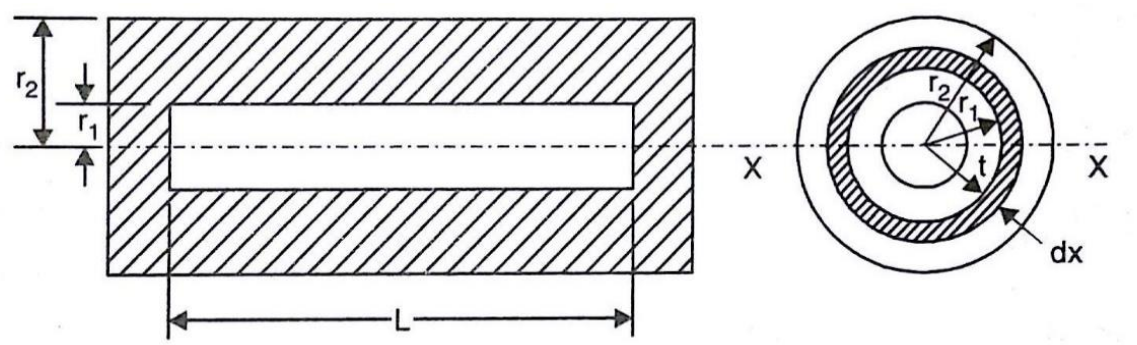
5.13. INTRODUCTION OF THICK CYLINDER

In the last chapter, we have mentioned that if the ratio of thickness to internal diameter of a cylindrical shell is less than about 1/20,the cylinder shell is know as thin cylinders. For them it may be assumed with reasonable accuracy that the hoop and longitudinal stresses are constant over the thickness and the radial stress is small and can be neglected. If the ratio of thickness to internal diameter is more than 1/20,then cylinder shell is know as thick cylinders.

The hoop stress in case of a thick cylinder will not be uniform across the thickness. Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

5.14. STRESSES IN A THICK CYLINDERICAL SHELL

Fig. shows a thick cylinder subjected to an internal fluid pressure.



Let r_2 = External radius of the cylinder,

r_1 = Internal radius of the cylinder, and

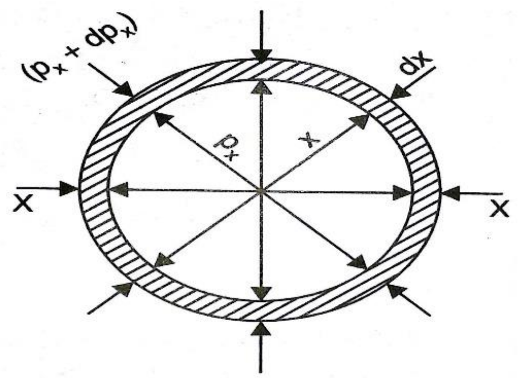
L = Length of cylinder.

Consider an elementary ring of the cylinder of radius x and thickness dx as shown in the figure

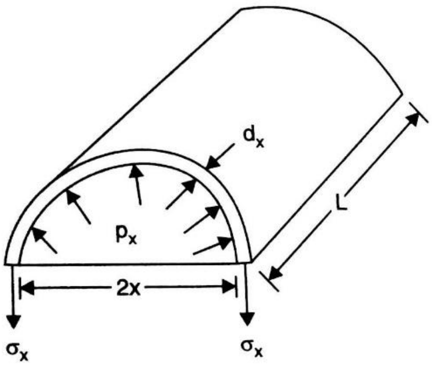
Let p_x = Radial pressure on the inner surface of the ring

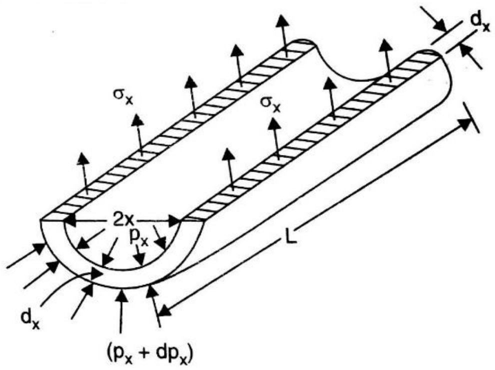
p_x + dp_x = Radial pressure on the outer surface of the ring

σ_x = Hoop stress induced in the ring.



Take a longitudinal section x-x and consider the equilibrium of half of the ring of figure.





Bursting force

$= p_x$ (2xL) – (p_x+ dp_x) × 2(x + dx). L

= 2L [p_x. x – (p_x.x + p_x.dx + x . dp_x + dp_x. dx)]

=2L [- p_x.dx - x . dp_x]

= - 2L (p_x.dx + x . dp_x) …. (1)

Resisting force = Hoop stress × Area on which it acts = σ_x × 2dx.L …. (2)

Equating the resisting force to the bursting force, we get

σ_x × 2dx.L = - 2L (p_x.dx + x . dp_x)

Or $\sigma_x = -p_x - x\,\frac{dp_x}{dx}$ …. (3)

The longitudinal strain at any point in the section is constant and is independent of the radius. This mean that cross- sections remain plane after straining and this is true for sections, remote from any end fixing. As longitudinal strain is constant, hence longitudinal stress will also be constant.

Let σ_2 = Longitudinal stress.

Hence at any point at a distance x from the centre, three principle stresses are acting :

They are :

1. the radial compressive stress, p_x
2. the hoop (or circumferential) tensile stress, σ_x
3. the longitudinal tensile strain σ_2.

