4.1 GENERAL WAVE BEHAVIOUR ALONG UNIFORM PARALLEL PLANES (or) APPLICATION OF RESTICTIONS TO MAXWELL'S EQUATION (or) WAVES BETWEEN PARALLEL PLANES OF PERFECT CONDUCTORS:

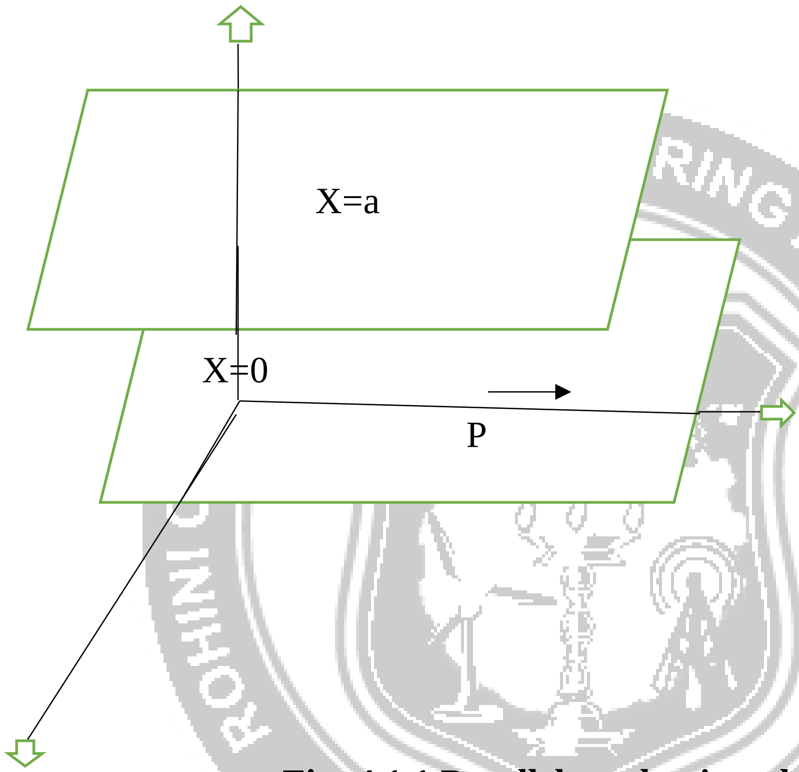

Fig: 4.1.1 Parallel conducting planes

In Fig 4.1.1 consider an electromagnetic wave propagate between a pair of parallel perfectly conducting planes of infinite incident in the plane of Y and Z direction the Maxwell equation for long conducting rectangular region is given by,

$\nabla \times H = j\omega\, \varepsilon E$ …….(1)

$\nabla \times E = -j\omega\, \mu H$ …….(2)

$\nabla^2 E = \gamma^2 E$ …….(3)

$\nabla^2 H = \gamma^2 H$ ……..(4)

Where,

$\gamma^2 = -\omega^2 \mu\, \varepsilon$

For non conducting in medium

∇^2 E = $-\omega^2\mu\,\varepsilon$ E …….(5)

∇^2 H = $-\omega^2\mu\,\varepsilon$ H ……..(6)

It can be written as,

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2\mu\,\varepsilon\, E \qquad ……(7)$$

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2\mu\,\varepsilon\, H \qquad ……(8)$$

From the properties of vector algebra,

$$\nabla \times H = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \vec{a_x}\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - \vec{a_y}\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] + \vec{a_z}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] \qquad …..(9)$$

Equ (1) can be written as,

$$\nabla \times H = j\omega\,\varepsilon\left[E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z}\right]$$

$$\nabla \times H = j\omega\,\varepsilon\, E_x \vec{a_x} + j\omega\,\varepsilon\, E_y \vec{a_y} + j\omega\,\varepsilon\, E_z \vec{a_z} \qquad …….(10)$$

Equate equ (9) and (10),

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\,\varepsilon\, E_x \qquad ……(11)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\,\varepsilon\, E_y \qquad ………(12)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\,\varepsilon\, E_z \qquad ……….(13)$$

$$\nabla \times E = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= \underset{a_x}{\rightarrow} \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] - \underset{a_y}{\rightarrow} \left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + \underset{a_z}{\rightarrow} \left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right] \quad .....(14)$$

Equ (2) can be written as,

$$\nabla \text{ x E} = \text{-j}\omega\ \mu \left[H_x \underset{a_x}{\rightarrow} + H_y \underset{a_y}{\rightarrow} + H_z \underset{a_z}{\rightarrow}\right]$$

$$\nabla \text{ x E} = \text{-j}\omega\ \mu\ H_x \underset{a_x}{\rightarrow} + \text{j}\omega\ \varepsilon\ H_y \underset{a_y}{\rightarrow} + \text{j}\omega\ \varepsilon\ H_z \underset{a_z}{\rightarrow} \quad ....(15)$$

Equate equ (14) & (15)

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = \text{-j}\omega\ \mu\ E_x \quad ......(16)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = \text{-j}\omega\mu\ E_y \quad ....(17)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\text{j}\omega\ \mu\ E_z \quad ....(18)$$

It is assumed that the propagation is in z direction.

The radiation of component in this z-direction may be expressed interms of $e^{-\gamma z}$

where γ is propagation constant ,

$\gamma = \alpha + j\beta$

If α = 0 waves propagate without attenuation.

