2.3 NORTH WEST CORNER RULE

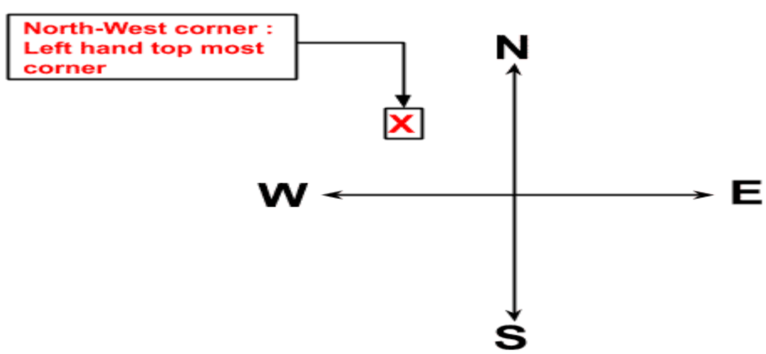
INTRODUCTION

The North West corner method is one of the methods to obtain a basic feasible solution of the transportation problems (special case of LPP). The procedure is given below:

Step 1: Balance the problem i.e. $\sum \textbf{Supply} = \sum \textbf{Demand}$

Step 2: Start allocating from North-West corner cell

We will start the allocation from the left hand top most corner (north-west) cell in the matrix and make allocation based on availability and demand.



Now, verify the smallest among the availability (Supply) and requirement (Demand), corresponding to this cell.

Step 3: Remove the row or column whose supply or demand is fulfilled and prepare a new matrix

As we have fulfilled the availability or requirement for that row or column respectively, remove that row or column and prepare a new matrix.

Step 4: Repeat the procedure until all the allocations are over

Repeat the same procedure of allocation of the new North-west corner until all allocations are over.

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Once all allocations are over, prepare the table with all allocations marked and calculate the transportation cost.

Problem 1:

A mobile phone manufacturing company has three branches located in three different regions, say Jaipur, Udaipur and Mumbai. The company has to transport mobile phones to three destinations, say Kanpur, Pune and Delhi. The availability from Jaipur, Udaipur and Mumbai is 40, 60 and 70 units respectively. The demand at Kanpur, Pune and Delhi are 70, 40 and 60 respectively. The transportation cost is shown in the matrix below (in Rs). Use the North-West corner method to find a basic feasible solution (BFS).

		Destinations			
		Kanpur	Pune	Delhi	Supply
sources	Jaipur	4	5	1	40
	Udaipur	3	4	3	60
	Mumbai	6	2	8	70
	Demand	70	40	60	170

Solution:

Step 1: Balance the problem

Σ Supply=Σ Demand

→ The given transportation problem is balanced.

Step 2: Start allocating from North-West corner cell

We will start the allocation from the left hand top most corner (north-west) cell in the matrix and make allocation based on availability and demand.

Destinations

		Kanpur	Pune	Delhi	Supply
	~~Jaipur~~	~~4 (40)~~	~~5~~	~~1~~	~~40~~ ~~0~~
sources	Udaipur	3	4	3	60
	Mumbai	6	2	8	70
	Demand	~~70~~ 30	40	60	170

Step 3: **Remove the row or column whose supply or demand is fulfilled and prepare a new matrix**

Destinations

		Kanpur	Pune	Delhi	Supply
	Udaipur	3	4	3	60
sources	Mumbai	6	2	8	70
	Demand	30	40	60	

Step 4: Repeat the procedure until all the allocations are over

Repeat the same procedure of allocation of the new North-west corner so generated and check based on the smallest value as shown below, until all allocations are over.

Destinations

		Kanpur	Pune	Delhi	Supply
	Udaipur	3 (30)	4	3	~~60~~ 30
sources	Mumbai	6	2	8	70
	Demand	~~30~~ 0	40	60	

Destinations

		Pune	Delhi	Supply
	~~Udaipur~~	~~4(30)~~	~~3~~	~~30~~ 0
sources	Mumbai	2	8	70
	Demand	~~40~~ 10	60	

Destinations

		Pune	Delhi	Supply
sources	Mumbai	2 (10)	8(60)	~~70~~ ~~60~~ 0
	Demand	~~10~~ 0	~~60~~ 0	

Step 5: After all the allocations are over, write the allocations and calculate the transportation cost

Destinations

		Kanpur	Pune	Delhi	Supply
	Jaipur	4 (40)	5	1	40
sources	Udaipur	3 (30)	4 (30)	3	60
	Mumbai	6	2 (10)	8 (60)	70
	Demand	70	40	60	

Therefore, Transportation Cost = (4*40) + (3*30) + (4*30) + (2*10) + (8*60) = Rs. 870.

