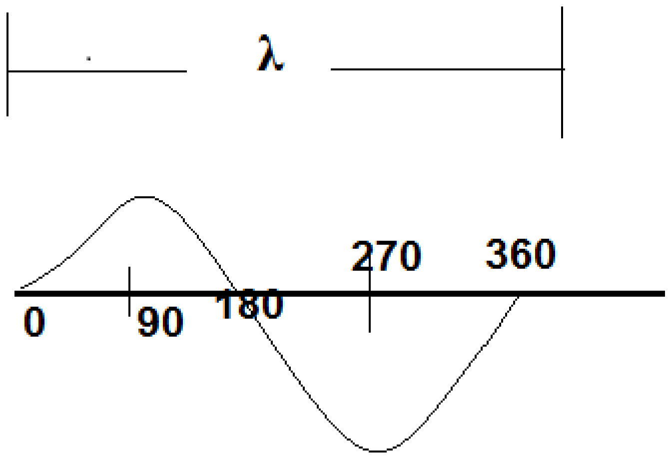
WAVELENGTH AND VELOCITY OF PROPAGATION

WAVELENGTH:



The distance with which the wave changes its phase by 2π radians is known as wavelength.

The distance the wave travels along the line where the phase angle is changing to 2π radians is known as wavelength.

It is denoted by λ,

$\lambda = \frac{2\pi}{\beta}$ ……(1)

and also we know,

$\lambda = \frac{v}{f}$

$v = \lambda . f$

v- velocity

f- frequency

$v = \frac{2\pi}{\beta} . f$

$v = \frac{\omega}{\beta}$ ………(2)

[$\lambda = \frac{2\pi}{\beta}$]

[$\omega = 2\pi f$]

VELOCITY OF PROPAGATION:

The velocity of propagation along the line depends on the change in the phase along the line. Therefore, this velocity is called phase velocity or wave velocity.

$\gamma = \ln\left(\frac{V_1}{V_2}\right) = \ln\left(\frac{I_1}{I_2}\right)$

In general,

$\gamma = \alpha + j\beta$

$\gamma = \sqrt{ZY}$ ………..(1)

where,

$Z = R + j\omega L$

$Y = G + j\omega C$ ………(2)

Sub equ (2) in equ (1)

$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$

$\gamma = \sqrt{RG + j\omega RC + j\omega LG - \omega^2 LC}$

$\alpha + j\beta = \sqrt{RG - \omega^2 LC + j(\omega RC + \omega LG)}$

Squaring on both sides,

$(\alpha + j\beta)^2 = RG - \omega^2 LC + j(\omega RC + \omega LG)$

$\alpha^2 + \beta^2 - 2j\alpha\beta = RG - \omega^2 LC + j(\omega RC + \omega LG)$

Equating real and imaginary parts,

$\alpha^2 + \beta^2 = RC - \omega^2 LG$ ………(3)

$2\alpha\beta = \omega RC + \omega LG$

$2\alpha\beta = \omega(RC + LG)$ ……….(4)

From equ (3),

$\alpha^2 = RG - \omega^2 LC + \beta^2$ ………(5)

Squaring equ (4),

$4\alpha^2\beta^2 = \omega^2 (RC + LG)^2$ ………(6)

Sub equ (5) in equ (6)

$4(RG - \omega^2 LC + \beta^2)\beta^2 = \omega^2 (RC + LG)^2$

$$4(RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4) = \omega^2 (RC + LG)^2$$

$$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 = \frac{\omega^2}{4}(RC + LG)^2$$

$$RG\beta^2 - \omega^2 LC\,\beta^2 + \beta^4 - \frac{\omega^2}{4}(RC + LG)^2 = 0$$

$$\beta^4 + \beta^2(RG - \omega^2 LC) - \frac{\omega^2}{4}(RC + LG)^2 = 0 \qquad \text{.......(7)}$$

The above equation os of the form of ax^4+bx^2+c = 0

$$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a=1, b= $RG - \omega^2 LC$, c= - $\frac{\omega^2}{4}(RC + LG)^2$

$$\beta^2 = \frac{-(RG - \omega^2 LC) \pm \sqrt{(RG - \omega^2 LC)^2 - 4\left(-\frac{\omega^2}{4}(RC+LG)^2\right)}}{2}$$

Neglect the negative value,

$$\beta^2 = \frac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\beta = \sqrt{\frac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \qquad \text{........(8)}$$

Sub β^2 value in equ (5),

$$\alpha^2 = RG - \omega^2 LC + \left(\frac{(\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}\right)$$

$$\alpha^2 = \frac{2\,(RG - \omega^2 LC) + (\omega^2 LC - RG) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha^2 = \frac{(RG - \omega^2 LC) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}$$

$$\alpha = \sqrt{\frac{(RG - \omega^2 LC) + \sqrt{(RG - \omega^2 LC)^2 + (\omega^2 (RC+LG)^2)}}{2}} \qquad \text{......(9)}$$

In a perfectly matched line R=0 and G=0,

Sub the above condition in β.

From equ (8),

$$\beta = \sqrt{\frac{(\omega^2 LC) + \sqrt{(-\,\omega^2 LC)^2}}{2}}$$

$\beta = \omega \sqrt{LC}$

The velocity of propagation of a ideal line is,

$v = \frac{\omega}{\beta}$

$v = \frac{\omega}{\omega \sqrt{LC}}$

$v = \frac{1}{\sqrt{LC}}$

The velocity of propagation is constant for a given L and C.