2 Laminar Flow Forced Convection Heat Transfer

2.1Forced Convection Heat Transfer Principles

The mechanism of heat transfer by convection requires mixing of one portion of fluid with another portion due to gross movement of the mass of the fluid. The transfer of heat energy from one fluid particle or a molecule to another one is still by conduction but the energy is transported from one point in space to another by the displacement of fluid.

When the motion of fluid is created by the imposition of external forces in the form of pressure differences, the process of heat transfer is called'forced convection'. And, the motion of fluid particles may be either laminar or turbulent and that depends upon the relative magnitude of inertia and viscous forces, determined by the dimensionless parameter Reynolds number. In free convection, the velocity of fluid particle is very small in comparison with the velocity of fluid particles in forced convection, whether laminar or turbulent. In forced convection heat

transfer, $Gr/Re^2 \ll 1$, in free convection heat transfer, $GrRe^2 \gg 1$ and we have combined free and forced convection when $Gr/Re^2 \approx 1$.

2.2. Methods for Determining Heat Transfer Coefficient

The convective heat transfer coefficient in forced flow can be evaluated by: (a) Dimensional Analysis combined with experiments;

(b) Reynolds Analogy _ an analogy between heat and momentum transfer; (c) Analytical Methods _ exact and approximate analyses of boundary layer equations.

2.3. Method of Dimensional Analysis

As pointed out in Chapter 5, dimensional analysis does not yield equations which can be solved. It simply combines the pertinent variables into non-dimensional numbers which facilitate the interpretation and extend the range of application of experimental data. The relevant variables for forced convection heat transfer phenomenon whether laminar or turbulent, are

(b) The properties of the fluid – density p, specific heat capacity C_p, dynamic or absolute viscosity μ thermal conductivity k.

(ii) The properties of flow–flow velocity Y, and the characteristic dimension of the system L.

As such, the convective heat transfer coefficient, h, is written as h = f (ρ, V, L, μ, Cp, k) = 0 (5.14)

Since there are seven variables and four primary dimensions, we would expect three dimensionless numbers. As before, we choose four independent or core variables as ρ ,V, L, k, and calculate the dimensionless numbers by applying Buckingham π method:

$$\pi_1 = \rho^a V^b L^c K^d h = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(MT^{-3}\theta^{-1}\right)$$

$$= M^o L^o T^o \theta^o$$

Equating the powers of M, L, T and θ on both sides, we get

M : a + d + 1 = O

L : - 3a + b + c + d = 0

T : - b – 3d – 3 = 0 By solving them, we have

θ : - d – 1= 0. D = -1, a = 0, b = 0, c = 1.

Therefore, π_1= hL/k is the Nusselt number.

$$\pi_2 = \rho^a V^b L^c K^d \mu = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(ML^{-1}T^{-1}\right)$$

$$= M^o L^o T^o \theta^o$$

Equating the powers of M, L, T and on both sides, we get

$M : a + d + 1 = 0$

$L : -3a + b + c + d = 1 = 0$

$T : -b - 3d - 1 = 0$

$\theta : -d = 0.$

By solving them, d = 0, b = - 1, a = - 1, c = - 1

and $\pi_2 = \mu / \rho VL; or, \pi_3 = \frac{1}{\pi_2} = \frac{\rho VL}{\mu}$

(Reynolds number is a flow parameter of greatest significance. It is the ratio of inertia forces to viscous forces and is of prime importance to ascertain the conditions under which a flow is laminar or turbulent. It also compares one flow with another provided the corresponding length and velocities are comparable in two flows. There would be a similarity in flow between two flows when the Reynolds numbers are equal and the geometrical similarities are taken into consideration.)

$$\pi_4 = \rho^a V^b L^c k^d C_p = \left(ML^{-3}\right)^a \left(LT^{-1}\right)^b (L)^c \left(MLT^{-3}\theta^{-1}\right)^d \left(L^2T^{-2}\theta^{-1}\right)$$

$M^o L^o T^o \theta^o$

Equating the powers of M, L, T, on both Sides, we get

$M : a + d = 0;$ $\quad L : -3a + b + c + d + 2 = 0$

$T : -b - 3d - 2 = 0;$ $\quad \theta : -d - 1 = 0$

By solving them,

d =- 1,a = l, b = l, c = l,

$$\pi_4 = \frac{\rho VL}{k} C_p; \quad \pi_5 = \pi_4 \times \pi_2$$

$$= \frac{\rho VL}{k} C_p \times \frac{\mu}{\rho VL} = \frac{\mu C_p}{k}$$

$\therefore \pi_5$ is Prandtl number.

