4.2 GENERAL WAVE BEHAVIOUR ALONG UNIFORM GUIDING STRUCTURES (or) APPLICATION OD MAXWELL'S EQUATIONS TO THE RECTANGULAR WAVEGUIDE:

In rectangular waveguide, the propagation of energy takes place in the Z-direction, with the length of the guide infinite in the Z-direction.

The field components of electric field and magnetic field are obtained by solving Maxwell's equation and wave equations applying appropriate boundary conditions.

The general equations for field components is determined from Maxwell's curl equations.

∇ x H = jω εE ....(1)

∇ x E = -jω μH ....(2)

Expanding equation (1),

$$\nabla \text{ x H} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\ \varepsilon[E_x\ \hat{x} + E_y\ \hat{y} + E_z\ \hat{z}]$$

Equating x, y, z components,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\ \varepsilon\ E_x \qquad ......(3a)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\ \varepsilon\ E_y \qquad ....(3b)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\ \varepsilon\ E_z \qquad ....(3c)$$

Similarly

Expanding equation (2),

$$\nabla \times E = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial y} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\,\mu\left[H_x \underset{a_x}{\rightarrow} + H_y \underset{a_y}{\rightarrow} + H_z \underset{a_z}{\rightarrow}\right]$$

Equating x, y, z components,

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\,\mu\ H_x \qquad \ldots\ldots(3d)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu\, H_y \qquad \ldots\ldots\ldots(3e)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\,\mu\, H_z \qquad \ldots\ldots\ldots(3f)$$

The wave equations are written as,

For non conducting in medium

$$\nabla^2 E = -\omega^2 \mu\,\varepsilon\, E \qquad \ldots\ldots.(4)$$

$$\nabla^2 H = -\omega^2 \mu\,\varepsilon\, H \qquad \ldots\ldots..(5)$$

It can be written as,

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = -\omega^2 \mu\,\varepsilon\, H \qquad \ldots\ldots(6)$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = -\omega^2 \mu\,\varepsilon\, E \qquad \ldots\ldots(7)$$

, $H_y = H_y^o\, e^{-\gamma z}$ …….(8)

Diff w.r.to ‘z’

$$\frac{\partial H_y}{\partial z} = H_y^o\, e^{-\gamma z}\,(-\gamma)$$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H_y^o\, e^{-\gamma z}$$

$$\frac{\partial H_y}{\partial z} = -\gamma\, H_y \qquad \ldots\ldots..(9)$$

$$\frac{\partial H_x}{\partial z} = -\gamma\, H_x \qquad \ldots\ldots\ldots.(10)$$

And also let,

$$E_y = E_y^o\, e^{-\gamma z} \qquad \ldots\ldots.(11)$$

Diff w.r.to ‘z’

$\frac{\partial E_y}{\partial z} = E_y^o \ e^{-\gamma z} \ (-\gamma)$

$\frac{\partial E_y}{\partial z} = -\gamma \ E_y^o \ e^{-\gamma z}$

$\frac{\partial E_y}{\partial z} = -\gamma \ E_y$ ……..(12)

$\frac{\partial E_x}{\partial z} = -\gamma \ E_x$ ……….(13)

Sub the equ (9), (10), (12), (13) in equ (3),

$\frac{\partial H_z}{\partial y} + \gamma \ H_y = j\omega \ \varepsilon \ E_x$ ………(14a)

$-\gamma \ H_x - \frac{\partial H_z}{\partial x} = j\omega \ \varepsilon \ E_y$

$\gamma \ H_x + \frac{\partial H_z}{\partial x} = - j\omega \ \varepsilon \ E_y$ ……..(14b)

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega \ \varepsilon \ E_z$ ……..(14c)

$\frac{\partial E_z}{\partial y} + \gamma \ E_y = - j\omega \ \mu \ H_x$ …….(14d)

$\gamma \ E_x + \frac{\partial E_z}{\partial x} = j\omega\mu \ H_y$ ………(14e)

