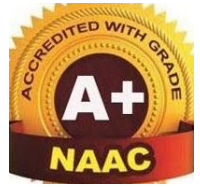




ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY



DEPARTMENT OF MECHANICAL ENGINEERING

UNIT I – FUNDAMENTAL CONCEPTS IN DESIGN

1.4 Impact Stress

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as *impact stress*. Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig.

Let A = Cross-sectional area of the bar,

E = Young's modulus of the material of the bar,

l = Length of the bar,

δl = Deformation of the bar,

P = Force at which the deflection δl is produced,

σ_i = Stress induced in the bar due to the application of impact load, and

h = Height through which the load falls.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

And potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\begin{aligned} \frac{1}{2} \times P \times \delta l &= W(h + \delta l) \\ \frac{1}{2} \sigma_i \times A \times \frac{\sigma_i \times l}{E} &= W \left(h + \frac{\sigma_i \times l}{E} \right) \quad \dots \left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E} \right] \\ \therefore \frac{A l}{2 E} (\sigma_i)^2 - \frac{W l}{E} (\sigma_i) - W h &= 0 \end{aligned}$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A} \left(1 + \sqrt{1 + \frac{2 h A E}{W l}} \right) \quad \dots \text{ [Taking +ve sign for maximum value]}$$

When $h = 0$, then $\sigma_i = 2W/A$. This means that the stress in the bar when the load is applied suddenly is double of the stress induced due to gradually applied load.

Problem:

An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm² in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take $E = 200 \text{ kN/mm}^2$.

Solution. Given : $h = 10 \text{ mm}$; $l = 3 \text{ m} = 3000 \text{ mm}$; $A = 600 \text{ mm}^2$; $\delta l = 2 \text{ mm}$;
 $E = 200 \text{ kN/mm}^2 = 200 \times 10^3 \text{ N/mm}^2$

Stress in the bar

Let $\sigma = \text{Stress in the bar.}$

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon} = \frac{\sigma \cdot l}{\delta l}$$

$$\therefore \sigma = \frac{E \cdot \delta l}{l} = \frac{200 \times 10^3 \times 2}{3000} = \frac{400}{3} = 133.3 \text{ N/mm}^2 \text{ Ans.}$$

Value of the unknown weight

Let $W = \text{Value of the unknown weight.}$

We know that
$$\sigma = \frac{W}{A} \left[1 + \sqrt{1 + \frac{2hAE}{Wl}} \right]$$

$$\frac{400}{3} = \frac{W}{600} \left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}} \right]$$

$$\frac{400 \times 600}{3W} = 1 + \sqrt{1 + \frac{800\,000}{W}}$$

$$\frac{80\,000}{W} - 1 = \sqrt{1 + \frac{800\,000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \text{ or } \frac{6400 \times 10^2}{W} = 96$$

$$\therefore W = 6400 \times 10^2 / 96 = 6666.7 \text{ N Ans.}$$

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as **strain energy**. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as **resilience** and the maximum energy which can be stored in a body up to the elastic limit is called **proof resilience**. The proof resilience per unit volume of a material is known as

modulus of resilience. It is an important property of a material and gives capacity of the material to bear impact or shocks. Mathematically, strain energy stored in a body due to tensile or compressive load or resilience,

$$U = \frac{\sigma^2 \times V}{2E}$$

And Modulus of resilience

$$= \frac{\sigma^2}{2E}$$

Where σ = Tensile or compressive stress,

V = Volume of the body, and

E = Young's modulus of the material of the body.

When a body is subjected to a shear load, then modulus of resilience (shear)

$$= \frac{\tau^2}{2C}$$

Where τ = Shear stress, and

C = Modulus of rigidity.

When the body is subjected to torsion, then modulus of resilience

$$= \frac{\tau^2}{4C}$$

Problem:

A wrought iron bar 50 mm in diameter and 2.5 m long transmits shock energy of 100 N-m.

Find the maximum instantaneous stress and the elongation. Take $E = 200 \text{ GN/m}^2$.

Solution. Given : $d = 50 \text{ mm}$; $l = 2.5 \text{ m} = 2500 \text{ mm}$; $U = 100 \text{ N-m} = 100 \times 10^3 \text{ N-mm}$;
 $E = 200 \text{ GN/m}^2 = 200 \times 10^3 \text{ N/mm}^2$

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4} (50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25 \sigma^2$$

$\therefore \sigma^2 = 100 \times 10^3 / 12.25 = 8163$ or $\sigma = 90.3 \text{ N/mm}^2$ Ans.