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But longitudinal strain is constant

$$\frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant.

$$\sigma_x - p_x = \text{constant}$$

$$= 2a \text{ where a is constant}$$

$$\sigma_x = p_x + 2a \quad \text{.... (4)}$$

Equating the two values of σ_x given by equation (3) and (4), we get

$$p_x + 2a = -p_x - x\frac{dp_x}{dx}$$

$$x\frac{dp_x}{dx} = -p_x - p_x - 2a = -2p_x - 2a$$

$$\frac{dp_x}{dx} = -\frac{2p_x}{x} - \frac{2a}{x} = \frac{-2(p_x+a)}{x}$$

$$\frac{dp_x}{(p_x+a)} = -\frac{2d_x}{x}$$

Integrating the above equation, we get

$$\log_e(p_x + a) = -\log_e x^2 + \log_e b$$

$$= \log_e \frac{b}{x^2}$$

$$p_x + a = \frac{b}{x^2}$$

$$p_x = \frac{b}{x^2} - a \quad \text{.... (5.1)}$$

Substituting the values of p_x in equation (4), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a \quad \text{.... (5.2)}$$

Equation (5.1) gives the radial pressure p_x and equation (5.2) gives the hoop stress at any radius x. These two equations are called Lame's equations. The constants 'a' and 'b' are obtained from boundary conditions, which are :

i. at x = r_1, $p_x = p_0$ or the pressure of fluid inside the cylinder, and
ii. at x = r_2, $p_x = 0$ or atmosphere pressure.

After knowing the value of 'a' and 'b' , the hoop stress can be calculated at any radius

Problem 5.24: Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2. Also sketch the radial pressure distribution and hoop stress distribution across the section.

Sol. Given:

Internal dia, = 400 mm

Internal radius, $r_1 = \frac{400}{2} = 200$ mm

Thickness, = 100 mm

External dia, = 400 +2 × 100 = 600 mm

External radius, $r_2 = \frac{600}{2} = 300$ mm

Fluid pressure, $p_0 = 8$ N/mm^2

or at x = r_1, $p_x = p_0 = 8$ N/mm^2

The radial pressure (p_x) is given by equation (18.1) as

$$p_x = \frac{b}{x^2} - a \qquad \text{.... (1)}$$

Now apply the boundary conditions to the above equation. The boundary conditions are : At x = r_1 = 200 mm , $p_x = 8$ N/mm^2

1) At x = r_2 = 300 mm , $p_x = 0$

Substituting these boundary conditions in equation (1), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \qquad \text{.... (2)}$$

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \qquad \text{.... (3)}$$

and

Subtracting equation (3) from equation (2), we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b-4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 576000$$

Substituting this value in equation (3), we get

$$0 = \frac{576000}{90000} - a \text{ or } a = \frac{576000}{9000} = 6.4$$

The values of 'a' and 'b' are substituted in the hoop stress.

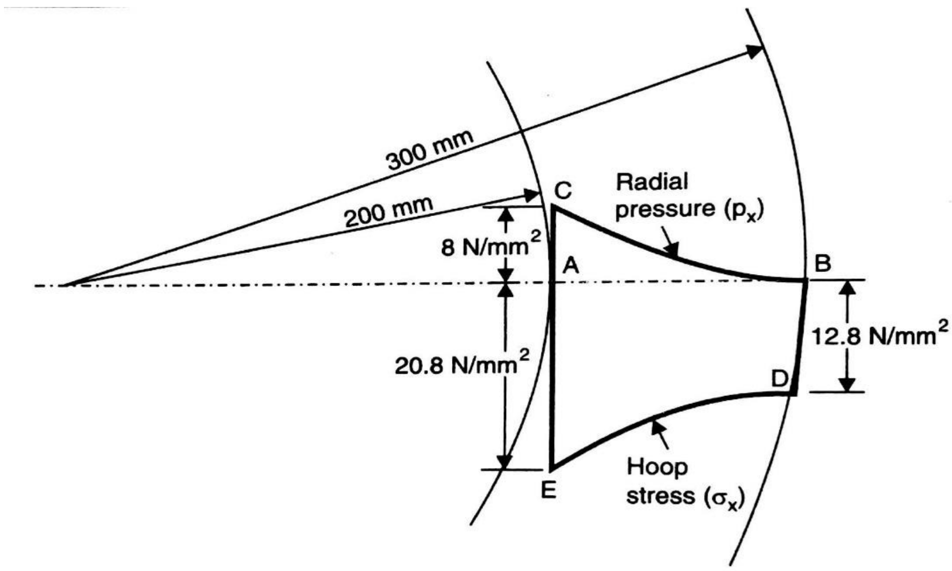
Now hoop stress at any radius x is given by equation as

$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

At x = 200 mm, $\sigma_{200} = \frac{576000}{200^2} + 6.4$ = 14.4 + 6.4 = **20.8 N/mm^2.Ans.**

At x = 300 mm, $\sigma_{300} = \frac{576000}{300^2} + 6.4$ = 6.4 + 6.4 = **12.8 N/mm^2.Ans.**

Figure shows the radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = 8 N/mm^2 . The variation between B and



C is parabolic. The curve BC shows the variation of radial pressure across AB.

The curve DE which is also parabolic, shows the variation of hoop stress across AB. Values BD = 12.8 N/mm^2. The radial pressure is compressive whereas the hoop stress is tensile.

Problem 5.25: Find the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N/mm^2. The maximum hoop stress in the section is not to exceed 35 N/mm^2.