Problem : 2

Find Solution using North-West Corner method

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Demand	5	8	7	14	

Solution:

Σ Supply=Σ Demand

→ The given transportation problem is balanced.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19	30	50	10	7
$S2$	70	30	40	60	9
$S3$	40	8	70	20	18
Demand	5	8	7	14	

The rim values for $S1$=7 and $D1$=5 are compared.

The smaller of the two i.e. min(7,5) = **5** is assigned to $S1$ $D1$

This meets the complete demand of $D1$ and leaves 7 - 5=2 units with $S1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30	50	10	2
$S2$	70	30	40	60	9
$S3$	40	8	70	20	18
Demand	0	8	7	14	

The rim values for $S1$=2 and $D2$=8 are compared.

The smaller of the two i.e. min(2,8) = **2** is assigned to $S1$ $D2$

This exhausts the capacity of $S1$ and leaves 8 - 2=6 units with $D2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30**(2)**	50	10	0
$S2$	70	30	40	60	9
$S3$	40	8	70	20	18
Demand	0	6	7	14	

The rim values for $S2$=9 and $D2$=6 are compared.

The smaller of the two i.e. min(9,6) = **6** is assigned to $S2$ $D2$

This meets the complete demand of $D2$ and leaves 9 - 6=3 units with $S2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30**(2)**	50	10	0
$S2$	70	30**(6)**	40	60	3
$S3$	40	8	70	20	18
Demand	0	0	7	14	

The rim values for $S2$=3 and $D3$=7 are compared.

The smaller of the two i.e. min(3,7) = **3** is assigned to $S2$ $D3$

This exhausts the capacity of $S2$ and leaves 7 - 3=4 units with $D3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30**(2)**	50	10	0
$S2$	70	30**(6)**	40**(3)**	60	0
$S3$	40	8	70	20	18
Demand	0	0	4	14	

The rim values for $S3$=18 and $D3$=4 are compared.

The smaller of the two i.e. min(18,4) = **4** is assigned to $S3$ $D3$

This meets the complete demand of $D3$ and leaves 18 - 4=14 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30**(2)**	50	10	0
$S2$	70	30**(6)**	40**(3)**	60	0
$S3$	40	8	70**(4)**	20	14
Demand	0	0	0	14	

The rim values for $S3$=14 and $D4$=14 are compared.
The smaller of the two i.e. min(14,14) = **14** is assigned to $S3$ $D4$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19**(5)**	30**(2)**	50	10	0
$S2$	70	30**(6)**	40**(3)**	60	0
$S3$	40	8	70**(4)**	20**(14)**	0
Demand	0	0	0	0	

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	19 **(5)**	30 **(2)**	50	10	7
$S2$	70	30 **(6)**	40 **(3)**	60	9
$S3$	40	8	70 **(4)**	20 **(14)**	18
Demand	5	8	7	14	

The minimum total transportation cost $=19\times5+30\times2+30\times6+40\times3+70\times4+20\times14=1015$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6

∴ This solution is non-degenerate.

Problem: 3

The Amulya Milk Company has three plants located throughout a state with production capacity 50, 75 and 25 gallons. Each day the firm must furnish its four retail shops R_1, R_2, R_3, & R_4 with at least 20, 20 , 50, and 60 gallons respectively. The transportation costs (in Rs.) are given below.

Plant	**Retail Shop**				**Supply**
	R_1	**R_2**	**R_3**	**R_4**	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

The economic problem is to distribute the available product to different retail shops in such a way so that the total transportation cost is minimum

Solution.

Starting from the North west corner, we allocate min (50, 20) to P_1R_1, i.e., 20 units to cell P_1R_1. The demand for the first column is satisfied. The allocation is shown in the following table.

Plant	Retail Shop				Supply
	R_1	**R_2**	**R_3**	**R_4**	
P_1	3 20	5	7	6	~~50~~ 30
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	20	50	60	

Now we move horizontally to the second column in the first row and allocate 20 units to cell P_1R_2. The demand for the second column is also satisfied.

Plant	Retail Shop				Supply
	R_1	**R_2**	**R_3**	**R_4**	
P_1	3 20	5 20	7	6	~~50~~ ~~30~~ 10
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	~~20~~	50	60	

Proceeding in this way, we observe that $P_1R_3 = 10$, $P_2R_3 = 40$, $P_2R_4 = 35$, $P_3R_4 = 25$. The initial basic feasible solution is shown below.