Elongation produced

Let $\delta l =$ Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm Ans.}$$

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to **torsion**. The stress set up by torsion is known as **torsional shear stress**. It is zero at the centroidal axis and maximum at the outer surface. Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. As a result of this torque, every cross-section of the shaft is subjected to torsional shear stress. We have discussed above that the torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \text{ ----- (i)}$$

Where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,

r = Radius of the shaft,

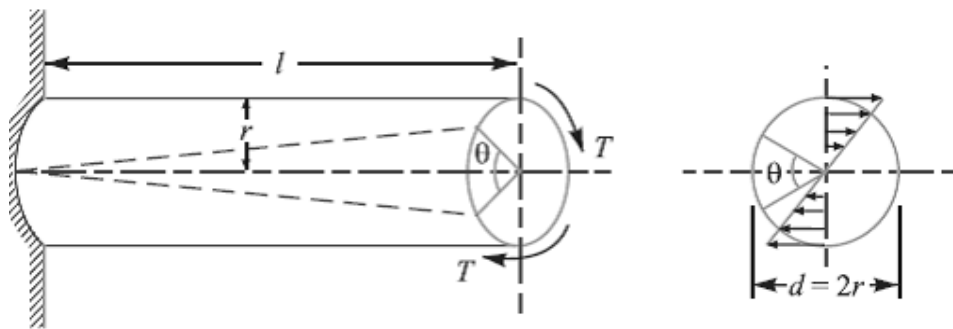
T = Torque or twisting moment,

J = Second moment of area of the section about its polar axis or polar moment of inertia,

C = Modulus of rigidity for the shaft material,

l = Length of the shaft, and

θ = Angle of twist in radians on a length l .



The above equation is known as **torsion equation**. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The twist along the length of the shaft is uniform.
3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Note: 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (i), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

Therefore,

$$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32} [(d_o)^4 - (d_i)^4] \quad \text{and} \quad r = \frac{d_o}{2}$$

$$\begin{aligned} T &= \tau \times \frac{\pi}{32} [(d_o)^4 - (d_i)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o} \right] \\ &= \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4) \quad \dots \left(\text{Substituting, } k = \frac{d_i}{d_o} \right) \end{aligned}$$

3. The expression ($C \times J$) is called **torsional rigidity** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2 \pi N \cdot T}{60} = T \cdot \omega \quad \dots \left(\because \omega = \frac{2 \pi N}{60} \right)$$

Where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Problem:

A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

Solution. Given : $P = 100 \text{ kW} = 100 \times 10^3 \text{ W}$; $N = 160 \text{ r.p.m}$; $T_{max} = 1.25 T_{mean}$; $\tau = 70 \text{ MPa} = 70 \text{ N/mm}^2$

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and
 d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2 \pi N \cdot T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76 T_{mean}$$

$$\therefore T_{mean} = 100 \times 10^3 / 16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75 d^3$$

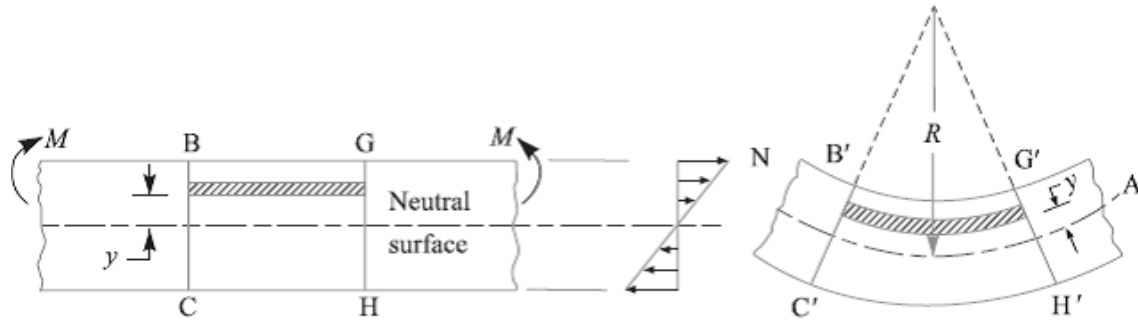
$$\therefore d^3 = 7458 \times 10^3 / 13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm Ans.}$$

Bending Stress

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment M as shown in Fig.

The following assumptions are usually made while deriving the bending formula.

