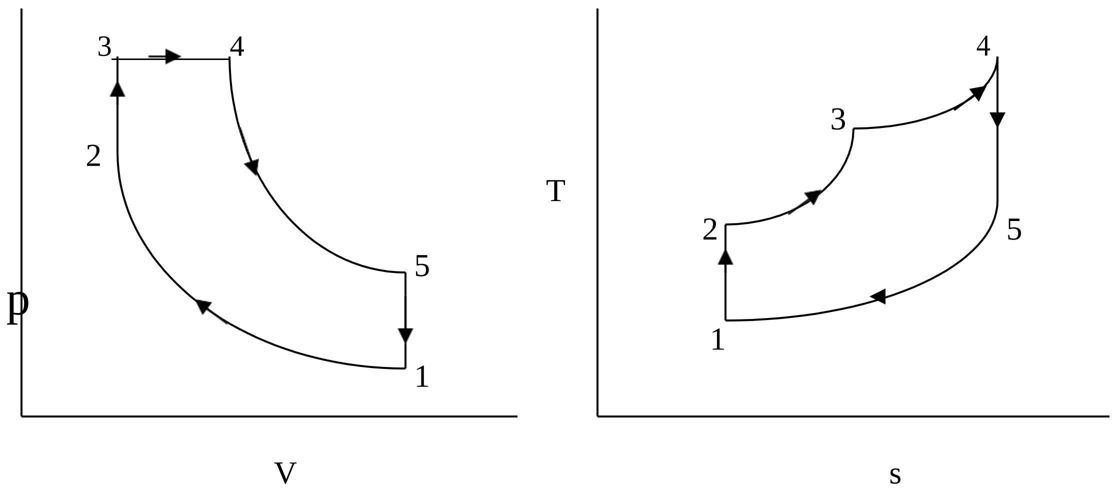
MODULE III

Dual Cycle (Limited Pressure or Mixed Cycle)

This cycle is a combination of Otto and Diesel cycles. In this cycle the heat is added partially at constant volume and partially at constant pressure. The advantage of this cycle is increased time to fuel for injection.



1 – 2 → Isentropic compression

2 – 3 → Constant volume heat addition

3 – 4 → Constant pressure heat addition

4 – 5 → isentropic expansion

5 – 1 → Constant volume heat rejection

ρ = Cut-off ratio = V_4/V_3

r_p = Explosion ratio or Pressure ratio = p_3/p_2

The cycle is the equivalent air cycle for reciprocating high speed compression ignition engines. The P-V and T-s diagrams are shown in Figs.6 and 7. In the cycle, compression and expansion processes are isentropic; heat addition is partly at constant volume and partly at constant pressure while heat rejection is at constant volume as in the case of the Otto and Diesel cycles.

The heat supplied, Qs per unit mass of charge is given by

$$c_v(T_3 - T_2) + c_p(T_{3'} - T_2) \qquad (32)$$

Whereas the heat rejected, Qr per unit mass of charge is given by

cv (T4 – T1) and

The thermal efficiency is given by

$$\eta_{th} = 1 - \frac{c_v(T_4 - T_1)}{c_v(T_3 - T_2) + c_p(T_{3'} - T_2)} \qquad (33A)$$

$$= 1 - \left\{ \frac{T_1\left(\frac{T_4}{T_1} - 1\right)}{T_2\left(\frac{T_3}{T_2} - 1\right) + \gamma T_3\left(\frac{T_{3'}}{T_3} - 1\right)} \right\} \qquad (33B)$$

$$= 1 - \frac{\frac{T_4}{T_1} - 1}{\frac{T_2}{T_1}\left(\frac{T_3}{T_2} - 1\right) + \frac{\gamma T_3}{T_2}\frac{T_2}{T_1}\left(\frac{T_{3'}}{T_3} - 1\right)} \qquad (33C)$$

From thermodynamics

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = r_p \qquad (34)$$

the explosion or pressure ratio and

$$\frac{T_{3'}}{T_3} = \frac{V_{3'}}{V_3} = r_c \qquad (35)$$

the cut-off ratio.