Sol. Given :

Internal dia, = 160 mm

Internal radius, $r_1 = \frac{160}{2}$ = 80 mm

Internal pressure, = 8 N/mm^2

This means at x =80 mm, p_x= 8 N/mm^2

Maximum hoop stress, σ_x = 35 N/mm^2

The maximum hoop stress is at the inner radius of the shell.

Let r_2 = External radius.

The radial pressure and hoop stress at any radius x are given by equation (18.1) and (18.2) as

$$p_x = \frac{b}{x^2} - a \quad \text{.... (1)}$$

and

$$\sigma_x = \frac{b}{x^2} + a \quad \text{.... (2)}$$

Let us now apply the boundary conditions. The boundary conditions are :

At x = 80 mm, p_x = 8 N/mm^2 and σ_x = 35 N/mm^2

Substituting x = 80 mm and p_x = 8 N/mm^2 in equation (1),we get

$$8 = \frac{b}{80^2} - a = \frac{b}{6400} - a \quad \text{.... (3)}$$

Substituting x = 80 mm and σ_x = 35 N/mm^2 in equation (2),we get

$$35 = \frac{b}{80^2} + a = \frac{b}{6400} + a \quad \text{.... (4)}$$

Subtracting equation (3) from equation (4),we get

$$27 = 2a \quad \text{or} \quad a = \frac{27}{2} = 13.5$$

Substituting the value of a in equation (3),we get

$$8 = \frac{b}{6400} - 13.5$$

$$b = (8 + 13.5) \times 6400 = 21.5 \times 6400$$

Substituting the values of ‘a’ and ‘b’ in equation (1),

$$p_x = \frac{21.5 \times 6400}{x^2} - 13.5$$

But at the outer surface, the pressure is zero. Hence at x = r_2, $p_x = 0$.Substituting these values in the above equation , we get

$$0 = \frac{21.5 \times 6400}{r_2^2} - 13.5$$

$$r_2^2 = \frac{21.5 \times 6400}{13.5} \quad \text{or} \quad r_2 = \sqrt{\frac{21.5 \times 6400}{13.5}} = 100.96 \text{ mm}$$

Thickness of the shell, t = $r_2 - r_1$

= 100.96 – 80 = **20.96 mm.Ans.**

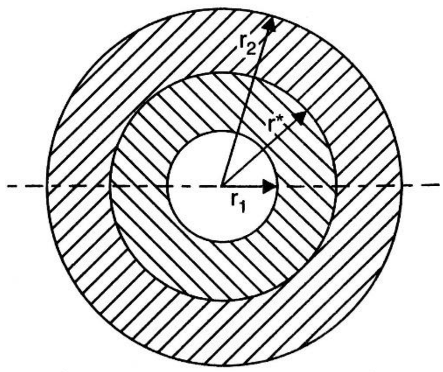
5.15. STRESSES IN COMPOUND THICK CYLINDERS

From the problem, we find that the hoop stress is maximum at the inner radius and it decreases towards the outer radius. The hoop stress is tensile in nature and it is caused by the internal fluid pressure inside the cylinder. This maximum hoop stress at the inner radius is always greater than the internal fluid pressure. Hence the maximum fluid pressure inside the cylinder is limited corresponding to the condition that the hoop stress at the inner radius reaches the permissible value. In case of cylinders which have to carry high internal fluid pressure, some methods of reducing the hoop stress have to be devised.

One method is to wind* strong steel wire under tension on the cylinder. The effect of the wire is to put the cylinder wall under an initial compressive stress.

Another method is to shrink one cylinder over the other. Due to this , the inner cylinder will be put into initial compression whereas the outer cylinder will be put into initial tension. If now the compound cylinder is subjected to internal fluid pressure, both the inner and outer cylinders will be subjected to hoop tensile stress. The net effect of the Initial stresses due to shrinking and those due to internal fluid pressure is to make the resultant stresses more or less uniform.

Figure shows a compound thick cylinder made up of two cylinders.



Let r_2 = Outer radius of compound cylinder

r_1 = Inner radius of compound cylinder

r^*= Radius at the junction of the two cylinders

(i.e.., outer radius of inner cylinder or inner radius of outer

cylinder)

p^* = Radial pressure at the junction of the two cylinders.

Let us now apply Lame's equation for the initial conditions (i.e., after shrinking the outer cylinderover the inner cylinder and fluid under pressure is not admitted into the inner cylinder).

1) For outer cylinder

The Lame's equation at a radius x for outer cylinder are given by

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (1)} \qquad \sigma_x = \frac{b_1}{x^2} + a_1 \text{.... (2)}$$

where a_1,b_1 are constants for outer cylinder.

At x = r_2, p_x = 0. And at x = r*, $p_x = p*$

Substituting these conditions in equation (1), we get

$$0 = \frac{b_1}{r_2^2} - a_1 \quad \text{.... (3)} \qquad p* = \frac{b_1}{r*^2} + a_1 \text{ .... (4)}$$

From equation (3) and (4), the constants a_1 and b_1 can be determined. These values are substituted in equation (2). And then hoop stresses in the outer cylinder due to shrinking can be obtained.

2) For inner cylinder

The Lame's equations for inner cylinder at a radius x are given by

$$p_x = \frac{b_2}{x^2} - a_2 \ , \sigma_x = \frac{b_2}{x^2} + a_2$$

Where a_2, b_2 are constants for inner cylinder.