1. The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).
2. The material of the beam obeys Hooke's law.
3. The transverse sections (*i.e.* BC or GH) which were plane before bending remain plane after bending also.
4. Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.
5. The Young's modulus (E) is the same in tension and compression.
6. The loads are applied in the plane of bending.



A little consideration will show that when a beam is subjected to the bending moment, the fibres on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated due to tension. It may be seen that somewhere between the top and bottom fibres there is a surface at which the fibres are neither shortened nor lengthened. Such a surface is called **neutral surface**. The intersection of the neutral surface with any normal cross-section of the beam is known as **neutral axis**. The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where M = Bending moment acting at the given section,

σ = Bending stress,

I = Moment of inertia of the cross-section about the neutral axis,

y = Distance from the neutral axis to the extreme fibre,

E = Young's modulus of the material of the beam, and

R = Radius of curvature of the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since E and R are constant, therefore within elastic limit, the stress at any point is directly proportional to y , *i.e.* the distance of the point from the neutral axis.

Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio I/y is known as **section modulus** and is denoted by Z .

Notes: 1. the neutral axis of a section always passes through its centroid.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fibre from the neutral axis

is $y = d / 2$, where d is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

Problem:

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

Solution. Given: $W = 400 \text{ N}$; $L = 300 \text{ mm}$;
 $\sigma_b = 40 \text{ MPa} = 40 \text{ N/mm}^2$; $h = 2b$

The beam is shown in Fig. 5.7.

Let $b =$ Width of the beam in mm, and
 $h =$ Depth of the beam in mm.

\therefore Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b (2b)^2}{6} = \frac{2 b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2 b^3} = \frac{180 \times 10^3}{b^3}$$

$$\therefore b^3 = 180 \times 10^3 / 40 = 4.5 \times 10^3 \text{ or } b = 16.5 \text{ mm Ans.}$$

and

$$h = 2b = 2 \times 16.5 = 33 \text{ mm Ans.}$$

Problem:

A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

Solution. Given : $P = 10 \text{ kW} = 10 \times 10^3 \text{ W}$; $N = 400 \text{ r.p.m}$; $D = 1.2 \text{ m} = 1200 \text{ mm}$ or $R = 600 \text{ mm}$; $\sigma_b = 15 \text{ MPa} = 15 \text{ N/mm}^2$

Let $T =$ Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2 \pi N \cdot T}{60} = \frac{2 \pi \times 400 \times T}{60} = 42 T$$

$$\therefore T = 10 \times 10^3 / 42 = 238 \text{ N-m} = 238 \times 10^3 \text{ N-mm}$$

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$$2a = \text{Major axis in mm} = 2 \times 2b = 4b$$

...(Given)

\therefore Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4} (2b)^2 \times b = \pi b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8 \text{ mm}$

\therefore Minor axis, $2b = 2 \times 10.8 = 21.6 \text{ mm Ans.}$

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2 \text{ mm Ans.}$

Principal Stresses and Principal Planes

In the previous chapter, we have discussed about the direct tensile and compressive stress as well as simple shear. Also we have always referred the stress in a plane which is at right angles to the line of action of the force. But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as *principal planes* and the direct stresses along these planes are known as *principal stresses*. The planes on which the maximum shear stress act are known as planes of maximum shear.

Determination of Principal Stresses for a Member Subjected to Bi-axial Stress

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

Consider a rectangular body $ABCD$ of uniform cross-sectional area and unit thickness subjected to normal stresses σ_1 and σ_2 as shown in Fig. (a). In addition to these normal stresses, a shear stress τ also acts. It has been shown in books on '*Strength of Materials*' that the normal stress across any oblique section such as EF inclined at an angle θ with the direction of σ_2 , as shown in Fig. (a), is given by

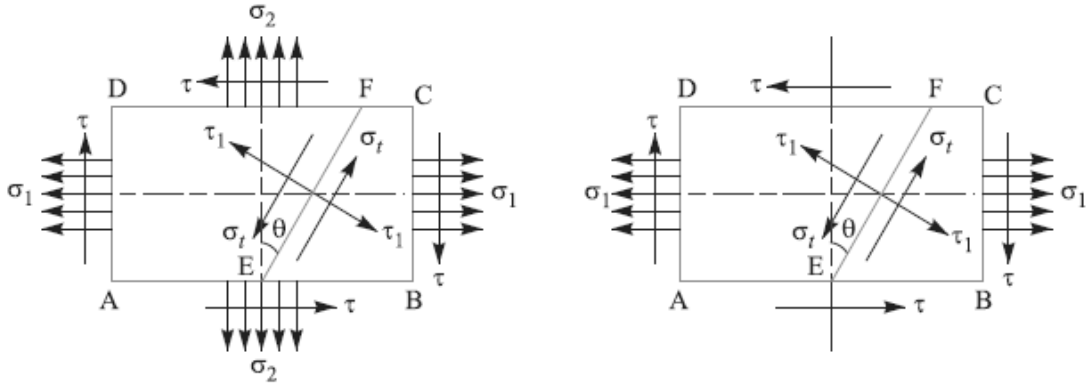
$$\sigma_f = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \sin 2\theta \quad \dots(i)$$