Therefore, the functional relationship is expressed as:

$Nu = f(Re, Pr)$; or $Nu = C\ Re^m\ Pr^n$ (5.15)

Where the values of c, m and n are determined experimentally.

2.4. Principles of Reynolds Analogy

Reynolds was the first person to observe that there exists a similarity between the exchange of momentum and the exchange of heat energy in laminar motion and for that reason it has been termed 'Reynolds analogy'. Let us consi der the motion of a fluid where the fluid is flowing over a plane wall. The X-coordinate is measured parallel to the surface and the V-coordinate is measured normal to it. Since all fluids are real and viscous, there would be a thin layer, called momentum boundary layer, in the vicinity of the wall where a velocity gradient normal to the direction of flow exists. When the temperature of the surface of the wall is different than the temperature of the fluid stream, there would also be a thin layer, called thermal boundary layer, where there is a variation in temperature normal to the direction of flow. Fig. 2.6 depicts the velocity distribution and temperature profile for the laminar motion of the fluid flowing past a plane wall.

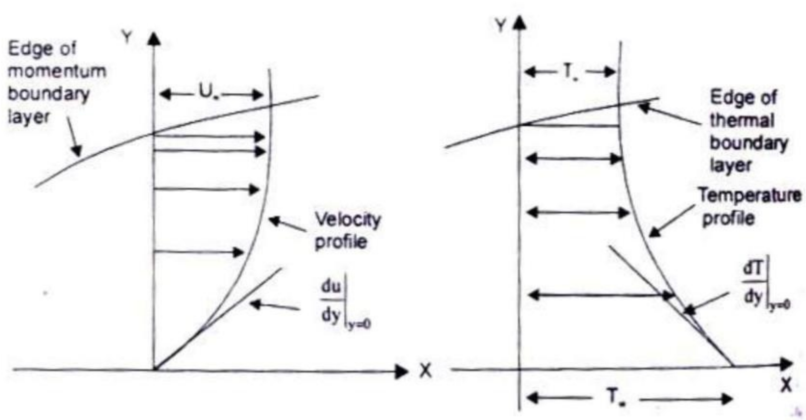


Fig. 2.6 velocity distribution and temperature profile for laminar motion of the fluid over a plane surface

In a two-dimensional flow, the shearing stress is given by $\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0}$

and the rate of heat transfer per unit area is given by $\frac{\dot{Q}}{A} = \frac{\tau_w k}{\mu}\frac{dT}{du}$

For Pr = $\mu C_p/k = 1$, we have $k/\mu = C_p$ and therefore, we can write after separating the variables,

$$\frac{\dot{Q}}{A\tau_w C_p} du = -dT \qquad (5.16)$$

Assuming that Q and τ_w are constant at any station x, we integrate equation (5.16) between the limits: u = 0 when T = T_w, and u = U_∞ when T = T_∞, and we get,

$$\dot{Q}/\left(A\tau_w C_p\right) \times U_\infty = \left(T_w - T_\infty\right)$$

Since by definition, $\dot{Q}/A = h_x\left(T_w - T_\infty\right)$, and $\tau_w = C_{fx} \times \rho U_\infty^2/2$,

Where C_{fx}, is the skin friction coefficient at the station x. We have

$$C_{fx}/2 = h_x/(C_p \rho U_\infty) \qquad (5.17)$$

Since $h_x/C_p\rho U_\infty = \left(h_x x/k\right) \times \left(\mu/\rho \times U_\infty\right) \times \left(k/\mu C_p\right) = Nu_x/\left(Re.Pr\right)$,

$$Nu_x/Re.Pr = C_{fx}/2 = \text{Stantonnumer}, St. \qquad (5.18)$$

Equation (5.18) is satisfactory for gases in which Pr is approximately equal to unity. Colburn has shown that Eq. (5.18) can also be used for fluids having Prandtl numbers ranging from 0.6 to about 50 if it is modified in accordance with experirnental results.

$$\text{Or, } \frac{Nu_x}{Re_x Pr}.Pr^{2/3} = St_x Pr^{2/3} = C_{fx}/2 \qquad (5.19)$$

Eq. (5.19) expresses the relation between fluid friction and heat transfer for laminar flow over a plane wall. The heat transfer coefficient could thus be determined by making measurements of the frictional drag on a plate under conditions in which no heat transfer is involved.

