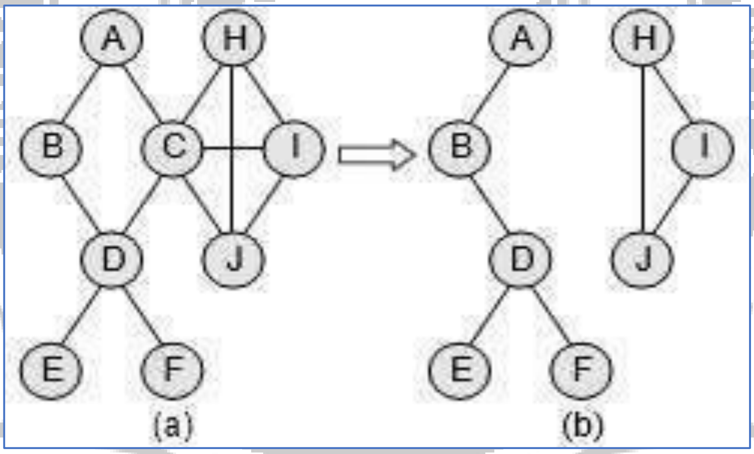
BI- CONNECTIVITY

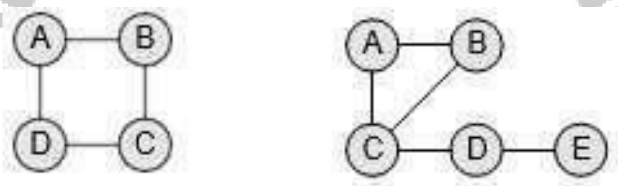
A vertex v of G is called an articulation point, if removing v along with the edges incident on v, results in a graph that has at least two connected components.

A bi-connected graph is defined as a connected graph that has no articulation vertices. That is, a bi-connected graph is connected and non-separable in the sense that even if we remove any vertex from the graph, the resultant graph is still connected. By definition,

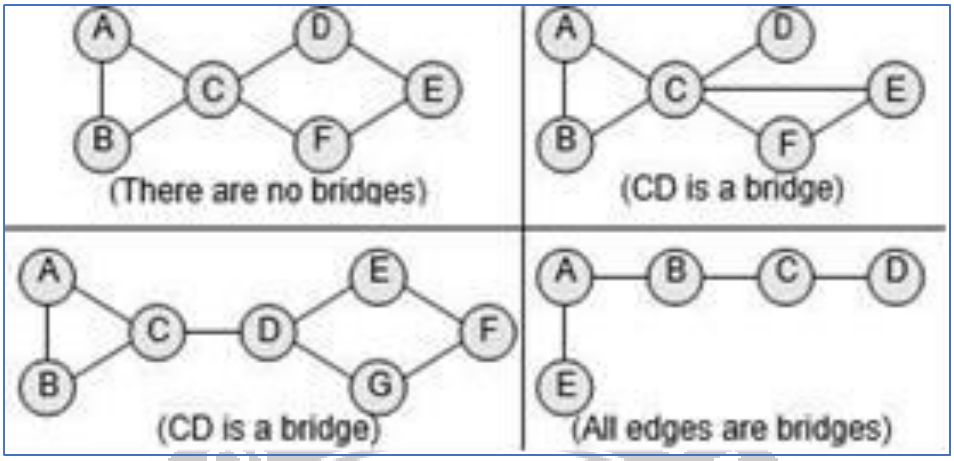
- A bi-connected undirected graph is a connected graph that cannot be broken into disconnected pieces by deleting any single vertex.
- In a bi-connected directed graph, for any two vertices v and w, there are two directed paths from v to w which have no vertices in common other than v and w.



Note that the graph is not a bi-connected graph, as deleting vertex C from the graph results in two disconnected components of the original graph



Biconnected Graph



Graph with Bridges

As for vertices, there is a related concept for edges. An edge in a graph is called a bridge if removing that edge results in a disconnected graph. Also, an edge in a graph that does not lie on a cycle is a bridge. This means that a bridge has at least one articulation point at its end, although it is not necessary that the articulation point is linked to a bridge.

CUT VERTEX

Articulation point

The vertices whose removal would disconnect the graph are known as articulation points.

Steps to find Articulation point

1. Find DFS spanning tree

2. Number the vertex in the order in which they are visited. This number is referred as Num(v)

3. Compute the lowest numbered vertex for every vertex v in the DFS spanning tree which we call as Low(w)(i.e) reachable from v by taking 0 or more tree edges and then possible one back edge. By definition Low(v) is the

a. Minimum of Num(v)

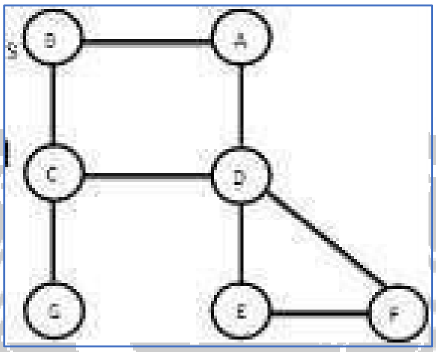
b. The lowest Num(w) among all back edges

c. The lowest Low(w) among all tree edges (i.e) min(Num(v),Num(w),Low(w))

Use post order traversal to calculate Low(v). The root is articulation if and only if it has more than 2 children. Any vertex V other than root is an articulation point if and only if V has some children such that Low(w)≥Num(v).

Example

Checking whether finding the articulation point



Low(v)=min(Num(v),Num(w),Low(w))

Low(F)=min(Num(F),Num(D),Low(D)) =min(6,4) =4

Low(E)=min(Num(E),Num(F),Low(F)) =min(5,6,4) =4

Low(D)=min(Num(D),Num(E),Low(E),Num(A),Low(A)) =min(4,5,4,1) =1

Low(G)=min(Num(G)) =min(7) =7

Low(C)=min(Num(C),Num(D),Low(D),Num(G),Low(G)) =min(3,4,1,7,7) =1

Low(B)=min(Num(B),Num(C),Low(C)) =min(2,3,1) =1

Low(A)=min(Num(A),Num(B),Low(B)) =min(1,2,1) =1

At vertex F Low(W) ≥Num(V) 1 ≥ 6

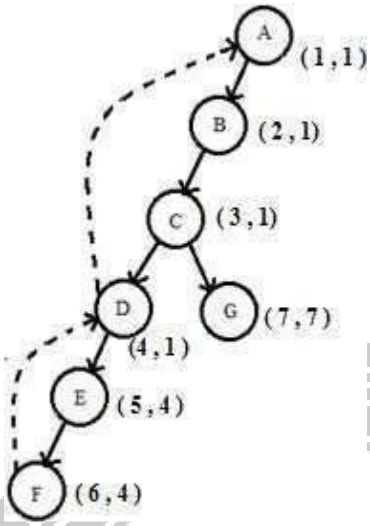
At vertex D Low(A) ≥ Num(D) 1 ≥ 4

Low(E) ≥ Num(D) 4 ≥ 4(articulation point)

At vertex E Low(F) ≥ Num(E) 4 ≥5

At vertex C Low(D) ≥ Num(C) 1 ≥3

Low(G) ≥Num(C) 6 ≥ 3(articulation point)



DFS Spanning Tree