And tangential stress (*i.e.* shear stress) across the section EF ,

$$\tau_1 = \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta \quad \dots(ii)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating $\tau_1 = 0$ in the above equation (ii), *i.e.*

$$\begin{aligned} \frac{1}{2} (\sigma_1 - \sigma_2) \sin 2\theta - \tau \cos 2\theta &= 0 \\ \tan 2\theta &= \frac{2\tau}{\sigma_1 - \sigma_2} \quad \dots(iii) \end{aligned}$$



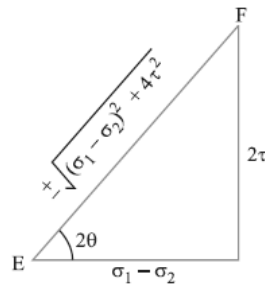
(a) Direct stress in two mutually perpendicular planes accompanied by a simple shear stress.

(b) Direct stress in one plane accompanied by a simple shear stress.

Fig. Principal stresses for a member subjected to bi-axial stress

We know that there are two principal planes at right angles to each other. Let θ_1 and θ_2 be the inclinations of these planes with the normal cross-section. From the following Fig., we find that

$$\sin 2\theta = \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$



$$\therefore \sin 2\theta_1 = + \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

and
$$\sin 2\theta_2 = - \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

Also
$$\cos 2\theta = \pm \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

$$\therefore \cos 2\theta_1 = + \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

and
$$\cos 2\theta_2 = - \frac{\sigma_1 - \sigma_2}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of $\sin 2\theta$ and $\cos 2\theta$ in equation (i).

So, Maximum principal (or normal) stress,

$$\sigma_{r1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \quad \dots(iv)$$

And minimum principal (or normal) stress,

$$\sigma_{r2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \quad \dots(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by **one-half the algebraic difference between the principal stresses**, i.e.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4 \tau^2} \quad \dots(vi)$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting $\sigma_2 = 0$ in equation (iv), (v) and (vi).

$$\begin{aligned} \sigma_{r1} &= \frac{\sigma_1}{2} + \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \\ \sigma_{r2} &= \frac{\sigma_1}{2} - \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \\ \tau_{max} &= \frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right] \end{aligned}$$

2. In the above expression of σ_{r2} , the value of $\frac{1}{2} \left[\sqrt{(\sigma_1)^2 + 4 \tau^2} \right]$ is more than $\sigma_1/2$. Therefore the nature of σ_{r2} will be opposite to that of σ_{r1} , i.e. if σ_{r1} is tensile then σ_{r2} will be compressive and *vice-versa*.

Application of Principal Stresses in Designing Machine Members

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts

normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

1. Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2} \left[\sqrt{(\sigma_t)^2 + 4 \tau^2} \right]$$

2. Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2} \left[\sqrt{(\sigma_c)^2 + 4 \tau^2} \right]$$

3. Maximum shear stress,

$$\tau_{max} = \frac{1}{2} \left[\sqrt{(\sigma_t)^2 + 4 \tau^2} \right]$$

Where σ_t = Tensile stress due to direct load and bending,

σ_c = Compressive stress, and

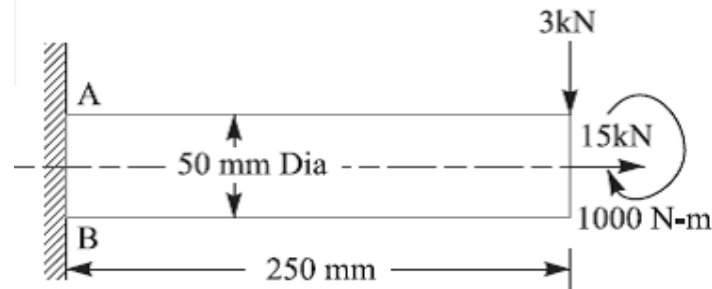
τ = Shear stress due to torsion.