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega \ \mu \ H_z$ ……….(14f)

The wave equations (6) and (7) can also be written as,

$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \ \varepsilon \ H_z$

$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \ \varepsilon \ E_z$ ……(15)

Solving Equations (14a) and (14d),

From (14d),

$\frac{\partial E_z}{\partial y} + \gamma \ E_y = - j\omega \ \mu \ H_x$

$H_x = \frac{1}{- j\omega \ \mu} \left[\frac{\partial E_z}{\partial y} + \gamma \ E_y\right]$

Sub the H_x value in equ (14b),

From (14b),

$\gamma\, H_x + \frac{\partial H_z}{\partial x} = - j\omega\, \varepsilon\, E_y$

$\gamma\, \left(\frac{1}{-j\omega\,\mu}\, \left[\frac{\partial E_z}{\partial y} + \gamma\, E_y\right]\right) + \frac{\partial H_z}{\partial x} = - j\omega\, \varepsilon\, E_y$

$\frac{\gamma}{-j\omega\,\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma^{\,2} E_y}{j\omega\,\mu} + \frac{\partial H_z}{\partial x} = - j\omega\, \varepsilon\, E_y$

$\frac{\partial H_z}{\partial x} - \frac{\gamma}{j\omega\,\mu}\frac{\partial E_z}{\partial y} = E_y\left[\frac{\gamma^{\,2}}{j\omega\,\mu} - j\omega\,\varepsilon\right]$

Multiply throughout by $j\omega\,\mu$,

$j\omega\,\mu\,\frac{\partial H_z}{\partial x} - \gamma\,\frac{\partial E_z}{\partial y} = E_y\,[\gamma^{\,2} + \omega^2\mu\,\varepsilon]$

$\gamma^{\,2} + \omega^2\mu\,\varepsilon = h^{\,2}$

$j\omega\,\mu\,\frac{\partial H_z}{\partial x} - \gamma\,\frac{\partial E_z}{\partial y} = E_y\, h^{\,2}$

$E_y = \frac{1}{h^{\,2}}\left[j\omega\,\mu\,\frac{\partial H_z}{\partial x} - \gamma\,\frac{\partial E_z}{\partial y}\right]$

$E_y = \frac{j\omega\,\mu}{h^{\,2}}\frac{\partial H_z}{\partial x} - \frac{\gamma}{h^{\,2}}\frac{\partial E_z}{\partial y}$ ……(16a)

Similarly,

$H_x = -\frac{\gamma}{h^{\,2}}\frac{\partial H_z}{\partial x} + \frac{j\omega\,\varepsilon}{h^{\,2}}\frac{\partial E_z}{\partial y}$ ………(16b)

From equ (14a),

$\frac{\partial H_z}{\partial y} + \gamma\, H_y = j\omega\,\varepsilon\, E_x$

$E_x = \frac{1}{j\omega\,\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma\, H_y\right]$

Sub the E_x value in equ (14e),

$\gamma\, E_x + \frac{\partial E_z}{\partial x} = j\omega\mu\, H_y$

$$\gamma \left(\frac{1}{j\omega\,\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma\, H_y\right]\right) + \frac{\partial E_z}{\partial x} = j\omega\mu\, H_y$$

$$\frac{\gamma}{j\omega\,\varepsilon}\frac{\partial H_z}{\partial y} + \frac{\gamma^{2}}{j\omega\,\varepsilon} H_y + \frac{\partial E_z}{\partial x} = j\omega\mu\, H_y$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\,\varepsilon}\frac{\partial H_z}{\partial y} = H_y\left(j\omega\mu - \frac{\gamma^{2}}{j\omega\,\varepsilon}\right)$$