Now, $\frac{T_4}{T_1} = \frac{p_4}{p_1} = \frac{p_4}{p_{3'}}\frac{p_{3'}}{p_3}\frac{p_3}{p_2}\frac{p_2}{p_1}$

Also $\frac{p_4}{p_{3'}} = \left(\frac{V_{3'}}{V_4}\right)^\gamma = \left(\frac{V_{3'}}{V_3}\frac{V_3}{V_4}\right)^\gamma = \left(r_c\frac{1}{r}\right)^\gamma$

And $\frac{p_2}{p_1} = r^\gamma$

Thus $\frac{T_4}{T_1} = r_p r_c^\gamma$

Also $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^\gamma = r^{\gamma-1}$

Therefore, the thermal efficiency of the dual cycle is

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_p r_c^\gamma - 1}{(r_p - 1) + \gamma r_p (r_c - 1)}\right] \qquad (36)$$

1. An air standard dual cycle has a compression ratio of 16 and compression begins at 1bar and 50^0 C. The maximum pressure is 70bar. The heat transferred to air at constant Pressure is equal to heat transferred at constant volume. Find the temperature at all cardinal Points, cycle efficiency and mean effective pressure. Take C_p = 1.005kJ/kgK; C_V=0.718kJ/kgK.

Given data:

r = 16

P_1 = 1bar

$T_1 = 50^0C = 323K$

P_3 = 70bar

$$Q_{S_1} = Q_{S_2}$$

C_p = 1.005kJ/kgK

C_V = 0.718kJ/kgK

Solution:

Specific volume, $v_1 = \frac{RT_1}{P_1} = \frac{287 \times 323}{1 \times 10^5}$

= 0.92701m3/*kg*

$V_2 = 0.05794 m^3/kg$

1-2 ⇒ Isentropic compression process

$$p_2 = (r)^{\gamma} \times p_1 = (16)^{1.4} \times 1$$

= 48.5bar **Ans.**

$$T_2 = (r)^{\gamma-1} \times T_1 = (16)^{1.4-1} \times 323$$

= 979K **Ans.**

2-3 ⇒ Constant volume heat addition process

$$T_3 = \left(\frac{p_3}{p_2}\right) \times T_2 = \frac{70}{48.5} \times 979$$

= 1413 K **Ans.**

$$Q_{S_1} = C_V (T_3 - T_2)$$

= 0.718 (1413 – 979)

= 311.612kJ/kg

3-4 $\Rightarrow$ ***Constant pressure heat addition***

$$Q_{S_1} = Q_{S_2} = C_P (T_4 - T_3)$$

311.612 = 1.005 (T_4 – 1413)

T_4 = 1723K **Ans.**

$$v_4 = \frac{T_4}{T_3} \times v_3 = \frac{1723}{1413} \times 0.05794 \qquad (\because v_2 = v_3)$$

= 0.070652*m³/kg*

Expansion ratio, $r_e = \frac{v_4}{v_1} = \frac{0.070652}{0.92701}$

r_e = 0.076215

4-5 $\Rightarrow$ ***isentropic expansion process***

$$p_5 = (r_e)^\gamma \times p_4 = (0.076215)^{1.4} \times 70$$

= *1.9063bar* **Ans.**

$\therefore$ **Cut off ratio,** $\rho = \frac{v_4}{v_3} = \frac{0.070652}{0.05744}$

= 1.2194

$\therefore$ **Pressure ratio** $(k) = \left(\frac{p_3}{p_2}\right) = \left(\frac{70}{48.5}\right)$

= 1.4433

The cycle efficiency,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}} \left[\frac{(k\rho^\gamma - 1)}{(k-1) + k \times \gamma \times (\rho - 1)} \right]$$

$$=1-\frac{1}{(16)^{1.4-1}}\left[\frac{\left(1.4433\times(1.2194)^{1.4}-1\right)}{(1.4433-1)+1.4\times1.4433\times(1.2194-1)}\right]$$