If γ = real then β = 0, there is no wave propagation

Let, $H_y = H_y^o\ e^{-\gamma z}$ ....(19)

Diff w.r.to 'z'

$$\frac{\partial H_y}{\partial z} = H_y^o\ e^{-\gamma z}\ (-\gamma)$$

$$\frac{\partial H_y}{\partial z} = -\gamma\ H_y^o\ e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma\ H_y \quad ....(20)$$

$$\frac{\partial H_x}{\partial z} = -\gamma\ H_x \quad ....(21)$$

And also let,

$E_y = E_y^o\ e^{-\gamma z}$ ....(22)

Diff w.r.to 'z'

$\frac{\partial E_y}{\partial z} = E_y^o \, e^{-\gamma z} \, (-\gamma)$

$\frac{\partial E_y}{\partial z} = -\gamma \, E_y^o \, e^{-\gamma z}$

$\frac{\partial E_y}{\partial z} = -\gamma \, E_y$ ……..(23)

$\frac{\partial E_x}{\partial z} = -\gamma \, E_x$ ……….(24)

There is no attenuation in y direction. Hence the derivative of y is zero.

Let E = $E_o \, e^{-\gamma z}$

Diff w. r. to 'z'

$\frac{\partial E}{\partial z} = E_o \, e^{-\gamma z} \, (-\gamma)$

Again diff w. r. to 'z'

$\frac{\partial^2 E}{\partial z^2} = E_o \, e^{-\gamma z} \, (-\gamma) \, (-\gamma)$

$\frac{\partial^2 E}{\partial z^2} = E_o \, e^{-\gamma z} \, \gamma^2$

$\frac{\partial^2 E}{\partial z^2} = \gamma^2 \mathrm{E}$

From equ (7),

$\frac{\partial^2 E}{\partial x^2} + 0 + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu \, \varepsilon \, \mathrm{E}$

$\frac{\partial^2 E}{\partial x^2} + \gamma^2 \mathrm{E} = -\omega^2 \mu \, \varepsilon \, \mathrm{E}$ ……(25)

From equ (8),

$\frac{\partial^2 H}{\partial x^2} + 0 + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu \, \varepsilon \, \mathrm{H}$

$\frac{\partial^2 H}{\partial x^2} + \gamma^2 \mathrm{H} = -\omega^2 \mu \, \varepsilon \, \mathrm{H}$ ……(26)

Sub equ (20) & (21) in (11), (12) & (13)

From equ (11),

$-(-\gamma \, H_y) = \mathrm{j}\omega \, \varepsilon \, E_x$

$\gamma \, H_y = \mathrm{j}\omega \, \varepsilon \, E_x$ ………(27)

From equ (12),

$-\gamma\, H_x - \frac{\partial H_z}{\partial x} = \mathrm{j}\omega\, \varepsilon\, E_y$ ……..(28)

From equ (13),

$\frac{\partial H_y}{\partial x} = \mathrm{j}\omega\, \varepsilon\, E_z$ ……..(29)

Sub equ (23) & (24) in (16), (17) & (18)

From equ (16),

$-(-\gamma\, E_y) = -\, \mathrm{j}\omega\, \mu\, H_x$

$\gamma\, E_y = -\, \mathrm{j}\omega\, \mu\, H_x$ …….(30)

From equ (17),

$(-\gamma\, E_x) - \frac{\partial E_z}{\partial x} = -\mathrm{j}\omega\mu\, H_y$

$\gamma\, E_x + \frac{\partial E_z}{\partial x} = \mathrm{j}\omega\mu\, H_y$ ………(31)

From equ (18),

$\frac{\partial E_y}{\partial x} = -\mathrm{j}\omega\, \mu\, H_z$ ……….(32)

From equ (30),

$H_x = \frac{-\gamma\, E_y}{\mathrm{j}\omega\, \mu}$ ……..(33)

From equ (28),

$E_y = \frac{-1}{\mathrm{j}\omega\, \varepsilon}\left(\gamma\, H_x + \frac{\partial H_z}{\partial x}\right)$ ……(34)

Sub equ (34) in equ (33)

$H_x = \frac{-\gamma}{\mathrm{j}\omega\, \mu}\left(\frac{-1}{\mathrm{j}\omega\, \varepsilon}\left(\gamma\, H_x + \frac{\partial H_z}{\partial x}\right)\right)$

$H_x = \frac{\gamma}{j^2\omega^2 \mu\, \varepsilon}\left(\gamma\, H_x + \frac{\partial H_z}{\partial x}\right)$

[j^2 = -1]

$H_x = \frac{-\gamma}{\omega^2 \mu\, \varepsilon}\left(\gamma\, H_x + \frac{\partial H_z}{\partial x}\right)$

$H_x = \frac{-\gamma^2}{\omega^2 \mu\, \varepsilon} H_x - \frac{\gamma}{\omega^2 \mu\, \varepsilon}\frac{\partial H_z}{\partial x}$