Example 2.4 Glycerine at 35°C flows over a 30cm by 3Ocm flat plate at a velocity of 1.25 m/s. The drag force is measured as 9.8 N (both Side of the plate). Calculate the heat transfer for such a flow system.

Solution: From tables, the properties of glycerine at 35°C are:

ρ = 1256 kg/m^3, C_p = 2.5 kJ/kgK, μ = 0.28 kg/m-s, k = 0.286 W/mK, Pr = 2.4 Re= $\rho VL/\mu$ = 1256 × 1.25 × 0.30/0.28 = 1682.14, a laminar flow.*

Average shear stress on one side of the plate = drag force/area

= 9.8/(2 × 0.3 × 0.3) = 54.4

and shear stress = $C_f \rho U^2/2$

$\therefore$ The average skin friction coefficient, Cr/ 2 = $\frac{\tau}{\rho U^2}$

= 54.4/(1256 ×1.25 × 1.25) = 0.0277

From Reynolds analogy, $C_f/2 = St. Pr^{2/3}$

or, h = $\rho\ C_p\ U \times C_f/2 \times Pr^{-2/3} = \frac{1256 \times 2.5 \times 1.25 \times 0.0277}{(2.45)^{0.667}}$ =59.8 kW/m^{2}K.

2.5. Analytical Evaluation of 'h' for Laminar Flow over a Flat Plat – Assumptions

As pointed out earlier, when the motion of the fluid is caused by the imposition of external forces, such as pressure differences, and the fluid flows over a solid surface, at a temperature different from the temperature of the fluid, the mechanism of heat transfer is called 'forced convection'. Therefore, any analytical approach to determine the convective heat transfer coefficient would require the temperature distribution in the flow field surrounding the body. That is, the theoretical analysis would require the use of the equation of motion of the viscous fluid flowing over the body along with the application of the principles of conservation of mass and energy in order to relate the heat energy that is convected away by the fluid from the solid surface.

For the sake of simplicity, we will consider the motion of the fluid in 2 space dimension, and a steady flow. Further, the fluid properties like viscosity, density, specific heat, etc are constant in the flow field, the viscous shear forces m the Y–direction is negligible and there are no variations in pressure also in the Y –direction.

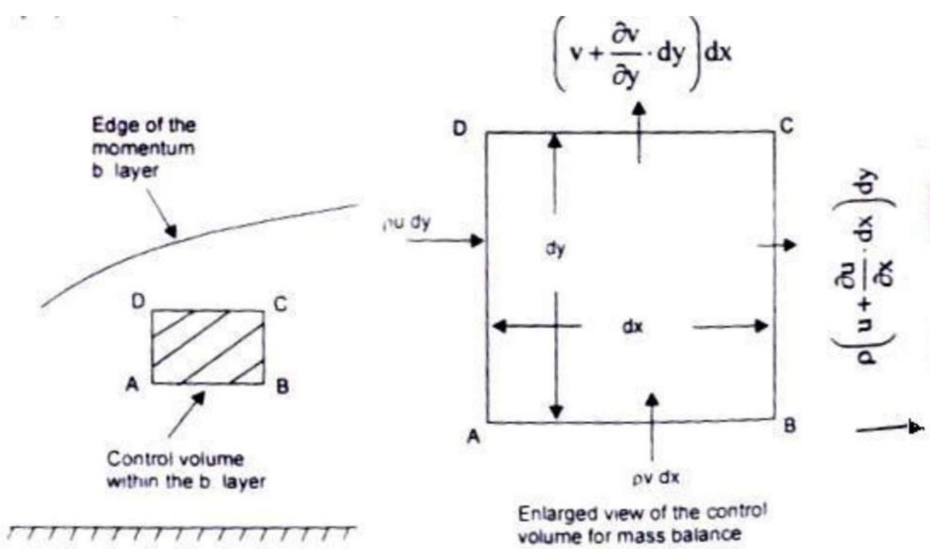
2.6. Derivation of the Equation of Continuity–Conservation of Mass