= 66.34% **Ans.**

Net heat supplied to the cycle,

$$Q_S = Q_{S_1} + Q_{S_2}$$

= 311.612 + 311.612
= 623.224*kJ/kg*

Net work done of the cycle,

$$W = Q_S \times \eta$$

= 623.224 x 0.6634

= 413.45*kJ/kg*

The mean effective pressure,

$$\rho_m = \frac{W}{v_1 - v_2} = \frac{413.45}{0.92701 - 0.05794}$$

= 4.75*bar* **Ans.**

2. **In engine working on dual cycle, the temperature and pressure at the beginning of the Cycle of the are 90^0C and 1bar. The compression ratio is 9. The maximum pressure is limited to 68bar and total heat supplied per kg of air is 1750kJ. Determine air standard efficiency and mean effective pressure.**

Given data:

$p_1 = 1$ *bar*

$T_1 = 90^0C = 363K$

$p_3 = p_4 = 68bar$

$r=9$

$Q_S = 1750kJ/kg$

Solution:

1-2 $\Rightarrow$ isentropic comp. process

$$p_2 = (r)^{\gamma} \times p_1 = (9)^{1.4} \times 1$$

= 21.67bar

$$T_2 = (r)^{\gamma - 1} \times T_1 = (9)^{0.4} \times 363$$

= 874K

2-3 $\Rightarrow$ Constant volume heat addition process

$$T_3 = \left(\frac{p_3}{p_2}\right) \times T_2 = \left(\frac{68}{21.67}\right) \times 874 = 2743\,K$$

3-4 $\Rightarrow$ Constant pressure heat addition process

$$Q_S = C_V(T_3 - T_2) + C_p(T_4 - T_3)$$

1750 = 0.718 (2743 – 874) + 1.005 (T_4 – 2743)

T_4 = 3149K

$$v_1 = \frac{RT_1}{p_1} = \frac{287 \times 363}{1 \times 105} = 1.04181\, m3/kg$$

$$v_3 = v_2 = \frac{v_1}{r} = \frac{1.04181}{9}$$

$=0.11576 m^3/kg$

$$v_4 = \left(\frac{T_4}{T_3}\right) \times v_3 = \left(\frac{3149}{2743}\right) \times 0.11576$$

= 0.132894m3/kg

Cut off ration, $\rho = \frac{v_4}{v_3} = \frac{0.13289}{0.11576}$

= 1.148

Pressure ratio, $k = \frac{p_3}{p_2} = \frac{68}{21.67}$

= 3.138

Efficiency of the cycle,

$$\eta = 1 - \frac{1}{(r)^{\gamma-1}}\left[\frac{k\rho^{\gamma}-1}{(k-1)+k\times\gamma(\rho-1)}\right]$$

$$1 - \frac{1}{(9)^{1.4-1}}\left[\frac{3.138\times1.148^{1.4}-1}{(3.13-1)+3.138\times1.4(1.148)}\right]$$

= 58.19 % **Ans.**

Net work done of the cycle,

$$W_{net} = \eta \times Q_S$$

= 0.5819 x 1750

= 1018.33*kJ/kg*

Mean effective pressure,

$$p_m = \frac{W_{net}}{v_1 - v_2}$$

$$= \frac{1018.33}{1.04181 - 0.11576} = 10.98 bar$$ **Ans.**

3. An air standard dual cycle has a compression ratio of 16 and compression begins at 1 bar and 50ºC. The maximum pressure is 70 bar. The heat transformed to air at constant pressure is equal to heat transferred at constant volume. Find the temperature at all cardial points, cycle efficiency and mean effective pressure take Cp= 1.005KJ/kgK, Cv = 0.718KJ/kgK.