$H_x + \frac{\gamma^2}{\omega^2 \mu\, \varepsilon} H_x = \frac{-\gamma}{\omega^2 \mu\, \varepsilon}\frac{\partial H_z}{\partial x}$

$$H_x\left(1 + \frac{\gamma^2}{\omega^2\mu\,\varepsilon}\right) = \frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{\frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}}{\left(1 + \frac{\gamma^2}{\omega^2\mu\,\varepsilon}\right)}$$

$$H_x = \frac{\frac{-\gamma}{\omega^2\mu\,\varepsilon}\frac{\partial H_z}{\partial x}}{\left(\frac{\omega^2\mu\,\varepsilon + \gamma^2}{\omega^2\mu\,\varepsilon}\right)}$$

$$H_x = \left(\frac{-\gamma}{\omega^2\mu\,\varepsilon + \gamma^2}\right)\frac{\partial H_z}{\partial x}$$

It is given that,

$$\omega^2\mu\,\varepsilon + \gamma^2 = h^2$$

$$H_x = \left(\frac{-\gamma}{h^2}\right)\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x} \qquad \text{……..(35)}$$

To find H_y, we need to solve equ (27) & (31)

From equ (27),

$$\gamma\, H_y = \mathrm{j}\omega\,\varepsilon\; E_x$$

$$H_y = \frac{\mathrm{j}\omega\,\varepsilon\; E_x}{\gamma} \qquad \text{…….(36)}$$

From equ (31),

$$\gamma\, E_x + \frac{\partial E_z}{\partial x} = \mathrm{j}\omega\mu\; H_y$$

$$\gamma\, E_x = \mathrm{j}\omega\mu\; H_y - \frac{\partial E_z}{\partial x}$$

$$E_x = \frac{1}{\gamma}\left(\mathrm{j}\omega\mu\; H_y - \frac{\partial E_z}{\partial x}\right) \qquad \text{……(37)}$$

Sub equ (37) in equ (36),

$$H_y = \frac{\mathrm{j}\omega\,\varepsilon}{\gamma}\frac{1}{\gamma}\left(\mathrm{j}\omega\mu\; H_y - \frac{\partial E_z}{\partial x}\right)$$

$$H_y = \frac{\mathrm{j}\omega\,\varepsilon}{\gamma^2}\left(\mathrm{j}\omega\mu\; H_y\right) - \frac{\mathrm{j}\omega\,\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-\omega^2\mu\,\varepsilon H_y}{\gamma^2} - \frac{\mathrm{j}\omega\,\varepsilon}{\gamma^2}\frac{\partial E_z}{\partial x}$$

$$H_y + \frac{\omega^2 \mu\, \varepsilon H_y}{\gamma^2} = \frac{-j\omega\, \varepsilon}{\gamma^2} \frac{\partial E_z}{\partial x}$$

$$H_y \left(1 + \frac{\omega^2 \mu\, \varepsilon}{\gamma^2}\right) = \frac{-j\omega\, \varepsilon}{\gamma^2} \frac{\partial E_z}{\partial x}$$

$$H_y = \frac{\frac{-j\omega\, \varepsilon}{\gamma^2} \frac{\partial E_z}{\partial x}}{\frac{\gamma^2 + \omega^2 \mu\, \varepsilon}{\gamma^2}}$$

$$H_y = \frac{-j\omega\, \varepsilon}{\gamma^2 + \omega^2 \mu\, \varepsilon} \frac{\partial E_z}{\partial x}$$

$$H_y = \frac{-j\omega\, \varepsilon}{h^2} \frac{\partial E_z}{\partial x} \qquad \text{……(38)}$$

To find E_x,

Solve equ (27) & (31),

From equ (27),

$\gamma\, H_y = j\omega\, \varepsilon\, E_x$

$$H_y = \frac{j\omega\, \varepsilon\, E_x}{\gamma} \qquad \text{……(39)}$$

From equ (31),

$$\gamma\, E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\, H_y$$

Sub equ (39) in equ (31)

$$\gamma\, E_x + \frac{\partial E_z}{\partial x} = j\omega\mu \left(\frac{j\omega\, \varepsilon\, E_x}{\gamma}\right)$$

$$\gamma\, E_x + \frac{\partial E_z}{\partial x} = \frac{-\omega^2 \mu\, \varepsilon E_x}{\gamma}$$

$$\gamma\, E_x + \frac{\omega^2 \mu\, \varepsilon E_x}{\gamma} = -\frac{\partial E_z}{\partial x}$$

$$E_x \left(\gamma + \frac{\omega^2 \mu\, \varepsilon}{\gamma}\right) = -\frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-\frac{\partial E_z}{\partial x}}{\gamma + \frac{\omega^2 \mu\, \varepsilon}{\gamma}}$$

$$E_x = \frac{-\frac{\partial E_z}{\partial x}}{\frac{\gamma^2 + \omega^2 \mu\, \varepsilon}{\gamma}}$$

$$E_x = \frac{-\frac{\partial E_z}{\partial x}}{\frac{h^2}{\gamma}}$$

$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) \qquad \text{........(40)}$$

To find E_y:

Solve equ (28) & (30),

From equ (30),

$\gamma\, E_y$ = - jω µ H_x

$$H_x = \frac{-\gamma\, E_y}{\text{j}\omega\ \mu} \qquad \text{.......(41)}$$

Sub equ (41) in equ (28),

$$-\gamma\, H_x - \frac{\partial H_z}{\partial x} = \text{j}\omega\ \varepsilon\ E_y$$

$$-\gamma\left(\frac{-\gamma\, E_y}{\text{j}\omega\ \mu}\right) - \frac{\partial H_z}{\partial x} = \text{j}\omega\ \varepsilon\ E_y$$

$$\frac{\gamma^2\, E_y}{\text{j}\omega\ \mu} - \frac{\partial H_z}{\partial x} = \text{j}\omega\ \varepsilon\ E_y$$

$$\frac{\gamma^2\, E_y}{\text{j}\omega\ \mu} - \text{j}\omega\ \varepsilon\ E_y = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\gamma^2}{\text{j}\omega\ \mu} - \text{j}\omega\ \varepsilon\right] = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{\gamma^2 + \omega^2 \mu\, \varepsilon}{\text{j}\omega\ \mu}\right] = \frac{\partial H_z}{\partial x}$$

$$E_y\left[\frac{h^2}{\text{j}\omega\ \mu}\right] = \frac{\partial H_z}{\partial x}$$

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{\text{j}\omega\ \mu}{h^2}\right] \qquad \text{........(42)}$$

The various components of electric and magnetic field strength in equ (35), (38), (40), (42) is expressed interms of E_z & H_z .

There will be z component either in E or H otherwise all the components should be zero.

In general both the E_z & H_z may nor present at the same time the solutions are divided into two cases.

Case (i):

If E_z is present and H_z= 0 , then the wave is called **transverse magnetic wave or TM wave or E wave** because the magnetic field strength is completely transverse to the direction of propagation z.

Case (ii):

If H_z is present and E_z= 0 , then the wave is called **transverse electric wave or TE wave or H wave,** because the electric field strength is completely transverse to the direction of propagation .

Case (iii):

Transverse Magnetic Waves or TEM waves are waves that contain neither E_z or H_z. Both the electric field and magnetic field components are transverse to the direction of propagation, z-direction.

TRANSMISSION OF TRANSVERSE ELECTRIC WAVES BETWEEN PARALLEL PLANES [E_z= 0]

The general field equations of equation(35), (38), (40), (42) for E_z= 0 is given by,

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\omega\,\varepsilon}{h^2}\frac{\partial E_z}{\partial x} = 0$$

$$E_x = \frac{-\gamma}{h^2}\left(\frac{\partial E_z}{\partial x}\right) = 0$$

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{j\omega\,\mu}{h^2}\right]$$

The field components E_x and H_y are zero.

The field components H_x , E_y and H_z are to determined.

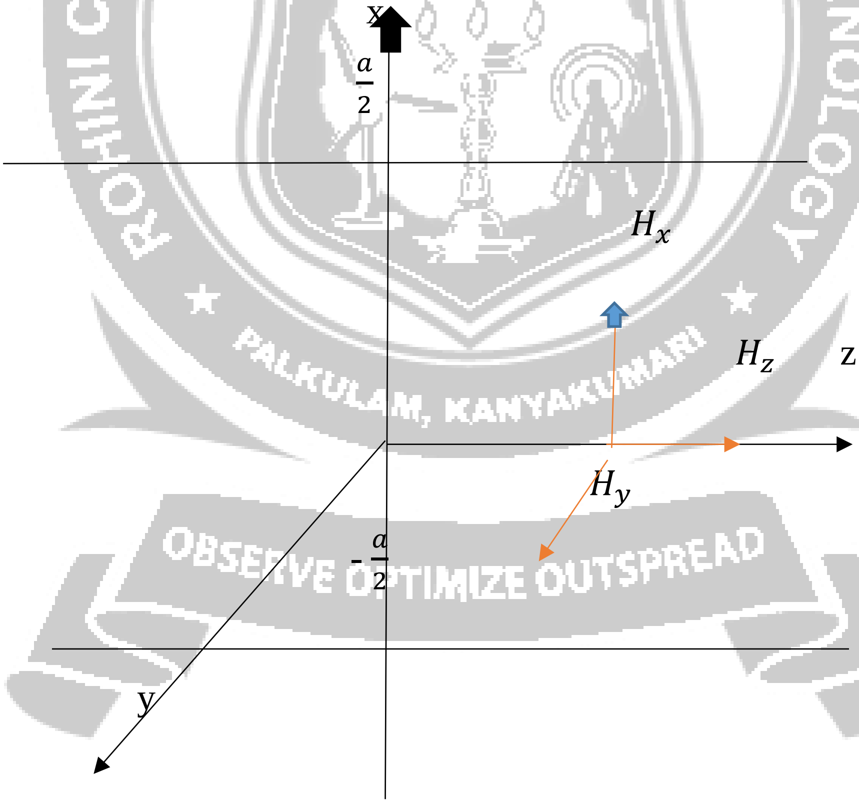

Fig: 4.1.2 Fields in TE waves (H-waves)

In the above Fig 4.1.2, E_x = E_z = 0 and the electric field E_y is made wholly transverse to the direction of propagation z.

The magnetic field components H_x and H_z, but H_y =0. The wave is called as **transverse electric wave or H-wave.**

The wave equation for the field component E_y can be written as,

From equ (25) ,

$\frac{\partial^2 E}{\partial x^2} + \gamma^2 \text{E} = -\omega^2 \mu\, \varepsilon\, \text{E}$

$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2\, E_y = -\omega^2 \mu\, \varepsilon\, E_y$

$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2\, E_y + \omega^2 \mu\, \varepsilon\, E_y = 0$

$\frac{\partial^2 E_y}{\partial x^2} + (\gamma^2 + \omega^2 \mu\, \varepsilon) E_y = 0$

$\omega^2 \mu\, \varepsilon + \gamma^2 = h^2$

$\frac{\partial^2 E_y}{\partial x^2} + h^2 E_y = 0$ …….(1)

Let $E_y = E_{yo}\, e^{-\gamma z}$

Equ (1) is a second order differential equation and the solution of this equation is given by,

$E_{yo} = C_1$ sin hx + C_2 coshx …….(2)

Where C_1 and C_2 are arbitrary constants.