Plant	Retail Shop				Supply
	R_1	R_2	R_3	R_4	
P_1	3 20	5 20	7 10	6	~~50~~
P_2	2	5	8 40	2 35	~~75~~
P_3	3	6	9	2 25	~~25~~
Demand	~~20~~	~~20~~	~~50~~	~~60~~	

Here, number of retail shops(n) = 4, and Number of plants (m) = 3

Number of basic variables = m + n – 1 = 3 + 4 – 1 = 6.

The total **transportation cost** is calculated by multiplying each x_{ij} in an occupied cell with the corresponding c_{ij} and adding as follows:

(20 X 3) + (20 X 5) + (10 X 7) + (40 X 8) + (35 X 2) + (25 X 2) = 670.

Problem : 4
Find Solution using North-West Corner method

Source \ To	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150

In this problem, three sources A, B and C with the production capacity of 50 units, 40 units, 60 units of product respectively is given. Every day the demand of three retailers D, E, F is to be furnished with at least 20 units, 95 units and 35 units of product respectively. The transportation costs are also given in the matrix.

Σ Supply=Σ Demand

→ The given transportation problem is balanced.

1. Select the north-west or extreme left corner of the matrix, assign as many units as possible to cell AD, within the supply and demand constraints. Such as 20 units are assigned to the first cell, that satisfies the demand of destination D while the supply is in surplus.
2. Now move horizontally and assign 30 units to the cell AE. Since 30 units are available with the source A, the supply gets fully saturated.
3. Now move vertically in the matrix and assign 40 units to Cell BE. The supply of source B also gets fully saturated.
4. Again move vertically, and assign 25 units to cell CE, the demand of destination E is fulfilled.
5. Move horizontally in the matrix and assign 35 units to cell CF, both the demand and supply of origin and destination gets saturated. Now the total cost can be computed.

Source \ To	D	E	F	Supply
A	5 20	8 30	4	50
B	6	6 40	3	40
C	3	9 25	6 35	60
Demand	20	95	35	150

The Total cost can be computed by multiplying the units assigned to each cell with the concerned transportation cost. Therefore,

Total Cost = 20*5+ 30*8+ 40*6+ 25*9+ 35*6 = Rs. 1015

Problem 5: Find Solution using North-West Corner method

	D1	D2	D3	D4	Supply
S1	11	13	17	14	250
S2	16	18	14	10	300
S3	21	24	13	10	400
Demand	200	225	275	250	

Solution:

Σ Supply=Σ Demand

→ The given transportation problem is balanced.

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11	13	17	14	250
$S2$	16	18	14	10	300
$S3$	21	24	13	10	400
Demand	200	225	275	250	

The rim values for $S1$=250 and $D1$=200 are compared.

The smaller of the two i.e. min(250,200) = 200 is assigned to $S1$ $D1$

This meets the complete demand of $D1$ and leaves 250 - 200=50 units with $S1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13	17	14	50
$S2$	16	18	14	10	300
$S3$	21	24	13	10	400
Demand	0	225	275	250	

The rim values for $S1$=50 and $D2$=225 are compared.

The smaller of the two i.e. min(50,225) = 50 is assigned to $S1$ $D2$

This exhausts the capacity of $S1$ and leaves 225 - 50=175 units with $D2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13(50)	17	14	0
$S2$	16	18	14	10	300
$S3$	21	24	13	10	400
Demand	0	175	275	250	

The rim values for $S2$=300 and $D2$=175 are compared.

The smaller of the two i.e. min(300,175) = 175 is assigned to $S2$ $D2$

This meets the complete demand of $D2$ and leaves 300 - 175=125 units with $S2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13(50)	17	14	0
$S2$	16	18(175)	14	10	125
$S3$	21	24	13	10	400
Demand	0	0	275	250	

The rim values for $S2$=125 and $D3$=275 are compared.

The smaller of the two i.e. min(125,275) = 125 is assigned to $S2$ $D3$

This exhausts the capacity of $S2$ and leaves 275 - 125=150 units with $D3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13(50)	17	14	0
$S2$	16	18(175)	14(125)	10	0
$S3$	21	24	13	10	400
Demand	0	0	150	250	

The rim values for $S3$=400 and $D3$=150 are compared.