Given data:
r = 16
P1 = 1 bar
T1 =50ºC = 323K
P3 = 70 bar Qs1 = Qs2
Cp = 1.005kJ/kgk
Cv = 0.718kJ/kgk

Solution:
Specific volume,

V1RT1/P1 = 287 x 323/1 x 105 V1= 0.92701m3/kg
V_2 = 0.05794m^3/kg

1-2 isentropic compression process:

P_2 = (r) ^{y} x P_1 = (16) $^{1.4}$ x 1 =

48.5 barT_2 = (r) $^{y-1}$ x T_1 = (16)

$^{1.4-1}$ x 323

T_2 = 979K

2-3 constant volume heat addition

process:T_3 = (P_3/P_2) x T_2

=70/48.5 x 979 T_3 = 1413K

Q_{s1} = Cv (T_3-T_2); 0.718(1413 – 979)

Q_{s1} = 311.612KJ/kg

3-4 constant pressure heat addition:Q_{s1} = Q_{s2} = C_p (T_4 – T_3) 311.62 = 1.005 (T_4 – 1413)

T_4 = 1723K

V_4 = T_4/T_3 x V_3 = 1723/1413 X

0.05794V_4 = 0.070652m^3/kg

Expansion ratio:

r_e = V_4/V_1 = 0.70652/0.92701 = 0.06215

4-5 isentropic expansion process:

P_5 = (r) x P_4 = (0.076215) $^{1.4}$

x 70P_5 = 1.9063 bar

T_5 = $(r)^{y-1}$x T_4

= $(0.076215)^{1.4-1}$ x 1723

= 567 K

Cut off ratio,

ρ= V4/ V3

=0.00652/0.05744

ρ = 1.2194

Pressure ratio (K) = (P3/P_2) =

(70/ 48.5) K = 1.4433

The cycle efficiency:

η= 1- 1/$(r)^{y-1}$ [(kp^y-1)/ (k-1 + K^y(p-1)]

= 66.34%

Net heat supplied to

the cycle:Q_s =

Q_{s1} + Q_{s2}

= 311.612 + 311.612

= 623.224 KJ/kg

The mean effective pressure:

$P_m = W/ V_1 – V_2 = 413.45/ (0.92701 -0.05794)$

$P_m = 4.75$ bar

4. In a air standard dual cycle, the compression ratio is 12 and the maximum pressure and temperature of the cycle are 1 bar and 300K. haet added during constant pressure process is upto 3% of the stroke. taking diameter as 25cm and stroke as 30cm, determine.

1) **The pressure and temperature at the end of compression**
2) **The thermal efficiency**
3) **The mean effective pressure**

Take, Cp =1.005KJ/kgK Cv =0.118KJ/kgk , = 1.4

Given data:

P_1 = 1 barr = 12

T_1 =300K

K = 3% of Vs = 0.03Vs

P_3 = 70 bar

D = 25 cmL =30cm

Solution:

Specific volumes,:

$V_1 \; RT_1/ P_1 = 287 \times 300/ 1 \times 10^5$

$= 0.861 \; m^3 /kg$

$V_3 = V_2 = V_1/r = 0.861/12$

$= 0.07175 m^3/kg$

$V_4 – V_3 = 0.03 (V_1 –V_2) V_4 = 0.0954275 \; m^3/kg$

$\rho = V_4 /V_3 = 0.054275/0.07175 \rho = 1.33$

1-2 isentropic compression process:

$P_2 = (r)^y \times P1 = (12)^{1.4} \times 1$

= 32.423 bar

V_2 = (r) y^{-1} x T_1 = (12) $^{1.4\ -1}$ x 300T_2 = 810.57K.

2-3 constant volume heat addition process

$P_3/T_3 = P_2/T_2$

T_3 = (P_3/P_2) x T_2 = (70 / 32.423) x

810.57T_3 = 1750K

3-4 constant pressure heat addition process:

T_4 = (V4/V_3) x T_3 = (0.0954275 / 0.07175)

x1750T_4 = 2327.5 K

Pressure ratio, K = (P_3/P_2) = 70/32.423 = 2.159

Net heat supplied to the cycle:

$Q_S = C_v (T_3 – T_2) + C_p (T_4-T_3)$

= 0.718 (1750 -810.57) + 1.005(2327.5-1759)