If E_y is expressed in time and direction $E_y = E_{yo}\, e^{-\gamma z}$, then solution becomes

$E_y = [C_1 \sin \text{hx} + C_2 \cosh\text{x}]\, e^{-\gamma z}$ ……(3)

The tangential component of E is zero at the surface of the conductors for all values of Z.

i. E_y = 0 at x = 0
ii. E_y = 0 at x = a

These are the boundary conditions to be applied.

Applying the boundary conditions E_y = 0 at x = 0 in equ (3)

$0 = [C_1 \sin \text{h}(0) + C_2 \cosh(0)]\, e^{-\gamma z}$

$C_2 = 0$ ……(4)

Sub equ (4) in equ (3),

E_y = C_1 sin hx $e^{-\gamma z}$ ……(5)

Applying the boundary conditions E_y = 0 at x = a in equ (5)

0 = C_1 sin ha $e^{-\gamma z}$

sin ha = 0

ha = $\sin^{-1}$ 0

ha = mπ

h = $\frac{m\pi}{a}$ where m = 1, 2 , 3 …….

Sub 'h' value in equ (5),

$E_y = C_1 \sin\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z}$ ……(7)

Sub E_y in equ (42),

$$E_y = \frac{\partial H_z}{\partial x}\left[\frac{j\omega\mu}{h^2}\right]$$

$$\frac{\partial H_z}{\partial x} = E_y \cdot \frac{h^2}{j\omega\mu}$$

$$H_z = \int E_y \cdot \frac{h^2}{j\omega\mu}\, .dx$$

$$H_z = \int E_y \cdot \frac{\left(\frac{m\pi}{a}\right)^2}{j\omega\mu}\, .dx$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{1}{j\omega\mu} \int E_y\, .dx$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{1}{j\omega\mu} \int C_1 \sin\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z}\, .dx$$

$$\int \sin ax = \frac{-\cos ax}{a}$$

$$H_z = \left(\frac{m\pi}{a}\right)^2 \cdot \frac{-1}{j\omega\mu} \cdot C_1 \frac{\cos\left(\frac{m\pi}{a}\right)x}{\left(\frac{m\pi}{a}\right)} \cdot e^{-\gamma z}$$

$H_z = \frac{-1}{j\omega\mu}\left(\frac{m\pi}{a}\right) C_1 \cos\left(\frac{m\pi}{a}\right) x\, e^{-\gamma z}$ ……(8)

Sub equ (8) in equ (35),

$$H_x = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$

$$H_x = \frac{-\gamma}{h^2}\frac{\partial}{\partial x}\left(\frac{-1}{\mathrm{j}\omega\,\mu}\left(\frac{\mathrm{m\pi}}{a}\right) C_1 cos\left(\frac{\mathrm{m\pi}}{a}\right) x\; e^{-\gamma z}\right)$$

cos ax= (-sinax) a

$$H_x = \frac{-\gamma}{\left(\frac{\mathrm{m\pi}}{a}\right)^2}\frac{-1}{\mathrm{j}\omega\,\mu}\left(\frac{\mathrm{m\pi}}{a}\right) C_1\left(-sin\left(\frac{\mathrm{m\pi}}{a}\right) x\right).\frac{\mathrm{m\pi}}{a}\, e^{-\gamma z}$$

$$H_x = \frac{-\gamma}{\mathrm{j}\omega\,\mu} C_1\; sin\left(\frac{\mathrm{m\pi}}{a}\right) x e^{-\gamma z} \qquad \text{…….(9)}$$

Each value of m specifies a particular field of configuration or mode and is designated as TE_{mo} mode.

The second subscript refers to another factor which varies with y, which is found in rectangular waveguides.

The smallest value of m=1, because m=0 makes all fields identically zero.

Therefore lowest order mode is TE_{10}. This is also called as the dominant mode in TE waves.

The propagation constant γ = α +jβ. If the wave propagates without attenuation ,α = 0 then γ = jβ.

sub γ = jβ in equation (7), (8), (9),

$$E_y = C_1 \sin\left(\frac{\mathrm{m\pi}}{a}\right) \mathrm{x}\; e^{-\mathrm{j}\beta z}$$

$$H_z = \frac{-1}{\mathrm{j}\omega\,\mu}\left(\frac{\mathrm{m\pi}}{a}\right) C_1 cos\left(\frac{\mathrm{m\pi}}{a}\right) x\; e^{-\mathrm{j}\beta z}$$

$$H_x = \frac{-\mathrm{j}\beta}{\mathrm{j}\omega\,\mu} C_1\, sin\left(\frac{\mathrm{m\pi}}{a}\right) x e^{-\mathrm{j}\beta z}$$

$$H_x = \frac{-\beta}{\omega\,\mu} C_1\, sin\left(\frac{\mathrm{m\pi}}{a}\right) x e^{-\mathrm{j}\beta z}$$

The above equations represent the field strength of TE waves between parallel conducting planes.