Multiply throughout by $j\omega\,\varepsilon$

$$j\omega\,\varepsilon\,\frac{\partial E_z}{\partial x} + \frac{\gamma\,\partial H_z}{\partial y} = -H_y\,(\gamma^{2} + \omega^2\mu\,\varepsilon)$$

$$j\omega\,\varepsilon\,\frac{\partial E_z}{\partial x} + \frac{\gamma\,\partial H_z}{\partial y} = -H_y\,h^{2}$$

$$H_y = \frac{-j\omega\,\varepsilon}{h^{2}}\frac{\partial E_z}{\partial x} - \frac{\gamma}{h^{2}}\frac{\partial H_z}{\partial y} \qquad ....(16c)$$

Similarly,

$$E_x = -\frac{\gamma}{h^{2}}\frac{\partial E_z}{\partial x} - \frac{j\omega\,\mu}{h^{2}}\frac{\partial H_z}{\partial y} \qquad ....(16d)$$

The above equations are in terms of E_z and H_z.

For wave propagation either E_z of H_z should exits.

If both E_z and H_z are zero, all the fields within guide will vanish.

Wave propagation within the guide is divided into two sets, TE waves and with E_z = 0 and TM waves with H_z = 0 shown in Fig 4.2.1.

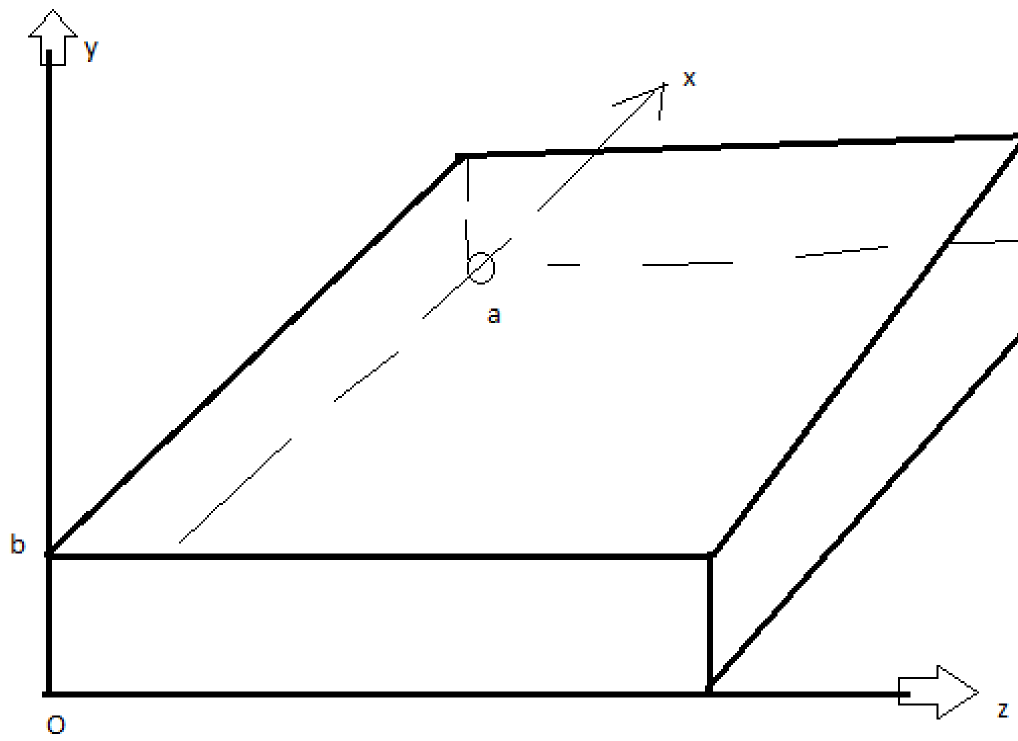

Fig: 4.2.1 Rectangular waveguide

FIELD COMPONENTS OF TRANSVERSE MAGNETIC WAVES IN RECTANGULAR WAVEGUIDE:

For TM waves, H_z = 0 and E_z is to be solved from eave equtions.