At x = r_1, $p_x = 0$ as fluid under pressure is not admitted into the inner cylinder. And at x = r* , $p_x = p*$.

Substituting these values in the above value of p_x, we get

$$0 = \frac{b_2}{x^2} - a_2 \quad \text{.... (5)} \quad \text{and} \quad p* = \frac{b_2}{r*^2} + a_2 \quad \text{.... (6)}$$

From equation (5) and (6) , the constants a_2 and b_2 can be determined. These values are substituted in σ_x. And then hoop stresses are obtained.

Hoop stresses in compound cylinder due to internal fluid pressure alone

When the fluid under pressure is admitted into the compound cylinder, the hoop stresses are set in the compound cylinder. To find these stresses, the inner cylinder and outer cylinder will together be considered as one thick shell, Let p = internal fluid pressure. Now the Lame's equations are applied, which are given by

$$p_x = \frac{B}{x^2} - A \quad \text{.... (7) and} \quad \sigma_x = \frac{B}{x^2} + A \quad \text{.... (8)}$$

where A and B are constants for single thick shell due to internal fluid pressure.

At x = r_2, $p_x = 0$.

Substituting these values in equation (7), we get

$$0 = \frac{B}{r_2^2} - A \quad \text{.... (9)}$$

At $x = r_1,\ p_x = p.$

Substituting these values in equation (7), we get

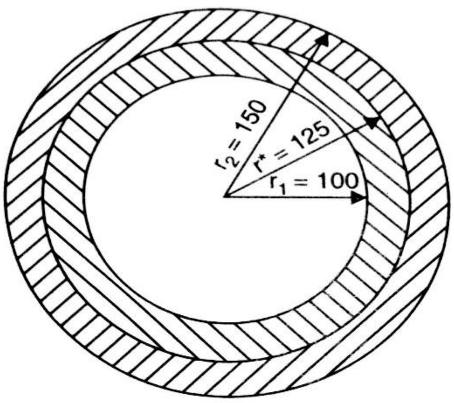
$$p = \frac{B}{r_1^2} - A \qquad \text{.... (10)}$$

From equation (9) and (10), the constants A and B can be determined. These values are substituted in equation (8). And then hoop stresses across the section can be obtained.

The resultant hoop stresses will be the algebraic sum of the hoop stresses caused due to shrinking and those due to internal fluid pressure.

Problem 5.26: A compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250 mm over another cylinder of external diameter 250 mm and internal diameter 200 mm. The radial pressure at the junction after shrinking is 8 N/mm^2. Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm^2.

Sol. Given :



For outer cylinder :

External diameter = 300 mm

External radius, $r_2 = \frac{300}{2} = 150$ mm

Internal diameter, = 250 mm

Radius at the junction, $r^* = \frac{250}{2} = 125$ mm

For inner cylinder :

Internal diameter, = 200 mm

Internal radius, $r_1 = \frac{200}{2} = 100$ mm

Radial pressure due to shrinking at the junction,

$$P^* = 8 \text{ N/mm}^2$$

Fluid pressure in the compound cylinder, p = 84.5 N/mm^2.

i. Stresses due to shrinking in the outer and inner cylinders before the fluid pressure is admitted.

(a) Lame's equations for outer cylinders are :

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (1)} \quad \text{and} \quad \sigma_x = \frac{b_1}{x^2} + a_1 \quad \text{.... (2)}$$

At x = 150 mm, p_x = 0.

Substituting these values in equation (1),

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1 \quad \text{.... (3)}$$

At x = r* = 125 mm, $p_x = p* = 8$ N/mm^2.

Substituting these values in equation (1), we get

$$8 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \quad \text{.... (4)}$$

Subtracting equation (3) from equation (4), we get

$$8 = -\frac{b_1}{22500} + \frac{b_1}{15625} = \frac{(-15625+22500)b_1}{22500 \times 15625}$$

$$b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)} = 409090.9$$

Substituting the value of b_1 in equation (3), we get

$$0 = \frac{409090.9}{22500} - a_1 \quad \text{or} \quad a_1 = \frac{409090.9}{22500} = 18.18$$

Substituting the value of a_1 and b_1 in equation (2), we get

$$\sigma_x = \frac{409090.9}{x^2} + 18.18$$

The above equation gives the hoop stress in the outer cylinder due to shrinking. The hoop stress at the outer and inner surface of the outer cylinder is obtained by substituting x = 150 mm and x = 125 mm respectively in the above equation.

$$\sigma_{150} = \frac{409090.9}{150^2} + 18.18 = 36.36 \text{ N/mm}^2 \text{ (tensile)}$$

and

$$\sigma_{125} = \frac{409090.9}{125^2} + 18.18 = 44.36 \text{ N/mm}^2 \text{ (tensile).}$$

(b) Lame's equation for the inner cylinder are :

$$p_x = \frac{b_2}{x^2} - a_2 \quad \text{.... (5)} \quad \text{and} \quad \sigma_x = \frac{b_2}{x^2} + a_2 \quad \text{.... (6)}$$

At x = r_1 = 100 mm , p_x = 0 (There is no fluid under pressure.)