TRANSMISSION OF TRANSVERSE ELECTROMAGNETIC WAVE BETWEEN PARALLEL PLANES (TEM WAVES)

Consider the electric field is totally along the x-axis (i.e., $E_x = E_y = 0$) and the magnetic field along the y-axis. (i.e., $H_x = H_y = 0$) shown in Fig 4.1.3.

Both the electric and magnetic field components are transverse to the direction of propagation on z, and the wave is said **transverse electromagnetic wave or principal wave.**

TEM wave is a **special case of transverse magnetic wave** in which the electric field E_z along the direction of propagation is zero.

The condition on E_z is obtained if m is made zero in TE waves.

TEM is also called as **Principal wave.**

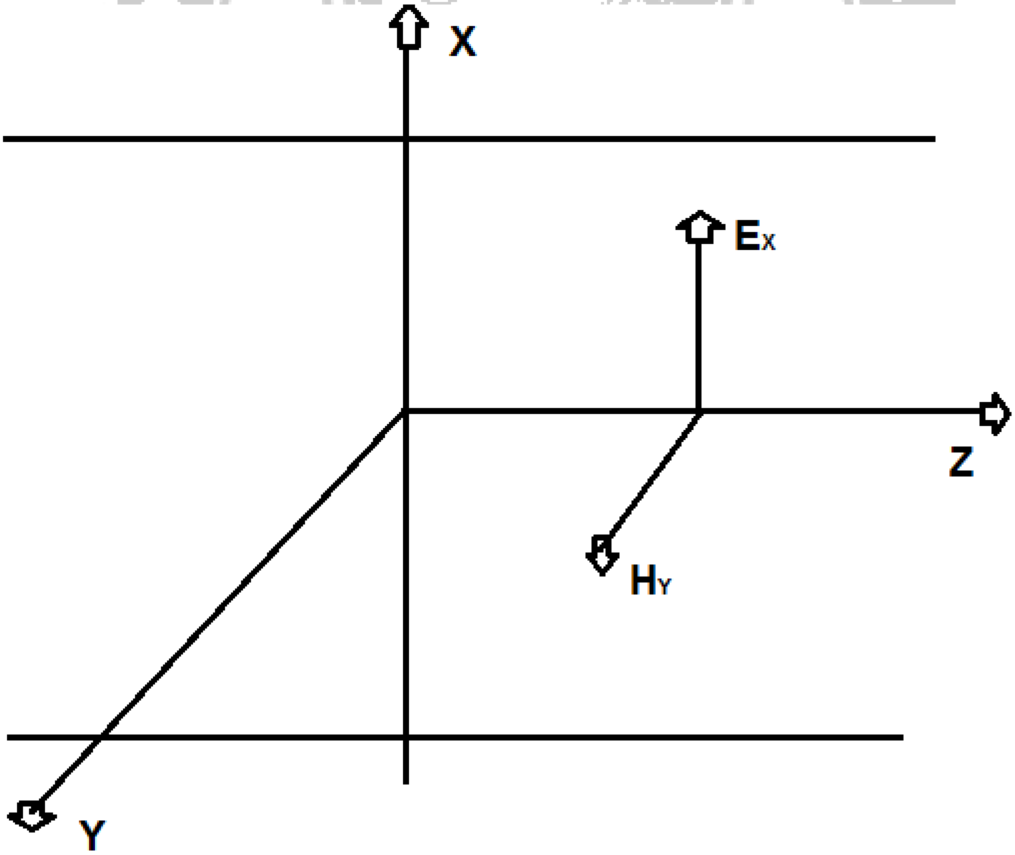

Fig: 4.1.3 Transverse Electromagnetic field vectors

Accordingly the TEM wave becomes a TM waves with m=0, the field equations of TM waves from equation are:

$H_y = C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$

$E_x = \frac{\beta}{\omega\varepsilon} C_4 \cos\left(\frac{m\pi}{a}\right)x\, e^{-j\beta z}$

$E_y = \frac{jm\pi}{\omega\varepsilon a} C_4 \cos\left(\frac{m\pi}{a}\right)xe^{-j\beta z}$

Putting m=0 in the above equations of TM waves, **the field equations of TEM waves are obtained**

$H_y = C_4 x e^{-j\beta z}$ ……(1)

$E_x = \frac{\beta}{\omega\varepsilon} C_4\, e^{-j\beta z}$ ……(3)

$E_y = 0$ ……(4)

These fields are not only transverse, but they are constant in amplitude across a cross section normal to the direction of propagation.