Wave equation for E_z is given by,

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = -\omega^2 \mu \, \varepsilon \, \mathrm{E} \qquad \ldots\ldots(1)$$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma\,^2 E_z = -\omega^2 \mu \, \varepsilon \, \mathrm{E} \qquad \ldots\ldots(2)$$

The wave equation is a partial differential equation that can be solved by the usual technique of assuming a product solution.

$E_z\ (x, y, z) = {E_z}^o\ (x, y)\ e^{-\gamma z}$

Let us assume a solution,

${E_z}^o\ (x, y) = \mathrm{X}(x)\ \mathrm{Y}(y) \qquad \ldots\ldots(3)$

Where X is the function of *x* alone.

Y is the function *y* alone.

Sub equ (3) in equ (2)

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + \gamma^2 XY = -\omega^2\mu\,\varepsilon\,XY \qquad .....(4)$$

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + [\gamma^2 + \omega^2\mu\,\varepsilon]XY = 0$$

$$\gamma^2 + \omega^2\mu\,\varepsilon = h^2$$

$$Y\frac{d^2X}{dx^2} + X\frac{d^2Y}{dy^2} + h^2 XY = 0$$

Dividing by XY,

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + h^2 = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = -\frac{1}{Y}\frac{d^2Y}{dy^2} \qquad ......(5)$$

This expression equates a function of x alone to a function of y alone and the only way for the above equation to be true is to have each of these functions equal to some constant A^2.

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 = A^2 \qquad ....(6)$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + h^2 - A^2 = 0$$

$$B^2 = h^2 - A^2$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + B^2 = 0 \qquad ......(7)$$

$$-\frac{1}{Y}\frac{d^2Y}{dy^2} = A^2 \qquad ....(8)$$

A solution of equation (7) is of the form

$X = C_1 \cos Bx + C_2 \sin Bx$

Where $B^2 = h^2 - A^2$

The solution of equation (8) is of the form

$$Y = C_3 \cos Ay + C_4 \sin Ay \qquad ......(9)$$

Wkt,

$E_z^{\,o}(x, y) = XY$

$E_z^{\,o} = (C_1 \cos Bx + C_2 \sin Bx)(C_3 \cos Ay + C_4 \sin Ay)$

$E_z^{\,o} = C_1 \cos Bx\, C_3 \cos Ay$

$E_z{}^o = C_2\ C_3$ sin Bx cos Ay + $C_2\ C_4$ sin Bx sin Ay ……(12)

Sub y=o in equ (8)

$E_z{}^o = C_2\ C_3$ sin Bx cos Ay = 0

x and B ≠ 0, either C_2 *or* C_3 has to zero. If C_2 = 0 ,then the equ (12) become zero.

Since C_3 = 0 ……(13)

Sub equ (13) in equ (12)

$E_z{}^o = C_2\ C_4$ sin Bx sin Ay ……(14)

If x = a, $E_z{}^o$ = 0, sub in (14)

$E_z{}^o = C_2\ C_4$ sin Ba sin Ay = 0

Since A ≠ 0

sin Ba = 0

Ba = mπ

B = $\frac{m\pi}{a}$ where m = 1,2,3,……. …….(15)

Sub equ (15) in equ (14)

$E_z{}^o = C_2\ C_4 \sin\left(\frac{m\pi}{a}\right)x$ sin Ay ……(16)

If y = a, $E_z{}^o$ = 0, sub in (16)

$E_z{}^o = C_2\ C_4 \sin\left(\frac{m\pi}{a}\right)x$ sin Ab = 0

sin Ab = 0

Ab = nπ

A = $\frac{n\pi}{b}$ where n = 1,2,3,……. …….(17)

Sub equ (17) in equ (16)

$E_z{}^o = C_2\ C_4 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$ ……(18)

C = $C_2\ C_4$

$E_z{}^o = C \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y$ ……(19)