Substituting these values in equation (5), we get

$$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2 \quad \text{.... (7)}$$

At x = r* =125 mm, p_x = P* = 8 N/mm^2. Substituting these values in equation (5), we get

$$8 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2 \qquad \text{.... (8)}$$

Subtracting equation (7) from equation (8), we get

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000}$$

$$= \frac{b_2(10000-15625)}{15625 \times 10000} = \frac{-5625\, b_2}{15625 \times 10000}$$

$$b_2 = -\frac{8 \times 15625 \times 10000}{5625} = -222222.2$$

Substituting the value of b_2 in equation (8), we get

$$0 = -\frac{222222.2}{10000} - a_2$$

$$a_2 = -22.22$$

Substituting the values of a_2 and b_2 in equation (7), we get

$$\sigma_x = -\frac{222222.2}{x^2} - 22.22$$

Hence the hoop stress for the inner cylinder is obtained by substituting x = 125 mm respectively in the above equation.

$$\sigma_{125} = -\frac{222222.2}{125^2} - 22.22$$

$$= -14.22 - 22.22 = -36.44 \text{ N/mm}^2 \text{ (Compressive)}$$

and

$$\sigma_{100} = -\frac{222222.2}{100^2} - 22.22$$

$$= -22.22 - 22.22 = -44.44 \text{ N/mm}^2 \text{ (Compressive)}$$

ii. Stresses due to fluid pressure alone

When the fluid under pressure is admitted inside the compound cylinder, the two cylinders together will be considered as one single unit. The hoop stresses are calculated by Lame’s equations, which are

$$p_x = \frac{B}{x^2} - A \quad \text{.... (9) and} \quad \sigma_x = \frac{B}{x^2} + A \quad \text{.... (10)}$$

Where A and B are constants.

At x = 100 mm , p_x = p = 84.5 N/mm^2. Substituting the values in equation (9),we get

$$84.5 = \frac{B}{100^2} - A = \frac{B}{10000} - A \qquad \text{.... (11)}$$

At x = 150 mm, $p_x = 0$. Substituting the values in equation (9),we get

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A \qquad \text{.... (12)}$$

Subtracting equation (12) from equation (11), we get

$$84.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$= \frac{B(22500 - 10000)}{1000 \times 22500} = \frac{12500 \times B}{10000 \times 22500}$$

$$B = \frac{84.5 \times 10000 \times 22500}{12500} = 1521000$$

Substituting this value in equation (12), we get

$$0 = \frac{1521000}{22500} - A \qquad \text{or } A = \frac{1521000}{22500} = 67.6$$

Substituting the values of A and B in equation (x), we get

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

Hence the hoop stresses due to internal fluid pressure alone are given by,

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 97.344 + 67.6 = 164.94 \text{ N/mm}^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 67.6 + 67.6 = 135.2 \text{ N/mm}^2$$

The resultant stresses will be the algebraic sum of the initial stresses due to shrinking and those due to internal fluid pressure.

Inner cylinder

F_{100} = σ_{100} due to shrinkage + σ_{100} due to internal fluid pressure

= $-$ 44.44 + 219.7 = 175.26 N/mm^2 (tensile). **Ans.**

F_{125} = σ_{125} due to shrinkage + σ_{125} due to internal fluid pressure

= $-$36.44 + 164.94 = 128.5 N/mm^2 (tensile). **Ans.**

Outer cylinder

F_{125} = σ_{125} due to shrinkage + σ_{125} due to internal fluid pressure

= 44.36 + 164.94 = 209.3 N/mm^2 (tensile). **Ans.**

F_{150} = σ_{150} due to shrinkage + σ_{150} due to internal fluid pressure

= 36.36 + 135.2 = 171.56 N/mm^2 (tensile). **Ans.**

5.16. INITIAL DIFFERENCE IN RADII AT THE JUNCTION OF A COMPOUND CYLINDER FOR SHRINKAGE

By shrinking the outer cylinder over the inner cylinder, some compressive stresses are produced in the inner cylinder . In order to shrink the outer cylinder over the inner cylinder, the inner diameter of the outer cylinder should be slightly less than the outer diameter of the inner cylinder. Now the outer cylinder shrinks over the inner cylinder. Thus inner cylinder is put into compression and outer cylinder is put into tension. After shrinking , the outer radius of inner cylinder decreases whereas the inner radius of outer cylinder increases from the initial values.

Let r_2 = Outer radius of the outer cylinder

r_1 = Inner radius of the inner cylinder

r* = Radius of junction after shrinking or it is common radius after shrinking

p* = Radial pressure at the junction after shrinking.

Before shrinking, the outer radius of the inner cylinder is slightly more than r* and inner radius of the cylinder is slightly less than r*.

For the outer and inner cylinder Lame's equation are used. These equations are

$$p_x = \frac{b}{x^2} - a \quad \text{and} \quad \sigma_x = \frac{b}{x^2} + a$$

The values of constants a and b will be different for each cylinder.

Let the constants for inner cylinder be a_2,b_2 and for outer cylinder a_1,b_1.

The radial pressure at the junction (i.e.,p*) is same for outer cylinder and inner cylinder.

At the junction ,x = r* and $p_x = p*$. Hence radial pressure at the junction.

$$P* = p* = \frac{b_1}{r*^2} - a_1 = \frac{b_2}{r*^2} + a_1 \quad \text{.... (A)}$$

or $$\frac{b_1 - b_2}{r*^2} = (a_1 - a_2) \quad \text{.... (B)}$$

or $$(b_1 - b_2) = r*^2(a_1 - a_2)$$

Now the hoop strain (or circumferential strain) in the cylinder at any point

$$= \frac{\sigma_x}{E} + \frac{p_x}{mE} \quad \text{.... (C)}$$

But circumferential strain

$$= \frac{Increase\ in\ circumference}{Original\ Circumference}$$

$$= \frac{2\pi(r+dr) - 2\pi r}{2\pi r} = \frac{dr}{r} \quad \text{.... (D)}$$

= Radial strain

Hence equating the two values of circumferential strain given by equation (C) and (D),we get

$$\frac{dr}{r} = \frac{\sigma_x}{E} + \frac{p_x}{mE} \quad \ldots. (1)$$

On shrinking, at the junction there is extension in the inner radius of the outer cylinder and compression in the outer radius of the inner cylinder.

