PUMPING LEMMA FOR CFL

If L is a context-free language, there is a pumping length p such that any string w ∈ L of length ≥ p can be written as w = uvxyz, where vy ≠ ε, |vxy| ≤ p, and for all i ≥ 0, uvixyiz ∈ L.

Applications of Pumping Lemma

Pumping lemma is used to check whether a grammar is context free or not. Let us take an example and show how it is checked.

Problem

Find out whether the language L = {xnynzn | n ≥ 1} is context free or not.

Solution

Let L is context free. Then, L must satisfy pumping lemma.

At first, choose a number n of the pumping lemma. Then, take z as 0n1n2n.

Break z into uvwxy, where

|vwx| ≤ n and vx ≠ ε.

Hence vwx cannot involve both 0s and 2s, since the last 0 and the first 2 are at least (n+1) positions apart. There are two cases −

Case 1 − vwx has no 2s. Then vx has only 0s and 1s. Then uwy, which would have to be in L, has n 2s, but fewer than n 0s or 1s.

Case 2 − vwx has no 0s.

Here contradiction occurs.

Hence, L is not a context-free language.

CFL Closure Property

Context-free languages are closed under −

- Union
- Concatenation
- Kleene Star operation

Union

Let L1 and L2 be two context free languages. Then L1 ∪ L2 is also context free.

Example

Let L1 = { anbn , n > 0}. Corresponding grammar G1 will have P: S1 → aAb|ab

Let L2 = { cmdm , m ≥ 0}. Corresponding grammar G2 will have P: S2 → cBb| ε

Union of L1 and L2, L = L1 ∪ L2 = { anbn } ∪ { cmdm }

The corresponding grammar G will have the additional production S → S1 | S2

Concatenation

If L1 and L2 are context free languages, then L1L2 is also context free.

Example

Union of the languages L1 and L2, L = L1L2 = { anbncmdm }

The corresponding grammar G will have the additional production S → S1 S2

Kleene Star

If L is a context free language, then L* is also context free.

Example

Let L = { anbn , n ≥ 0}. Corresponding grammar G will have P: S → aAb| ε

Kleene Star L1 = { anbn }*

The corresponding grammar G1 will have additional productions S1 → SS1 | ε

Context-free languages are not closed under −

- Intersection − If L1 and L2 are context free languages, then L1 ∩ L2 is not necessarily context free.
- Intersection with Regular Language − If L1 is a regular language and L2 is a context free language, then L1 ∩ L2 is a context free language.
- Complement − If L1 is a context free language, then L1’ may not be context free.