The general field components with $H_z = 0$ and $\gamma = j\beta$ is given by,

$E_x = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial x}$ ……(20a)

$H_x = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$ …….(20b)

$E_y = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial y}$ ……(20c)

$H_y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$ ……(20d)

Using equ (19) and equ (20a), 20b, 20c, 20d,

$E_x{}^0 = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial x}$

$E_x{}^0 = -\frac{j\beta}{h^2}\, C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\,\sin\left(\frac{n\pi}{b}\right)y$

$H_x{}^o = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}$

$H_x{}^o = \frac{j\omega\varepsilon}{h^2}\, C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x\,\cos\left(\frac{n\pi}{b}\right)y$

$E_y{}^0 = -\frac{j\beta}{h^2}\frac{\partial E_z}{\partial y}$

$E_y{}^0 = -\frac{j\beta}{h^2}\, C\left(\frac{n\pi}{b}\right)\sin\left(\frac{m\pi}{a}\right)x\,\cos\left(\frac{n\pi}{b}\right)y$

$H_y{}^o = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial x}$

$H_y{}^o = \frac{-j\omega\varepsilon}{h^2}\, C\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}\right)x\,\sin\left(\frac{n\pi}{b}\right)y$

Wkt,

$A = \frac{n\pi}{b}$ & $B = \frac{m\pi}{a}$

$E_z = E_z{}^0\, e^{-\gamma z}$

$E_z = E_z{}^0\, e^{-j\beta z}$

From equ (19)

$E_z = C\sin Bx\,\sin Ay\;\; e^{-j\beta z}$

$E_x = E_x{}^0\, e^{-j\beta z}$

$E_x = -\frac{j\beta}{h^2}\ BC \cos Bx \sin Ay\ e^{-j\beta z}$

$E_y = -\frac{j\beta}{h^2}\ \mathrm{AC}\ \sin Bx \cos Ay\ e^{-j\beta z}$

$H_x = \frac{j\omega\varepsilon}{h^2}\ \mathrm{AC}\ \sin Bx \cos Ay\ e^{-j\beta z}$

$H_y = \frac{j\omega\varepsilon}{h^2}\ \mathrm{AC}\ \cos Bx \sin Ay\ e^{-j\beta z}$

CHARACTERISTICS OF TE AND TM WAVES IN RECTANGULAR WAVEGUIDE:

$A^2 + B^2 = h^2$

$\mathrm{A} = \frac{n\pi}{b}$ & $\mathrm{B} = \frac{m\pi}{a}$

$h^2 = \gamma^2 + \omega^2\mu\,\varepsilon$

a = width of guide along x

b = width of guide along y

m , n = integers

i) PROPAGATION CONSTANT AND CUT OFF FREQUENCY:

$h^2 = \gamma^2 + \omega^2\mu\,\varepsilon = A^2 + B^2$

$\gamma = \sqrt{h^2 - \omega^2\mu\,\varepsilon}$

$\gamma = \sqrt{(A^2 + B^2) - \omega^2\mu\,\varepsilon}$

$\gamma = \sqrt{\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\,\varepsilon}$

This is the equation of propagation constant in a rectangular waveguide for TE and TM waves. Foe small frequencies $\gamma = \alpha$, γ is real and there is no wave propagation.

As frequency increases and reaches a particular value f_c , γ becomes zero.

Then for all values of f greater than f_c γ is imaginary, $\gamma = j\beta$, wave propagation takes place.