At the junction where x = r*, increase in the inner radius of outer cylinder

$$= r^* \left(\frac{\sigma_x}{E} + \frac{p_x}{mE}\right) \quad \ldots. (2)$$

But for outer cylinder at the junction, we have

$$\sigma_x = \frac{b_1}{r*^2} + a_1 \text{ and } \quad p_x = \frac{b_1}{r*^2} - a_1$$

Where a_1 and b_1 are constants for outer cylinders.

Substituting the values of σ_x and p_x in equation (2), we get

Increase in the inner radius of outer cylinder

$$= r^* \left[\frac{1}{E}\left(\sigma_x + \frac{p_x}{m}\right)\right] = r^* \left[\frac{1}{E}\left(\frac{b_1}{r*^2} + a_1\right) + \frac{1}{mE}\left(\frac{b_1}{r*^2} - a_1\right)\right]$$

Similarly, decrease in the outer radius of the inner cylinder is obtained from equation (1) as

$$= - \; r^* \left(\frac{\sigma_x}{E} + \frac{p_x}{mE}\right) \quad (-\text{ve sign is due to decrease}) \ldots. (3)$$

But for inner cylinder at the junction , we have

$$\sigma_x = \frac{b_2}{r*^2} + a_2 \text{ and } \quad p_x = \frac{b_2}{r*^2} - a_2$$

Substituting these values in equation (3), we get

Decrease in the outer radius of inner cylinder

$$= -r^* \left[\frac{1}{E}\left(\frac{b_2}{r*^2} + a_2\right) + \frac{1}{mE}\left(\frac{b_2}{r*^2} - a_2\right)\right] \quad \ldots. (4)$$

But the original difference in the outer radius of the inner cylinder and inner radius of the outer cylinder.

= Increase in inner radius of outer cylinder + Decrease in outer radius of the inner cylinder

$$= r^* \left[\frac{1}{E}\left(\frac{b_1}{r*^2} + a_1\right) + \frac{1}{mE}\left(\frac{b_1}{r*^2} - a_1\right)\right] - r^* \left[\frac{1}{E}\left(\frac{b_2}{r*^2} + a_2\right) + \frac{1}{mE}\left(\frac{b_2}{r*^2} - a_2\right)\right]$$

$$= \frac{r*}{E}\left[\left(\frac{b_1}{r*^2} + a_1\right) - \left(\frac{b_2}{r*^2} + a_2\right)\right] + \frac{r*}{mE}\left[\left(\frac{b_1}{r*^2} - a_1\right) - \left(\frac{b_2}{r*^2} - a_2\right)\right]$$

But from equation (A),

$\frac{b_1}{r*^2} - a_1 = \frac{b_2}{r*^2} - a_2.$

Hence second part of the above equation is zero. Hence above equation becomes as

Original difference of radii at the junction

$$= \frac{r*}{E}\left[\left(\frac{b_1}{r*^2} + a_1\right) - \left(\frac{b_2}{r*^2} + a_2\right)\right]$$

$$= \frac{r*}{E}\left[\frac{(b_1 - b_2)}{r*^2} + (a_1 - a_2)\right]$$

$$= \frac{r*}{E}\left[(a_1 - a_2) + (a_1 - a_2)\right] \left[From\ equation\ (B), \frac{b_1 - b_2}{r*^2} = a_1 - a_2\right]$$

$$= \frac{2r*}{E}(a_1 - a_2) \qquad \text{.... (18.3)}$$

The values of a_1 and a_2 are obtained from the given conditions. The value of a_1 is for outer cylinder whereas of a_2 is for inner cylinder.

Problem 5.27: A steel cylinder of 300 mm external diameter is to be shrunk to another steel cylinder of 150 mm internal diameter. After shrinking, the diameter at the junction is 250 mm and radial pressure at the common junction is 28 N/mm^2. Find the original difference in radii at the junction. Take E = 2×10^5 N/mm^2.

Sol. Given:

External dia. of outer cylinder = 300 mm

Radius, r_2 = 150 mm

Internal dia. of inner cylinder = 150 mm

Radius, r_1 = 75 mm

Diameter at the junction = 250 mm

Radius, r* = 125 mm

Radial pressure at the junction, p* = 28 N/mm^2

Value of E = 2×10^5 N/mm^2

Using equation (18.3), we get

Original difference of radii at the junction

$$= \frac{2r*}{E}(a_1 - a_2) \quad \text{.... (1)}$$

First find the values of a_1 and a_2 from the given conditions. These are the constants for outer cylinder and inner cylinder respectively. They are obtained by using Lame's equation.

For outer cylinder $p_x = \frac{b_1}{x^2} - a_1$

1) At junction, x = r* = 125 mm and $p_x = p* = 28$ N/mm^2
2) At x =150 mm , $p_x = 0$.

Substituting these two conditions in the above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \quad \text{.... (2)}$$

and

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1 \quad \text{.... (3)}$$

Solving equation (2) and (3), we get

b_1 = 1432000 and a_1 = 63.6.