We choose a control volume within the laminar boundary layer as shown in Fig. 6.2. The mass will enter the control volume from the left and bottom face and will leave the control volume from the right and top face. As such, for unit depth in the Z-direction,

$$\dot{m}_{AD} = \rho u dy \ ; \quad \dot{m}_{BC} = \rho\left(u + \frac{\partial u}{dx}.dx \right)dy;$$

$$\dot{m}_{AB} = \rho v dx \ ; \quad \dot{m}_{CD} = \rho\left(v + \frac{\partial u}{dy}.dy \right)dx;$$

For steady flow conditions, the net efflux of mass from the control volume is zero, therefore,



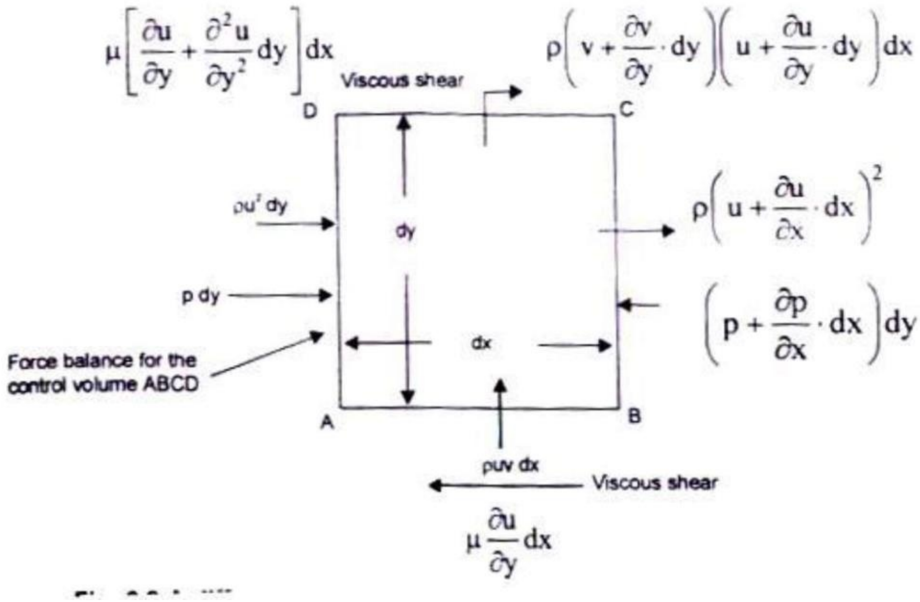


Fig. 2.7 a differential control volume within the boundary layer for laminar flow over a plane wall

$$\rho^{udy}+\rho^{xdx}=\rho^{udy}+\rho\frac{\partial^{u}}{\partial x}dxdy+\rho^{vdx}+\rho\frac{\partial^{v}}{\partial x}.dxdy$$

or, $\partial u/\partial x+\partial v+\partial y=0,$ the equation of continuity. (2.20)

Concept of Critical Thickness of Insulation

The addition of insulation at the outside surface of small pipes may not reduce the rate of heat transfer. When an insulation is added on the outer surface of a bare pipe, its outer radius, r_0 increases and this increases the thermal resistance due to conduction logarithmically whereas t he thermal resistance to heat flow due to fluid film on the outer surface decreases linearly with increasing radius, r_0. Since the total thermal resistance is proportional to the sum of these two resistances, the rate of heat flow may not decrease as insulation is added to the bare pipe.

Fig. 2.7 shows a plot of heat loss against the insulation radius for two different cases. For small pipes or wires, the radius r_l may be less than re and in that case, addition of insulation to the bare pipe will increase the heat loss until the critical radius is reached. Further addition of insulation will decrease the heat loss rate from this peak value. The insulation thickness (r^{*} r_l) must be added to reduce the heat loss below the uninsulated rate. If the outer pipe radius r_l is greater than the critical radius re any insulation added will decrease the heat loss.