Characteristics of TEM waves:

For m = 0 and dielectric is air.

i. **Propagation Constant**

$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu_o\, \varepsilon_o}$

$\gamma = \sqrt{-\omega^2 \mu_o\, \varepsilon_o}$

$\gamma = j\,\omega\sqrt{\mu_o\, \varepsilon_o}$

$\gamma = \alpha + j\beta$

$\gamma = j\,\omega\sqrt{\mu_o\, \varepsilon_o}$ ……(4)

Equating real and imaginary parts,

$\alpha = 0$

$\beta = \omega\sqrt{\mu_o\, \varepsilon_o}$ …..(5)

ii. **Guided Wavelength**

$\lambda_g = \frac{2\pi}{\beta}$

$\lambda_g = \frac{2\pi}{\omega\sqrt{\mu_o\,\varepsilon_o}}$

$$\omega = 2\pi f$$

$$v_o = \frac{1}{\sqrt{\mu_o\,\varepsilon_o}}$$

$\lambda_g = \frac{2\pi\, v_o}{2\pi f} = \lambda$ = Wavelength of free space …..(6)

iii. Velocity of Propagation

$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu_o\,\varepsilon_o}} = \frac{1}{\sqrt{\mu_o\,\varepsilon_o}} = C$ (7)

Velocity of TEM is independent of frequency and has a familiar free space value, C = 3x10^8 m/s.

iv. From equ (7), cut off frequency is given by,

$f_c = \frac{m}{2a\sqrt{\mu_o\,\varepsilon_o}}$

For m = 0

$f_c = 0$ …….(8)

Cut off frequency of the TEM waves is zero, indicating all the frequencies down to zero can propagate along the guide.

v. The ratio of the amplitudes of E to H between planes is defined as **characteristic wave impedance** given by

$\frac{E_x}{H_y} = \frac{\beta}{\omega\varepsilon} = \frac{\omega\sqrt{\mu_o\,\varepsilon_o}}{\omega\varepsilon_o} = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \eta$ …….(9)

Ƞ is the intrinsic impedance of the dielectric medium existing between the planes.

$E_x = \eta H_y$ ………(8)

vi. The total power propagating in the Z-direction is calculated using Poynting theorem

$\gamma = \iint E \; X \; H \; dx\; dy$

$P = \int_{x=-\frac{a}{2}}^{x=+\frac{a}{2}} \int_{y=0}^{1} \left(\frac{E_x}{\sqrt{2}}\right)\left(\frac{H_y}{\sqrt{2}}\right) dx\; dy$ for 1 meter width along y direction

P = $\frac{1}{2}\, E_x\, H_y\, [x]_{-\frac{a}{2}}^{+\frac{a}{2}}\, [y]_0^1$

P = $\frac{1}{2}\, E_x\, H_y$ a

$E_x = \eta H_y$

P = $\frac{1}{2}\, (\eta H_y)\, H_y$ a

P = $\frac{1}{2}\, \eta$ a $H_y{}^2$ watts / meter of width. ……(9)

CHARACTERISTICS OF TE AND TM WAVES:

The characteristics of TE and TM waves cab be studied by analyzing propagation constant γ.

$h^2 = \omega^2 \mu\, \varepsilon + \gamma^2$

$\gamma^2 = h^2 - \omega^2 \mu\, \varepsilon$

$\gamma = \sqrt{h^2 - \omega^2 \mu\, \varepsilon}$ ……(1)

i. Cut-off frequency (f_c):

Sub h = $\frac{m\pi}{a}$ in equ (1),

$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu\, \varepsilon} = \alpha + j\beta$ ……(2)

When $\omega^2 \mu\, \varepsilon > \left(\frac{m\pi}{a}\right)^2$. (i.e) at higher frequencies, γ becomes imaginary equal equal to jβ. Phase change for the wave occurs and hence the wave propagates.

At lower frequencies, $\omega^2 \mu\, \varepsilon < \left(\frac{m\pi}{a}\right)^2$ so that 'γ' becomes real equal to the attenuation constant 'α' and 'β' is zero. The wave completely attenuated and no propagation takes place.

As the frequency is decreased a critical frequency ω_c is reached when $\omega^2 \mu\, \varepsilon = \left(\frac{m\pi}{a}\right)^2$.

The frequency at which wave motion ceases or the frequency above which wave motion exits is called the cutoff frequency of the guide.

The system acts as a high pass filter with a cutoff frequency 'f_c' and is defined as the frequency at which the attenuation condition changes to the propagation condition.

At $f = f_c$, $\gamma = 0$,

From equ (2),

$$\sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega_c^2 \mu\,\varepsilon} = 0$$

$$\omega_c^2 \mu\,\varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c^2 = \frac{1}{\mu\,\varepsilon}\left(\frac{m\pi}{a}\right)^2$$

$$\omega_c = \sqrt{\frac{1}{\mu\,\varepsilon}}\left(\frac{m\pi}{a}\right)$$

$$\omega_c = 2\pi f_c$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\,\varepsilon}} \cdot \frac{m\pi}{a}$$

$$f_c = \frac{m}{2a\sqrt{\mu\,\varepsilon}} \qquad \text{……(3)}$$

Cutoff frequency is defined as the frequency at which propagation constant changes from being real to imaginary.

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu\,\varepsilon}$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{\omega^2 \mu\,\varepsilon}{\left(\frac{m\pi}{a}\right)^2}}$$

$$\omega_c^2 \mu\,\varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{\omega^2 \mu\,\varepsilon}{\omega_c^2 \mu\,\varepsilon}}$$

$$\omega_c = 2\pi f_c$$

$$\omega = 2\pi f$$

$$\gamma = \frac{m\pi}{a}\sqrt{1-\frac{f^2}{{f_c}^2}} \qquad \text{……..(4)}$$

$$\frac{m\pi}{a} = \omega_c\sqrt{\mu\,\varepsilon}$$

$$\gamma = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{1-\frac{f^2}{{f_c}^2}} \qquad \text{……(5)}$$

For frequencies below cutoff where $f < f_c$ and γ is real , γ = α , β = 0.