The smaller of the two i.e. min(400,150) = 150 is assigned to $S3$ $D3$

This meets the complete demand of $D3$ and leaves 400 - 150=250 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13(50)	17	14	0
$S2$	16	18(175)	14(125)	10	0
$S3$	21	24	13(150)	10	250
Demand	0	0	0	250	

The rim values for $S3$=250 and $D4$=250 are compared.

The smaller of the two i.e. min(250,250) = 250 is assigned to $S3$ $D4$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11(200)	13(50)	17	14	0
$S2$	16	18(175)	14(125)	10	0
$S3$	21	24	13(150)	10(250)	0
Demand	0	0	0	0	

Initial feasible solution is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	11 (200)	13 (50)	17	14	250
$S2$	16	18 (175)	14 (125)	10	300
$S3$	21	24	13 (150)	10 (250)	400
Demand	200	225	275	250	

The minimum total transportation cost $=11\times200+13\times50+18\times175+14\times125+13\times150+10\times250=12200$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate.

Problem: 6 Find Solution using North-West Corner method

	D1	D2	D3	Supply
S1	4	8	8	76
S2	16	24	16	82
S3	8	16	24	77
Demand	72	102	41	

Solution: Problem Table is

	$D1$	$D2$	$D3$	Supply
$S1$	4	8	8	76
$S2$	16	24	16	82
$S3$	8	16	24	77
Demand	72	102	41	

Here Total Demand = 215 is less than Total Supply = 235. So We add a dummy demand constraint with 0 unit cost and with allocation 20. The modified table is

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4	8	8	0	76
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	72	102	41	20	

The rim values for $S1$=76 and $D1$=72 are compared.

The smaller of the two i.e. min(76,72) = 72 is assigned to $S1$ $D1$

This meets the complete demand of $D1$ and leaves 76 - 72=4 units with $S1$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4(72)	8	8	0	4
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	0	102	41	20	

The rim values for $S1$=4 and $D2$=102 are compared.

The smaller of the two i.e. min(4,102) = 4 is assigned to $S1$ $D2$

This exhausts the capacity of $S1$ and leaves 102 - 4=98 units with $D2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4(72)	8(4)	8	0	0
$S2$	16	24	16	0	82
$S3$	8	16	24	0	77
Demand	0	98	41	20	

The rim values for $S2$=82 and $D2$=98 are compared.

The smaller of the two i.e. min(82,98) = 82 is assigned to $S2$ $D2$

This exhausts the capacity of $S2$ and leaves 98 - 82=16 units with $D2$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4(72)	8(4)	8	0	0
$S2$	16	24(82)	16	0	0
$S3$	8	16	24	0	77
Demand	0	16	41	20	

The rim values for $S3$=77 and $D2$=16 are compared.

The smaller of the two i.e. min(77,16) = 16 is assigned to $S3$ $D2$

This meets the complete demand of $D2$ and leaves 77 - 16=61 units with $S3$

	$D1$	$D2$	$D3$	$D4$	Supply
$S1$	4(72)	8(4)	8	0	0
$S2$	16	24(82)	16	0	0
$S3$	8	16(16)	24	0	61
Demand	0	0	41	20	

The rim values for $S3$=61 and $D3$=41 are compared.

The smaller of the two i.e. min(61,41) = 41 is assigned to $S3$ $D3$

This meets the complete demand of $D3$ and leaves 61 - 41=20 units with $S3$

	*D*1	*D*2	*D*3	*D4*	Supply
*S*1	4(72)	8(4)	8	0	0
*S*2	16	24(82)	16	0	0
*S*3	8	16(16)	24(41)	0	20
Demand	0	0	0	20	

The rim values for *S*3=20 and *Ddummy*=20 are compared.
The smaller of the two i.e. min(20,20) = 20 is assigned to *S*3 *D4*

	*D*1	*D*2	*D*3	*D4*	Supply
*S*1	4(72)	8(4)	8	0	0
*S*2	16	24(82)	16	0	0
*S*3	8	16(16)	24(41)	0(20)	0
Demand	0	0	0	0	

Initial feasible solution is

	*D*1	*D*2	*D*3	*D4*	Supply
*S*1	4 (72)	8 (4)	8	0	76
*S*2	16	24 (82)	16	0	82
*S*3	8	16 (16)	24 (41)	0 (20)	77
Demand	72	102	41	20	

The minimum total transportation cost $=4\times72+8\times4+24\times82+16\times16+24\times41+0\times20=3528$

Here, the number of allocated cells = 6 is equal to m + n - 1 = 3 + 4 - 1 = 6
∴ This solution is non-degenerate