= 1254.9 KJ/kg

Efficiency of the cycle:

η = 1- 1/ (r) $^{y-1}$ [(K x P^y-1)/(k-1) + Ky(p-1)]

= 61.92%

Net work done of the cycle:

W = η x Q_S

= 0.6192 x 1254.9

= 777.1 KJ/kg

Mean effective pressure,

P_m = W/ $V_1 – V_2$

= 777.1/ 0.361 – 0.07115

= 984.6 Kpa

P_m = 9.846 bar

5. The compression ratio of a dual cycle is 10. The pressure and temperature at the beginning of the cycle are 1 bar and 27°C. the maximum pressure of the cycle is limited to 70 bar and heat supplied is limited to 1675KJ/kg fair find thermal efficiency.

GIven data:

r= 10

P_1 = 1 bar

T_1 = 27ºC = 300K

P_3 = 70 bar

Qs = 1675 KJ/kg

Solution:

Specific volumes:

$V_1 = RT_1/P_1$ = 287 x

300/1 x $10^5 V_2 = V_1/r$

= 0.861/10

1-2 isentropic compression process:

$P_1 = (r)^{y} \times P_1 = (10)^{1.4} \times 1$ = 25.12 bar

$T_2 = (r)^{y-1} \times T_1 = (10)^{1.4-1} \times 300$ = 753.57K

2-3 constant volume heat addition process:

$T_3 = (P_3/P_2) \times T_2$ = (70 / 25.12) x 753.37 = 2100K

Total heat supplied to the cycle:

$Q_s = C_v (T_3 – T_2) + C_p (T_4 – T_3)$

1675 = 0.718 (2100 -753.57) + 1.005 (T4 –

2100)T_4 = 2804.6 K

Cut off ratio:

$\rho = V_4/V_3 = T_4/T_3$ =

2804.6/2300ρ= 1.3356

Pressure ratio:

K = P3/P2 = 70/25.12 = 2.787

Efficiency of the cycle:

η= 1- 1/ $(r)^{y-1}$ [(K x P^{y}-1)/(k-1) + Ky(p-1)]

= 59.13%

Explain the working principle of brayton cycle and draw the pv and T-s diagram .Also derive the air standard efficiency.

The Brayton cycle is also referred to as the Joule cycle or the gas turbine air cycle because all modern gas turbines work on this cycle. However, if the Brayton cycle is to be used for reciprocating piston engines, it requires two cylinders, one for compression and the other for expansion. Heat addition may be carried out separately in a heat exchanger or within the expander itself.

The pressure-volume and the corresponding temperature-entropy diagrams are shown in Figs 10 and 11 respectively.