For inner cylinder

$$p_x = \frac{b_2}{x^2} - a_2$$

1) At junction, x = r* = 125 mm and $p_x = p* = 28$ N/mm^2
2) At x =75 mm , $p_x = 0$.

Substituting these two conditions in the above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \quad \text{.... (4)}$$

and

$$0 = \frac{b_1}{75^2} - a_1 = \frac{b_1}{5625} - a_1 \quad \text{.... (5)}$$

Solving equation (4) and (5), we get

$b_2 = -246100$ and $a_1 = -43.75$

Now substituting the values of a_2 and a_1 in equation (1),we get

Difference of radii at the junction

$$= \frac{2 \times 125}{2 \times 10^5}[63.6 - (-43.75)]$$

$$= \frac{125}{10^5} \times 107.35 = 0.13 \text{ mm. } \textbf{Ans.}$$

Problem 5.28: A steel tube of 200 mm external diameter is to be shrunk onto another steel tube of 60 mm internal diameter. The diameter at the junction after shrinking is 120 mm. Before shrinking on , the difference of diameters at the junction is 0.08 mm. Calculate the radial pressure at the junction and the hoop stresses developed in the two tubes after shrinking on. Take E as 2×10^5 N/mm^2.

Sol. Given :

External dia. of outer tube = 200 mm

Radius , r_2 = 100 mm

Internal dia. of inner tube = 60 mm

Radius, r_1 = 30 mm

The diameter at the junction after shrinking = 120 mm

Radius, r*= 60 mm

Before shrinking on, the difference of dia. at the junction = 0.08 mm

Difference of original radii = 0.04 mm

Value of E = 2×10^5 N/mm^2

Let p* = Radial pressure at the junction

Using equation (18.3),

Original difference of radii at junction

$$= \frac{2r*}{E}(a_1 - a_2)$$

or $$0.04 = \frac{2 \times 60}{2 \times 10^5}(a_1 - a_2) \quad \text{or} \quad \frac{0.04 \times 2 \times 10^5}{2 \times 60} = (a_1 - a_2)$$

or $$(a_1 - a_2) = \frac{200}{3} \qquad \text{.... (1)}$$

Now using Lame's equation for outer tube

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (2)} \qquad \text{and} \qquad \sigma_x = \frac{b_1}{x^2} + a_1 \quad \text{.... (3)}$$

At x = 100 mm, $p_x = 0$.

Substituting these values in equation (2),

$$0 = \frac{b_1}{100^2} - a_1 = \frac{b_1}{10000} - a_1 \qquad \text{.... (4)}$$

At x = 60 mm, $p_x = p*$.

Substituting these values in equation (2),

$$p^* = \frac{b_1}{60^2} - a_1 = \frac{b_1}{3600} - a_1 \qquad \text{.... (5)}$$

Now applying Lame's equation for inner tube

$$p_x = \frac{b_2}{x^2} - a_2 \quad \text{.... (6)} \quad \text{and} \quad \sigma_x = \frac{b_2}{x^2} + a_2 \qquad \text{.... (7)}$$

At x = 30 mm, $p_x = 0$.

Substituting these values in equation (6),

$$0 = \frac{b_2}{30^2} - a_2 = \frac{b_2}{900} - a_2 \qquad \text{.... (8)}$$

At x = 60 mm, $p_x = p*$.

Substituting these values in equation (6),

$$p^* = \frac{b_2}{60^2} - a_2 = \frac{b_2}{3600} - a_2 \qquad \text{.... (9)}$$

Equating the two values of p*, given by equation (5) and (9)

$$\frac{b_2}{3600} - a_2 = \frac{b_1}{3600} - a_1$$

or $$\frac{b_2 - b_1}{3600} = a_2 - a_1 \qquad \text{.... (10)}$$

But from equation (4), $b_1 = 10000\, a_1$

and, from equation (8), $b_2 = 900\, a_2$

Substituting these values in equation (x), we get

$$\frac{900a_2 - 10000\, a_1}{3600} = a_2 - a_1$$

or $$900a_2 - 10000\, a_1 = 3600a_2 - 3600a_1$$

or $$900a_2 - 3600a_2 = -3600a_1 + 10000\, a_1$$

or $$-2700\, a_2 = 6400\, a_1$$

or $$a_1 = -\frac{2700}{6400}a_2 = -\frac{27}{64}a_2 \qquad \text{.... (11)}$$

Substituting these values of a_1 in equation (1), we get

$$-\frac{27}{64}a_2 - a_2 = \frac{200}{3}$$

or $$-\frac{(27a_2 + 64\, a_2)}{64} = \frac{200}{3} \text{ or } a_2 = -\frac{200 \times 64}{3 \times 91} = -46.88$$

Substituting these values in equation (11), we get

$$a_1 = +\frac{27}{64} \times 46.88 = +19.77$$

$$b_1 = 10000 \times a_1 = 10000 \times (19.77) = 197700$$

$$b_2 = 900 \times a_2 = -900 \times 46.88 = -42192$$

and

1) Radial pressure at the junction (p*)

Substituting these values of a_2 *and* b_2 in equation (9), we get

$$p^* = \frac{b_2}{3600} - a_2 = \frac{42192}{3600} + 46.88$$

$$= \frac{197700}{3600} - 19.77 = 54.916 - 19.77$$

$$= 35.146 \text{ N/mm}^2.$$ **Ans.**

2) Hoop stresses in the two tubes after shrinking on

The hoop stresses can be calculated from equations (3) and (7)