At frequencies above cutoff , $f > f_c$, γ is imaginary and α = 0. Thus propagation will occur and

γ = jβ

From equ (4),

$$j\beta = \frac{m\pi}{a}\sqrt{1-\frac{f^2}{{f_c}^2}}$$

$$j\beta = \frac{m\pi}{a}\sqrt{-1\left(\frac{f^2}{{f_c}^2}-1\right)}$$

$$j\beta = j\,\frac{m\pi}{a}\sqrt{\left(\frac{f^2}{{f_c}^2}-1\right)}$$

$$\frac{m\pi}{a} = \omega_c\sqrt{\mu\,\varepsilon}$$

$$j\beta = j\omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2}{{f_c}^2}-1\right)}$$

$$\beta = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2}{{f_c}^2}-1\right)} \qquad \text{…….(6)}$$

$$\beta = \omega_c\sqrt{\mu\,\varepsilon}\sqrt{\left(\frac{f^2-{f_c}^2}{{f_c}^2}\right)}$$

$$\beta = \frac{\omega_c\sqrt{\mu\,\varepsilon}}{f_c}\sqrt{\left(f^2-{f_c}^2\right)}$$

$$\omega_c = 2\pi f_c$$

$$\beta = \frac{2\pi f_c\sqrt{\mu\,\varepsilon}}{f_c}\sqrt{\left(f^2-{f_c}^2\right)}$$

$$\beta = 2\pi\sqrt{\mu\,\varepsilon}\sqrt{\left(f^2 - {f_c}^2\right)} \qquad \ldots\ldots.(7)$$

(or)

$$\gamma = j\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\,\varepsilon}$$

$$j\beta = \sqrt{-\left[\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2\right]}$$

$$j\beta = j\sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

$$\beta = \sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}$$

from equ (3),

Cut off frequency $f_c = \frac{m}{2a\sqrt{\mu\,\varepsilon}}$

$$f_c = \frac{m\,v}{2a.} \qquad \ldots\ldots(8)$$

$$v = \frac{1}{\sqrt{\mu\,\varepsilon}}$$

v is the velocity of propagation = 3 x 10^8 *m/s*

ii. Wavelength (λ) / Guided Wavelength ($\boldsymbol{\lambda_g}$):

The distance travelled by a wave to under go a phase shift of 2π radians is called wavelength. It is the wavelength in the direction of propagation and hence also called as guided wavelength.

$$\lambda = \frac{2\pi}{\beta} = \lambda_g$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2\mu\,\varepsilon - \left(\frac{m\pi}{a}\right)^2}} \qquad \ldots\ldots(9)$$

iii. Cut off Wavelength($\boldsymbol{\lambda_c}$):

Wavelength at cutoff frequency is called as cutoff wavelength.

$\lambda_c = \frac{v}{f_c}$

$\lambda_c = \frac{v}{\frac{m\,v}{2a.}}$

$\lambda_c = \frac{2a}{m}$ ……(10)

From equ (9),

$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu\, \varepsilon - \left(\frac{m\pi}{a}\right)^2}}$, at cutoff $\left(\frac{m\pi}{a}\right)^2 = \omega_c{}^2 \mu\, \varepsilon$

$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu\, \varepsilon - \omega_c{}^2 \mu\, \varepsilon}}$

$\lambda_g = \frac{2\pi}{\omega\sqrt{\mu\, \varepsilon}\sqrt{1 - \frac{\omega_c{}^2}{\omega^2}}}$

$\omega_c = 2\pi f_c$

$\omega = 2\pi f$

$\lambda_g = \frac{2\pi}{2\pi f\sqrt{\mu\, \varepsilon}\sqrt{1 - \frac{2\pi f_c{}^2}{2\pi f^2}}}$

$\lambda_g = \frac{1}{f\sqrt{\mu\, \varepsilon}\sqrt{1 - \frac{f_c{}^2}{f^2}}}$

$v = \frac{1}{\sqrt{\mu\, \varepsilon}}$

$\lambda_g = \frac{v}{f\sqrt{1 - \frac{f_c{}^2}{f^2}}}$

$\lambda = \frac{v}{f}$

$\lambda_g = \frac{\lambda}{\sqrt{1 - \frac{f_c{}^2}{f^2}}}$

$$f = \frac{v}{\lambda}$$

$$f_c = \frac{v}{\lambda_c}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\frac{v}{\lambda_c}}{\frac{v}{\lambda}}\right)^2}}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{\lambda_c}\right)^2}}$$

Squaring on both sides,

$$\lambda_g{}^2 = \frac{\lambda^2}{1-\left(\frac{\lambda}{\lambda_c}\right)^2}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \frac{\lambda^2}{\lambda_g{}^2}$$

$$1-\left(\frac{\lambda}{\lambda_c}\right)^2 = \left(\frac{\lambda}{\lambda_g}\right)^2$$

$$1 = \left(\frac{\lambda}{\lambda_g}\right)^2 + \left(\frac{\lambda}{\lambda_c}\right)^2$$

$$1 = \lambda^2\left[\frac{1}{\lambda_g{}^2} + \frac{1}{\lambda_c{}^2}\right]$$

$$\frac{1}{\lambda^2} = \frac{1}{\lambda_g{}^2} + \frac{1}{\lambda_c{}^2}$$

λ –Free space wavelength

λ_c – Cutoff wavelength

λ_g – Guide wavelength