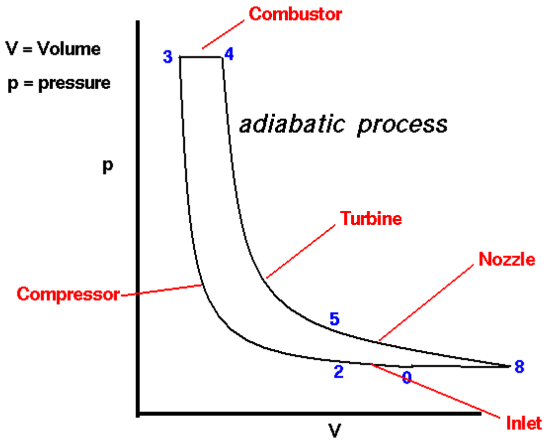


Fig. 10

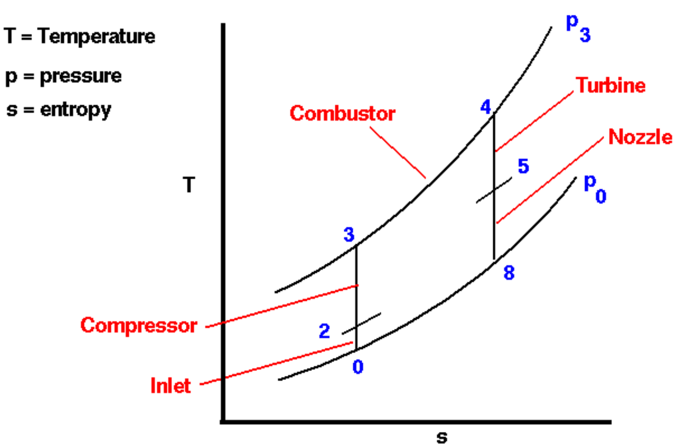


Fig. 11

The cycle consists of an isentropic compression process, a constant pressure heat addition process, an isentropic expansion process and a constant pressure heat rejection process. Expansion is carried out till the pressure drops to the initial (atmospheric) value.

Heat supplied in the cycle, Qs, is given by Cp(T3 – T2)

Heat rejected in the cycle, Qs, is given by Cp (T4 – T1)

Hence the thermal efficiency of the cycle is given by

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

$$= 1 - \frac{T_1}{T_2}\left\{\frac{\left(\frac{T_4}{T_1} - 1\right)}{\left(\frac{T_3}{T_2} - 1\right)}\right\} \qquad (42)$$

$$\text{Now } \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_3}{T_4}$$

$$\text{And since } \frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \textit{we have} \quad \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Hence, substituting in Eq. 62, we get, assuming that r_p is the pressure ratio p_2/p_1

$$\eta_{th} = 1 - \frac{T_1}{T_2}$$

$$= 1 - \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}}$$

$$= 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \qquad (43)$$

This is numerically equal to the efficiency of the Otto cycle if we put

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{1}{r}\right)^{\gamma-1}$$

$$\text{so that } \eta_{th} = 1 - \frac{1}{r^{\gamma-1}} \qquad (43A)$$

where r is the volumetric compression ratio.

1. In a gas turbine plant working on the Brayton cycle, the air at the inlet is at 23^0C, 0.1MPa. The pressure ratio is 6.75 and the maximum temperature is 750^0C. The turbine expansion is divided into two stages with reheat to 750^0C. The efficiency of compressor and two turbines are 82 % and 86% respectively. Determine the maximum power that can be Obtained from this plant, if the mass flow rate of air is 5kg/sec.

Given data:

p_1 = 0.1MPa = 1 bar

$T_1 = 23^0C = 296K$

$$R_p = \frac{p_2}{p_1} = 6.75$$

$T_3 = T_5 = 750^0C = 1023K$

$$\eta_C = 82\% = 0.82$$

$$\eta_{T1} = \eta_{T2} = 86\% = 0.86$$

Solution

For maximum work or power developed

$$p_4 = p_5 = p_4' = \sqrt{p_1 p_2} = \sqrt{1 \times 6.75} = 2.598\,bar$$

From process 1-2 isentropic compression

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 296(6.75)^{\frac{1.4-1}{1.4}} = 510.78K$$

$$\eta_C = \frac{T_2 - T_1}{T_2' - T_1} = \frac{510.78 - 296}{T_2' - 296} = 0.82$$

$$T_2' = 557.9K$$

From isentropic expansion of air in first turbine 3-4

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_2}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{6.75}{2.598}\right)^{\frac{1.4-1}{1.4}} \qquad (\because P_2 = P_1 \times 6.75 = 6.73)$$

$$\eta_{T_1} = \frac{T_3 - T_4^{'}}{T_3 - T_4} = \frac{1023 - T_4^{'}}{1023 - 778.75} = 0.86$$

$T_4^{'}$ = 812.94 K

Similarly, for turbine 2-process 5-6

$$\frac{T_5}{T_6} = \left(\frac{P_5}{P_6}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2.598}{1}\right)^{\frac{1.4-1}{1.4}} = 1.313$$

$$T_6 = \frac{T_5}{1.313} = \frac{1023}{1.313} = 778.76K$$

$$\eta_{T_2} = \frac{T_5^{'} - T_6^{'}}{T_5 - T_6} = \frac{1023 - T_6^{'}}{1023 - 778.76} = 0.86$$

