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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

VII Semester

AU3008 Sensors and Actuators

UNIT – I - INTRODUCTION TO MEASUREMENTS AND SENSORS

1.5 Limiting error- Probable Error-Propagation of error- Odds and uncertainty

1.5.1 Limiting Error:

- The accuracy of a measuring instrument is usually specified by its manufacturer as a percentage of the full-scale reading.
- However, the percentage error for various values of readings taken will be more than the percentage specified.
- The **limiting error** is defined as the maximum deviation in the reading. For example, the accuracy of a 0-150°C thermometer is specified as ± 1 % of full-scale reading. The limiting error of this thermometer is ± 1.5°C.
- Hence, when the thermometer reads 60 °C, the maximum possible deviation is ± 1.5°C and the percentage error at this reading is ± 1.5/60 x 100 = ± 2.5%. Limiting error is also called as **guarantee error**.
- The magnitude of a quantity having a nominal value As and a maximum error or limiting error of $\pm\delta A$ must have a magnitude Aa between the limits As - δA and As + δA or Actual value of quantity

$$Aa = As \pm \delta A$$

- For example, the nominal magnitude of a resistor is 100Ω with a limiting error of ± 10Ω. The magnitude of the resistance will be between the limits

Aa = 100 ±10Ω or

Aa > 90 Ω and **A**a <110 Ω

- In other words, the manufacturer guarantees that the value of resistance of the resistor lies between 90 Ω and 110 Ω.

- **RELATIVE (FRACTIONAL) LIMITING ERROR**:

  The relative (fractional) error is defined as the ratio of the error to the specified (nominal) magnitude of a quantity. Therefore

$$\text{Relative limiting error } \varepsilon_r = \frac{\delta A}{A_s} = \frac{\varepsilon_0}{A_s}$$

$$\varepsilon_0 = \delta A = \varepsilon_r A_s$$

$$A_a = A_s \pm \delta A = A_s \pm \varepsilon_r A_s = A_s(1 \pm \varepsilon_r)$$

$$\text{Percentage limiting error } \% \varepsilon_r = \varepsilon_r \times 100$$

In the example considered in we have

As =100 Ω and δA = ±10 Ω

.'. Relative limiting error $\varepsilon_r = \frac{\delta A}{A_s} = \pm \frac{10}{100} = \pm 0.1$

Percentage limiting error $\% \varepsilon_r = 0.1 \times 100 = \pm 10\%$

- In limiting errors the specified quantity *As* is taken as the true quantity, and the quantity which has the maximum deviation from *Aa* is taken as the erroneous quantity. Thus, we have

$$\delta A = A_a - A_s$$

$$\therefore \text{ Relative limiting error, } \varepsilon_r = \frac{A_a - A_s}{A_s}$$

$$= \frac{\text{actual value} - \text{nominal value}}{\text{nominal value}}$$

Problem 1: Calculate the limiting error of an Ammeter of range 0-25 A given also that it has a guaranteed accuracy of 1 % of full-scale reading. The current measured by the ammeter is 5A.

Full scale reading = 25 A

Guaranteed accuracy error = 1% of full-scale reading = 0.01 × 25 = 0.25 A

Measured value = 5 A

Limiting error $= \frac{0.25}{5} \times 100 = 5\%$

Problem 2: The value of capacitance of a capacitor is specified as 1 pF ±5% by the manufacturer. Find the limits between which the value of the capacitance is guaranteed.

The guaranteed value of the capacitance lie within the limits :

$$A_a = A_s(1 \pm \varepsilon_r) = 1 \times (1 \pm 0.05) = 0.95 \text{ to } 1.05\ \mu\text{F.}$$

Problem 3: A 0-150 V voltmeter has a guaranteed accuracy of 1 percent of full scale reading. The voltage measured by this instrument is 75 V. Calculate the limiting error in percent. Comment upon the result

The magnitude of limiting error of instrument is

$$\delta A = \varepsilon_r A_s = 0.01 \times 150 = 1.5 \text{ V}$$

The magnitude of the voltage being measured is 75 V. The relative error at this voltage is