At f = f_c , $\gamma = 0$

$\omega_c{}^2\mu\,\varepsilon = h^2$

$$\omega_c{}^2\mu\,\varepsilon = \left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c{}^2 = \frac{1}{\mu\,\varepsilon}\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]$$

(or)

$$\omega_c = \frac{1}{\sqrt{\mu\,\varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

$$\omega_c = \frac{h}{\sqrt{\mu\,\varepsilon}}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\,\varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$$

The frequency f_c below which there is no wave propagation (or) thefrequency above which the wave propagation exits is called cut off frequency. The propagation constant can be given by,

$$\gamma = \sqrt{h^2 - \omega^2\mu\,\varepsilon}$$

$$\gamma = h\sqrt{1 - \frac{\omega^2\mu\,\varepsilon}{h^2}}$$

$$\gamma = h\sqrt{1 - \frac{\omega^2\mu\,\varepsilon}{\omega_c{}^2\mu\,\varepsilon}}$$

$$\gamma = h\sqrt{1 - \frac{f^2}{f_c{}^2}}$$

$$\gamma = \omega\sqrt{\mu\,\varepsilon}\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

ii) ATTENUATION CONSTANT:

When $\left(\frac{f}{f_c}\right)^2 < 1$ (i.e) f < f_c, $\gamma = real$, $\gamma = \alpha$ No wave propagation

$$\gamma = \alpha = h\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$\gamma = \alpha = \sqrt{h^2 - \omega^2\mu\,\varepsilon}$$

iii) PHASE SHIFT:

$$\gamma = \sqrt{-(\omega^2\mu\,\varepsilon - h^2)}$$

$\gamma = j\beta = j\sqrt{(\omega^2\mu\,\varepsilon - h^2)}$

$\gamma = j\sqrt{\omega^2\mu\,\varepsilon - (A^2 + B^2)}$

$j\beta = j\sqrt{\omega^2\mu\,\varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$

$\beta = \sqrt{\omega^2\mu\,\varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}$

$\gamma = j\beta$

$\gamma = j\sqrt{(\omega^2\mu\,\varepsilon - h^2)}$

$\gamma = j\sqrt{(\omega^2\mu\,\varepsilon - \omega_c{}^2\mu\,\varepsilon)}$

$\gamma = j\,\omega\sqrt{\mu\,\varepsilon}\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2}$

$\gamma = j\,\omega\sqrt{\mu\,\varepsilon}\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$

iv) CUT- OFF WAVELENGTH:

It is the wavelength at cut – off frequency

$\lambda_c = \frac{v}{f_c}$

$\lambda_c = \frac{velocity}{cut-off\ frequency}$

$\lambda_c = \frac{v}{\frac{1}{2\pi\sqrt{\mu\,\varepsilon}}\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$

$\lambda_c = \frac{v\,2\pi\sqrt{\mu\,\varepsilon}}{\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$

$v = \frac{1}{\sqrt{\mu\,\varepsilon}}$

$\lambda_c = \frac{2\pi}{\sqrt{\left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$

$\lambda_c = \frac{2\pi}{\pi\sqrt{\left[\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2\right]}}$

$$\lambda_c = \frac{2}{\sqrt{\left[\left(\frac{n}{b}\right)^2 + \left(\frac{m}{a}\right)^2\right]}}$$

v) GUIDED WAVELENGTH (λ_g):

$$\lambda_g = \frac{v}{f} = \frac{2\pi}{\beta}$$

$$\lambda_g = \frac{2\pi}{\sqrt{\omega^2 \mu\, \varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$$

(or)

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

vi) PHASE VELOCITY(v_p):

$$v_p = \frac{\omega}{\beta}$$

$$v_p = \frac{\omega}{\sqrt{\omega^2 \mu\, \varepsilon - \left[\left(\frac{n\pi}{b}\right)^2 + \left(\frac{m\pi}{a}\right)^2\right]}}$$

At $\omega = \omega_c$

$$v_p = \frac{\omega_c}{\omega_c \sqrt{\mu\, \varepsilon} \sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

$$v_p = \frac{v}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

vii) GROUP VELOCITY:

$$v_g = \frac{d\omega}{d\beta}$$

$$\beta = \omega \sqrt{\mu\, \varepsilon} \sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$\beta = \omega \sqrt{\mu\, \varepsilon} \sqrt{\omega^2 - \omega_c^2}$$

$$v_g = v \sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$