/L^2)$

= $[370t/(8800 \times 381 \times (0.25 \times 10^{-3})^2] = 1760t$

exp(-Bi Fo) = $(T - T_\infty)(T_s - T_\infty)$

= (90 - 35)/(150 - 35) = 0.478

Bi Fo = 0.738 = 6.76 × 10^{-5} × 1760 t; ∴ t = 6.2s

when h = 40 W/ m^2K, Bi = 2.7 × 10^{-5} and 2.7 × 10^{-5} ×1760 t = 0.738;

or, t = 15.53s.

Example 1.23 A metallic rod (mass 0.1 kg, C = 350 J/kgK, diameter 12.5 mm, surface area 40cm^2) is initially at 100°C. It is cooled in air at 25°C. If the temperature drops to 40°C in 100 seconds, estimate the surface heat transfer coefficient.

Solution: $hA/\rho CV = hA/mC = h \times 40 \times 10^{-4}/(0.1 \times 350) = 1.143\times10^{-4}h$

and, $hAt/\rho CV = 1.143 \times 10^{-4}\,h \times 100 = 1.143 \times 10^{-2}h$

$(T - T_\infty)/(T_s - T_\infty) = (40 - 25) / (100 - 25) = 0.2$

∴ $\exp(-1.143 \times 10^{-2}h) = 0.2$

or, $1.143 \times 10^{-2}h = 1.6094$, and h = 140W/$m^2$K.