The relative error at this voltage is

$$\varepsilon_r = \frac{\delta A}{A_s} = \frac{1.5}{75} = 0.02$$

$$A_a = A_s(1 \pm \varepsilon_r)$$
$$= 75(1 \pm 0.02)\text{V} = 75 \pm 1.5 \text{ V}$$

The percentage limiting error is

$$\% \varepsilon_r = \frac{(1.5)}{75} \times 100 = 2 \text{ percent.}$$

Propagation of Errors :

- When the readings from a number of instruments are used to compute some quantity, the overall limiting error should be computed from the individual limiting errors of the instruments.
- Propagation of error refers to the methods used to determine how the uncertainty in a calculated result is related to the uncertainties in the individual measurements.

When the readings from a number of instruments are used to compute some quantity, the overall limiting error should be computed from the individual limiting errors of the instruments.

Consider the problem of computing a quantity α from n independent measurements $u_1, u_2, \ldots, u_n$. Now α can be expressed as

$$\alpha = f(u_1, u_2, \ldots\ldots, u_n) \qquad (1.1)$$

.

Let the limiting errors in u_1, u_2, ..., u_n be $\pm \Delta u_1$, $\pm \Delta u_2$, ... , $\pm \Delta u_n$ respectively. These errors lead to an error of $\pm \Delta \alpha$ in the computed value of α. Therefore,

$$\alpha \pm \Delta \alpha = f(u_1 \pm \Delta u_1,\ u_2 \pm \Delta u_2, \ldots, u_n \pm \Delta u_n) \quad (1.2)$$

Expanding the function f in equation (1.2) using Taylor's series, we get

$$f(u_1 \pm \Delta u_1,\ u_2 \pm \Delta u_2, \ldots, u_n \pm \Delta u_n)$$
$$= f(u_1, u_2, \ldots u_n) + \Delta u_1 \frac{\partial f}{\partial u_1} + \Delta u_2 \frac{\partial f}{\partial u_2} + \ldots \Delta u_n \frac{\partial f}{\partial u_n}$$
$$+ \frac{1}{2}\left[\Delta u_1^2 \frac{\partial^2 f}{\partial u_1} + \Delta u_2^2 \frac{\partial^2 f}{\partial u_2} + \ldots + \Delta u_n^2 \frac{\partial^2 f}{\partial u_n}\right] \quad (1.3)$$

In actual practice, the limiting errors Δu_1, Δu_2 .... etc. are small quantities and hence the higher powers of Δu's are negligible. Hence equation 1.3 becomes

$$f(u_1 + \Delta u_1, u_2 + \Delta u_2, \ldots u_n + \Delta u_n)$$
$$= f(u_1, u_2, \ldots u_n) + \Delta u_1 \frac{\partial f}{\partial u_1} + \Delta u_2 \frac{\partial f}{\partial u_2} + \ldots \Delta u_n \frac{\partial f}{\partial u_n} \quad (1.4)$$

Hence the absolute error Δ is given by

$$\Delta = \left|\Delta u_1 \frac{\partial f}{\partial u_1}\right| + \left|\Delta u_2 \frac{\partial f}{\partial u_2}\right| + \ldots + \left|\Delta u_n \frac{\partial f}{\partial u_n}\right| \quad (1.5)$$

The absolute value signs are used because some of the partial derivatives might be negative and for a positive Δu such a term would reduce the total error.

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When the Δu's are considered not as absolute limits of error, but rather as statistical bounds such as $\pm 3\sigma$ limits, probable errors, or uncertainties, the formulae for computing overall errors must be modified. It can be shown that the proper method of combining such errors is according to the root-sum square (r.s.s) formula.

$$E_{a_{rss}} = \sqrt{\left(\Delta u_1 \frac{\partial f}{\partial u_1}\right)^2 + \left(\Delta u_2 \frac{\partial f}{\partial u_2}\right)^2 + \ldots + \left(\Delta u_n \frac{\partial f}{\partial u_n}\right)^2}$$

ODDS and uncertainties:

- The *probability of occurrence* can be stated in terms of ODDS.
- Odds is the *number of chances* that a particular reading will occur when the error limit is specified.
- For example, if the error limits are specified as ±0.6745σ
- The chances are that 50% of the observations will lie between the above limits or in other words we can say that odds are 1 to 1.
- The odds can be calculated as under

$$\frac{odds}{odds + 1} = \text{Probability of occurence}$$

The odds that lie observation lies between ± σ limits are

$$\frac{odds}{odds + 1} = 0.6828$$

$odds$ *are 2.15 :1*

Table gives the deviations, the probability of occurrence and the odds

Deviations	Probability	Odds
±0.6145σ	**0.5000**	**1 to 1**
± σ	**0.6828**	**2.15 to 1**
± 2σ	**0.9546**	**21 to 1**
± 3σ	**0·9974**	**256 to 1**

- ❒ The odds are defined as the probability that the event will occur divided by the probability that the event will not occur.
- ❒ If the probability of an event occurring is Y, then the probability of the event not occurring is 1-Y.
- ❒ (Example: If the probability of an event is 0.80 (80%), then the probability that the event will not occur is 1-0.80 = 0.20, or 20%.
- ❒ The odds of an event represent the ratio of the (probability that the event will occur) / (probability that the event will not occur). This could be expressed as follows:

Odds of event = Y / (1-Y)

- ❒ So, in this example, if the probability of the event occurring = 0.80, then the odds are 0.80 / (1-0.80) = 0.80/0.20 = 4 (i.e., 4 to 1).

Examples:

1. If the horse runs 100 races and wins 80, the probability of winning is 80/100 = 0.80 or 80%, and the odds of winning are 80/20 = 4 to 1.
2. If the horse runs 100 races and wins 50, the probability of winning is 50/100 = 0.50 or 50%, and the odds of winning are 50/50 = 1 (even odds).
3. If the horse runs 100 races and wins 5 and loses the other 95 times, the probability of winning is 0.05 or 5%, and the odds of the horse winning are 5/95 = 0.0526.

Uncertainties:

- ❒A measure of range of measurements from the average. Also called *deviation* or *error*.

What is uncertainty of measurement?

- Uncertainty of measurement is the doubt that exists about the result of any measurement.
- There is always a margin of doubt.

Expressing uncertainty of measurement

- How big is the margin?' and
- 'How bad is the doubt?'

Thus, two numbers are really needed in order to quantify an uncertainty

One is the width of the margin, or interval. The other is a confidence level, and states how sure we are that the 'true value' is within that margin.

For example: We might say that the length of a certain stick measures 20 centimetres plus or minus 1 centimetre, at the 95 percent confidence level. This result could be written:

20 cm ±1 cm, at a level of confidence of 95%.

The statement says that we are 95 percent sure that the stick is between 19 centimetres and 21 centimetres long

Error versus uncertainty:

- It is important not to confuse the terms 'error' and 'uncertainty'.
- **Error** is the difference between the measured value and the 'true value' of the thing being measured.
- **Uncertainty** is a quantification of the doubt about the measurement result.
- Whenever possible we try to correct for any known errors:

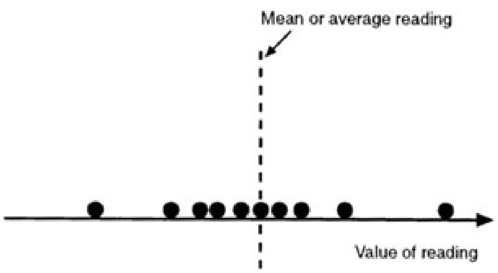
for example, by applying corrections from calibration certificates. But any error whose value we do not know is a source of uncertainty

The general kinds of uncertainty in any measurement:

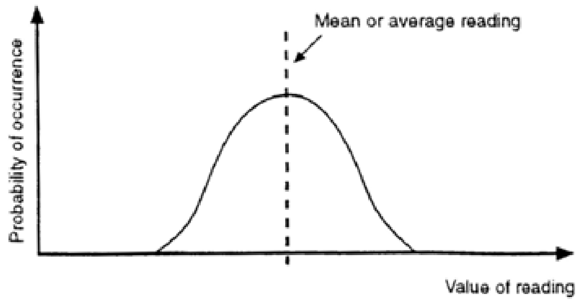
Random or systematic:

- **Random** - where repeating the measurement gives a randomly different result. If so, the more measurements you make, and then average, the better estimate you generally can expect to get.
- **systematic** - where the same influence affects the result for each of the repeated measurements (but you may not be able to tell). In this case, you learn nothing extra just by repeating measurements. Other methods are needed to estimate uncertainties due to systematic effects, e.g. different measurements, or calculations

Distribution - the 'shape' of the errors



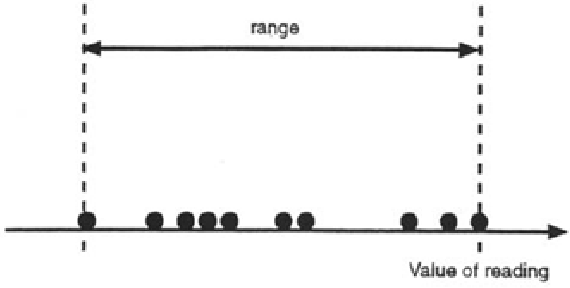
'Blob plot' of a set of values lying in a normal distribution



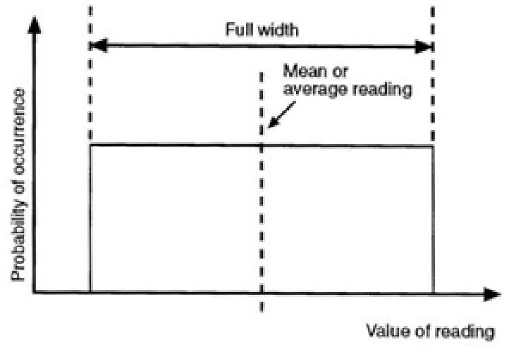
Sketch of a 'normal' distribution

Uniform or rectangular distribution

- When the measurements are quite evenly spread between the highest and the lowest values, a rectangular or uniform distribution is produced.
- Figure shows a set of 10 'random' values in an approximately rectangular distribution. A sketch of a rectangular distribution is shown in Figure.



'Blob plot' of a set of values lying in a rectangular distribution



Sketch of a rectangular distribution.

<u>The two ways to estimate uncertainties</u>:

'Type A' and 'Type B' evaluations

- Type A evaluations - uncertainty estimates using statistics (usually from repeated readings)

- Type B evaluations - uncertainty estimates from any other information. This could be information from past experience of the measurements, from calibration certificates, manufacturer's specifications, from calculations, from published information, and from common sense.

<u>Calculating standard uncertainty for a Type A evaluation</u>

- When a set of several repeated readings has been taken (for a Type A estimate of uncertainty), the mean, x , and estimated standard deviation, s, can be calculated for the set.
- From these, the estimated standard uncertainty, u, of the mean is calculated from:

$$u = \frac{s}{\sqrt{n}}$$

where n was the number of measurements in the set.

(*The standard uncertainty of the mean has historically also been called the standard deviation of the mean, or the standard error of the mean.*)

Calculating standard uncertainty for a Type B evaluation

- Where the information is more scarce (in some Type B estimates), you might only be able to estimate the upper and lower limits of uncertainty.
- You may then have to assume the value is equally likely to fall anywhere in between, i.e. a rectangular or uniform distribution.
- The standard uncertainty for a rectangular distribution is found from:

$$\frac{a}{\sqrt{3}},$$

where a is the semi-range (or half-width) between the upper and lower limits.

Combining standard uncertainties

Suppose you make two measurements,

x = xbest ± Δx

y = ybest ± Δy

What is the uncertainty in the quantity

q = x + y or

q = x – y?

To obtain the uncertainty we will find the lowest and highest probable value of q = x + y.

Note that we would like to state q in the standard form of

q = qbest ± Δq where qbest = xbest + ybest.

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