

4.4 BESSEL'S DIFFERENTIAL EQUATION AND BESSEL FUNCTION **& TM AND TE WAVES IN CIRCULAR WAVE GUIDES:**

A circular waveguide is a hollow metallic tube with circular cross section for propagating the electromagnetic waves by continuous reflections from the surfaces or walls of the guide.

The circular waveguides are avoided because of the following reasons:

- a) The frequency difference between the lowest frequency on the dominant mode and the next mode is smaller than in a rectangular waveguide, with $b/a = 0.5$
- b) The circular symmetry of the waveguide may reflect on the possibility of the wave not maintaining its polarization throughout the length of the guide.
- c) For the same operating frequency, circular waveguide is bigger in size than a rectangular waveguide.

The possible TM modes in a circular waveguide are: TM₀₁, TM₀₂, TM₁₁, TM₁₂.

The root values for the TM modes are:

- $(ha)_{01} = 2.405$ for TM₀₁
- $(ha)_{02} = 5.53$ for TM₀₂
- $(ha)_{11} = 3.85$ for TM₁₁
- $(ha)_{12} = 7.02$ for TM₁₂

The *dominant mode* for a circular waveguide is defined as the lowest order mode having the lowest root value.

The possible TE modes in a circular waveguide are: TE₀₁, TE₀₂, TE₁₁, TE₁₂.

The root values for the TE modes are:

- $(ha)_{01} = 3.85$ for TE₀₁
- $(ha)_{02} = 7.02$ for TE₀₂
- $(ha)_{11} = 1.841$ for TE₁₁

- $(ha)_{12} = 5.53$ for TE₁₂

The dominant mode for TE waves in a circular waveguide is the TE₁₁.v. Because it has the lowest root value of 1.841.

Since the root value of TE₁₁ is lower than TM₀₁, TE₁₁ is the dominant or the lowest order mode for a circular waveguide.

RECTANGULAR AND CIRCULAR CAVITY RESONATORS:

Resonator is a tuned circuit which resonates at a particular frequency at which the energy stored in the electric field is equal to the energy stored in the magnetic field.

Resonant frequency of microwave resonator is the frequency at which the energy in the resonator attains maximum value. i.e., twice the electric energy or magnetic energy.

At low frequencies upto VHF (300 MHz), the resonator is made up of the reactive elements or the lumped elements like the capacitance and the inductance.

The inductance and the capacitance values are too small as the frequency is increased beyond the VHF range and hence difficult to realize.

Transmission line resonator can be built using distributed elements like sections of coaxial lines. The coaxial lines are either opened or shunted at the end sections thus confining the electromagnetic energy within the section and acts as the resonant circuit having a natural resonant frequency.

At very high frequencies transmission line resonator does not give very high quality factor Q due to skin effect and radiation loss. So, transmission line resonator is not used as microwave resonator.

The performance parameters of microwave resonator are:

- (i) Resonant frequency

(ii) Quality factor

(iii) Input impedance

Quality Factor of a Resonator.:

- The quality factor Q is a measure of frequency selectivity of the resonator.
- It is defined as $Q = 2 \times \text{Maximum energy stored} / \text{Energy dissipated per cycle} = W / P$

Where,

- W is the maximum stored energy
- P is the average power loss

The methods used for constructing a resonator:

The resonators are built by,

- Using lumped elements like L and C
- Using distributed elements like sections of coaxial lines
- Using rectangular or circular waveguide

There are two types of cavity resonators.

- Rectangular cavity resonator
- Circular cavity resonator

Rectangular or circular cavities can be used as microwave resonators because they have natural resonant frequency and behave like a LCR circuit.

Cavity resonator can be represented by a LCR circuit as:

- The electromagnetic energy is stored in the entire volume of the cavity in the form of electric and magnetic fields.
- The presence of electric field gives rise to a capacitance value and the presence of magnetic field gives rise to an inductance value and the finite

conductivity in the walls gives rise to loss along the walls giving rise to a resistance value.

- Thus the cavity resonator can be represented by a equivalent LCR circuit and have a natural resonant frequency.

- * Cavity resonators are formed by placing the perfectly conducting sheets on the rectangular or circular waveguide on the two end sections and hence all the sides are surrounded by the conducting walls thus forming a cavity.

- * The electromagnetic energy is confined within this metallic enclosure and they acts as resonant circuits.