Notes: 1. When $\tau = 0$ as in the case of thin cylindrical shell subjected in internal fluid pressure, then $\sigma_{tmax} = \sigma_t$

2. When the shaft is subjected to an axial load (P) in addition to bending and twisting moments as in the propeller shafts of ship and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). This will give the resultant tensile stress or compressive stress (σ_t or σ_c) depending upon the type of axial load (*i.e.* pull or push).

Problem:

A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN. Calculate the stresses at A and B.



Solution. Given : $W = 3 \text{ kN} = 3000 \text{ N}$;
 $T = 1000 \text{ N-m} = 1 \times 10^6 \text{ N-mm}$; $P = 15 \text{ kN}$
 $= 15 \times 10^3 \text{ N}$; $d = 50 \text{ mm}$; $x = 250 \text{ mm}$

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2$$

$$= \frac{\pi}{4} (50)^2 = 1964 \text{ mm}^2$$

\therefore Tensile stress due to axial pulling at points A and B ,

$$\sigma_o = \frac{P}{A} = \frac{15 \times 10^3}{1964} = 7.64 \text{ N/mm}^2 = 7.64 \text{ MPa}$$

Bending moment at points A and B ,

$$M = W \cdot x = 3000 \times 250 = 750 \times 10^3 \text{ N-mm}$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32} (50)^3$$

$$= 12.27 \times 10^3 \text{ mm}^3$$

\therefore Bending stress at points A and B ,

$$\sigma_b = \frac{M}{Z} = \frac{750 \times 10^3}{12.27 \times 10^3}$$

$$= 61.1 \text{ N/mm}^2 = 61.1 \text{ MPa}$$

This bending stress is tensile at point A and compressive at point B .

\therefore Resultant tensile stress at point A ,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$

$$= 68.74 \text{ MPa}$$

and resultant compressive stress at point B ,

$$\sigma_B = \sigma_b - \sigma_o = 61.1 - 7.64 = 53.46 \text{ MPa}$$

We know that the shear stress at points A and B due to the torque transmitted,

$$\tau = \frac{16 T}{\pi d^3} = \frac{16 \times 1 \times 10^6}{\pi (50)^3} = 40.74 \text{ N/mm}^2 = 40.74 \text{ MPa} \quad \dots \left(\because T = \frac{\pi}{16} \times \tau \times d^3 \right)$$

Stresses at point A

We know that maximum principal (or normal) stress at point *A*,

$$\begin{aligned}\sigma_{A(max)} &= \frac{\sigma_A}{2} + \frac{1}{2} \left[\sqrt{(\sigma_A)^2 + 4 \tau^2} \right] \\ &= \frac{68.74}{2} + \frac{1}{2} \left[\sqrt{(68.74)^2 + 4 (40.74)^2} \right] \\ &= 34.37 + 53.3 = 87.67 \text{ MPa (tensile) Ans.}\end{aligned}$$

Minimum principal (or normal) stress at point *A*,

$$\begin{aligned}\sigma_{A(min)} &= \frac{\sigma_A}{2} - \frac{1}{2} \left[\sqrt{(\sigma_A)^2 + 4 \tau^2} \right] = 34.37 - 53.3 = -18.93 \text{ MPa} \\ &= 18.93 \text{ MPa (compressive) Ans.}\end{aligned}$$

and maximum shear stress at point *A*,

$$\begin{aligned}\tau_{A(max)} &= \frac{1}{2} \left[\sqrt{(\sigma_A)^2 + 4 \tau^2} \right] = \frac{1}{2} \left[\sqrt{(68.74)^2 + 4 (40.74)^2} \right] \\ &= 53.3 \text{ MPa Ans.}\end{aligned}$$

Stresses at point B

We know that maximum principal (or normal) stress at point *B*,

$$\begin{aligned}\sigma_{B(max)} &= \frac{\sigma_B}{2} + \frac{1}{2} \left[\sqrt{(\sigma_B)^2 + 4 \tau^2} \right] \\ &= \frac{53.46}{2} + \frac{1}{2} \left[\sqrt{(53.46)^2 + 4 (40.74)^2} \right] \\ &= 26.73 + 48.73 = 75.46 \text{ MPa (compressive) Ans.} \\ &= 48.73 \text{ MPa Ans.}\end{aligned}$$