$T_6^{'}$=812.95K

Work done by the turbine

$$W_T = mC_p\,[(T_3 - T_4^{'}) + (T_5 - T_6^{'})]$$

= 5 x 1.005 [(1023 – 812.94) + 1023 – 812.95)]

= 2111.1 kJ/s

Work required by compressor

$$W_C = m \times C_p(T_2^{'} - T_1) = 5 \times 1.005\,(557.9 - 296)$$

= 1316kJ/s

Net work,

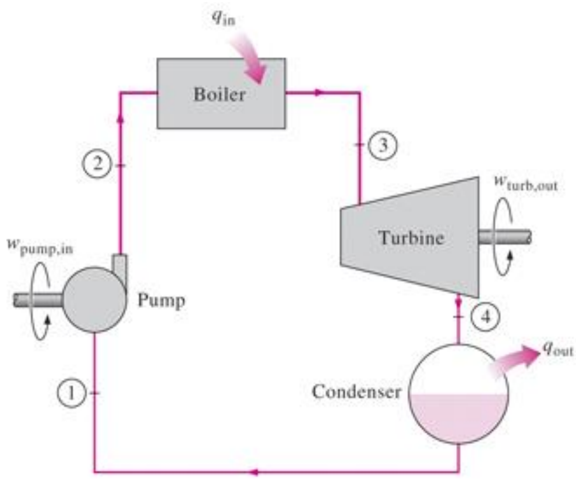
$$W = W_T - W_C = 2111.1 - 1316$$

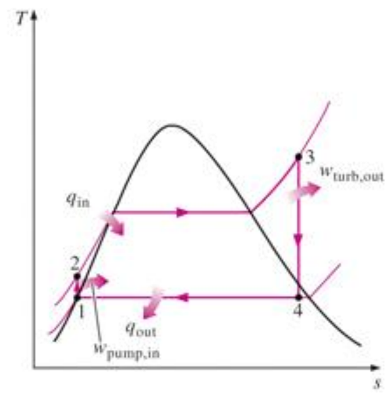
W* = 795.1*kJ/s or kW Ans.

Rankine Cycle

Rankine cycle is the thermodynamic cycle for steam power plant. The various processes are

- 1-2: Isentropic compression
- 2-3: Reversible constant pressure heat addition
- 3-4: Isentropic expansion
- 4-1: Reversible constant pressure heat rejection





Work output of turbine $W_T = (h_3 - h_4)$

Work input to pump $W_P = (h_2 - h_1)$

Heat supplied in boiler $Q_s = (h_3 - h_2)$

Efficiency of the cycle $\eta = \dfrac{W_T - W_P}{Qs}$

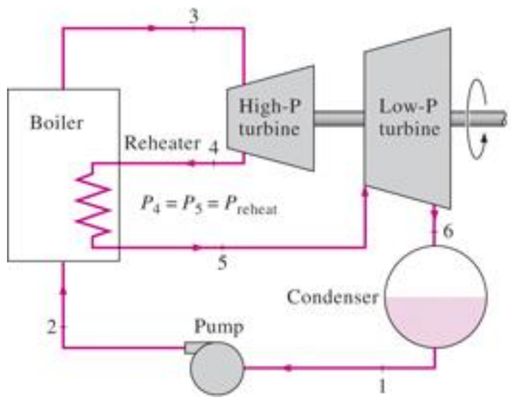
$$\text{Steam rate} = \frac{3600}{W_T - W_P} \text{ kg/kW hr}$$

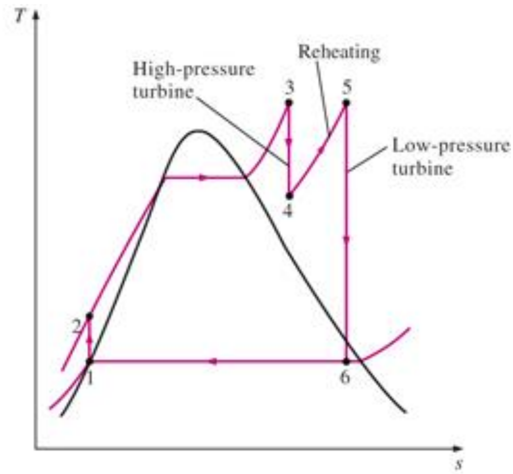
$$\text{Heat rate} = \frac{3600 \times Qs}{W_T - W_P} \text{ kJ/kW hr}$$

Reheat Cycle

The various processes in reheat cycle are:

- 1-2: Isentropic compression
- 2-3: Reversible constant pressure heat addition
- 3-4: Isentropic expansion in HP turbine
- 4-5: Reheating process
- 5-6: Isentropic expansion in LP turbine
- 6-1: Reversible constant pressure heat rejection





Work output of turbine $W_T = (h_3 – h_4) + (h_5 – h_6)$

Work input to pump $W_P = (h_2 – h_1)$

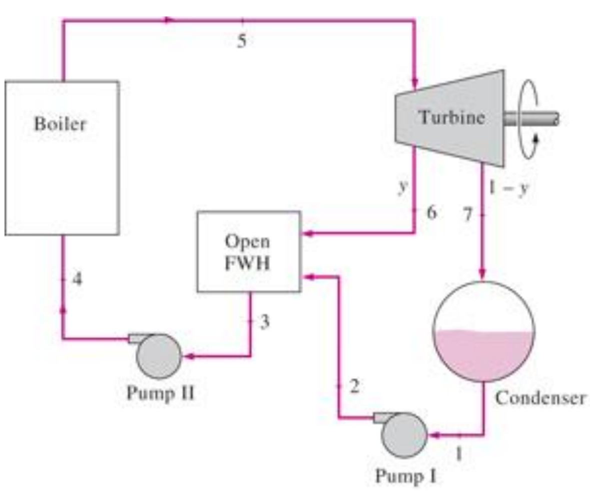
Heat supplied in boiler $Q_s = (h_3 – h_2) + (h_5 – h_4)$

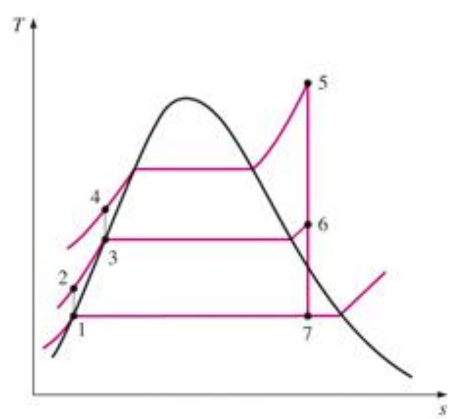
Efficiency of the cycle $\eta = \frac{W_T - W_P}{Qs}$

$$\text{Steam rate} = \frac{3600}{W_T - W_P} \text{ kg/kW hr}$$

$$\text{Heat rate} = \frac{3600 \times Qs}{W_T - W_P} \text{ kJ/kW hr}$$

Regenerative Cycle





The various processes are

1-2: Isentropic compression in Pump I

2-3: Reversible constant pressure heat addition in regenerator

3-4: Isentropic compression in Pump II
4-5: Reversible constant pressure heat addition in boiler
5-7: Isentropic expansion
7-1: Reversible constant pressure heat rejection

Work output of turbine $W_T = (h_5 - h_6) + (1 - y)(h_6 - h_7)$
Work input to pump $W_P = (1 - y)(h_2 - h_1) + (h_4 - h_3)$
Heat supplied in boiler $Q_s = (h_5 - h_4)$
Energy balance for regenerator $y h_6 + (1 - y) h_2 = h_3$ Where y = bleed steam

Efficiency of the cycle $\eta = \frac{W_T - W_P}{Qs}$

Steam rate $= \frac{3600}{W_T - W_P}$ kg/kW hr

Heat rate $= \frac{3600 \times Qs}{W_T - W_P}$ kJ/kW hr