(a) For outer tube

$$\sigma_x = \frac{b_1}{x^2} + a_1 = \frac{197700}{x^2} + 19.77 \quad (b_1 = 197700,\ a_1 = 19.77)$$

$$\sigma_{100} = \frac{197700}{100^2} + 19.77$$

$= 39.54$ N/mm^2 (tensile). **Ans.**

$$\sigma_{60} = \frac{197700}{60^2} + 19.77$$

$= 74.68$ N/mm^2 (tensile). **Ans.**

and

(b) For inner tube

$$\sigma_x = \frac{b_2}{x^2} + a_2 = -\frac{42192}{x^2} - 46.88 \quad (b_2 = -42192, a_2 = -46.88)$$

$$\therefore \quad \sigma_{60} = -\frac{42192}{60^2} - 46.88$$

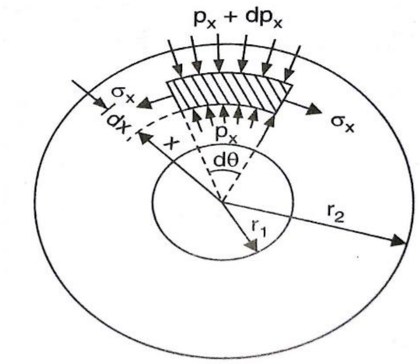
$= -58.6$ N/mm^2 (compressive). **Ans.**

and $\sigma_{30} = -\frac{42192}{30^2} - 46.88$

$= -93.76$ N/mm^2 (compressive). **Ans.**

5.17. THICK SPHERICAL SHELLS

Figure shows a spherical shell subjected to an internal fluid pressure p.



Let r_2 = External radius

r_1 = Internal radius.

Consider an elemental disc of the spherical shell of thickness dx at a radius x. Let this elemental disc subtend an angle dθ at the centre.

Due to internal fluid pressure, let the radius x increase to $(x + u)$ and increase in thickness dx be du.

Let e_y = Circumferential strain and

e_x = Radial strain

Now increase in radius = u

∴ Final radius = $x + u$

∴ Circumferential strain,

$$e_y = \frac{Final\ circumference - Original\ circumference}{Original\ circumference}$$

$$= \frac{2\pi(x+u) - 2\pi x}{2\pi x} = \frac{u}{x}\,. \qquad \ldots.\ (1)$$

Now original thickness of element = dx

Final thickness of element = dx + du

∴ Radial strain,

$$e_x = \frac{Final\ thickness\ of\ element - Original\ thickness}{Original\ thickness}$$

$$= \frac{(dx+du) - dx}{dx} = \frac{du}{dx}\,. \qquad \ldots.\ (2)$$

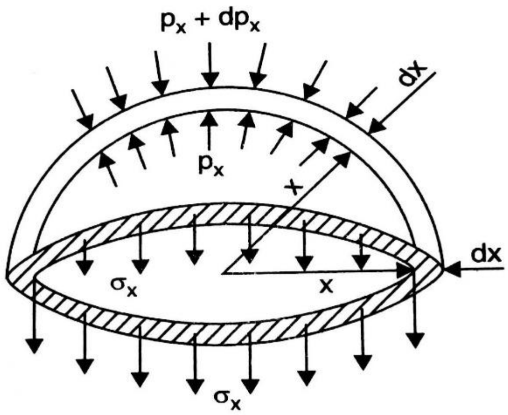
But from equation (1),

u = $x\,.\,e_y$

∴ Radial strain,

$$e_x = \frac{d}{dx}\left(x\,.\,e_y\right) = e_y + x\,.\frac{de_y}{dx} \qquad \ldots.\ (3)$$

Now consider an elemental spherical shell of radius x and thickness dx as shown in Fig. 18.7. Let p_x and $p_x + dp_x$ be the radial pressure at radii x and $x + dx$ respectively. And σ_x is the circumferential tensile stress which is equal in all direction in a spherical shell.



Consider the equilibrium of half of the elementary spherical shell on which the following external force are acting:

1) An upward force of $\pi x^2 \,.\, p_x$ due to internal radial pressure p
2) A downward force of $\pi(x + dx)^2 \,.\, (p_x + dp_x)$ due to radial pressure $p_x + dp_x$.
3) A downward resisting force $\sigma_x(2\pi x \,.\, dx)$.

Equating the upward and downward forces, we get

$$\pi x^2 p_x = \pi(x + dx)^2 \,.\, (p_x + dp_x) + 2\pi x \,.\, dx \,.\, \sigma_x$$

$$= \pi(x^2 + dx^2 + 2x \,.\, dx)(p_x + dp_x) + 2\pi x \,.\, dx \,.\, \sigma_x$$

$$x^2 \,.\, p_x = (x^2 + dx^2 + 2x \,.\, dx)(p_x + dp_x) + 2x \,.\, dx \,.\, \sigma_x$$

$$=(x^2 \,.\, p_x + dx^2 \,.\, p_x + 2x \,.\, dx \,.\, p_x + x^2 .\, dp_x + dx^2 \,.\, dp_x + 2x \,.\, dx \,.\, dp_x)$$

$$+ 2x \,.\, dx \,.\, \sigma_x$$

Neglecting squares and products of $dx \text{ and } dp_x$, we get

$$x^2 \,.\, p_x = x^2 \,.\, p_x + 0 + 2x \,.\, dx \,.\, p_x + x^2 .\, dp_x + 0 + 0 + 2x \,.\, dx \,.\, \sigma_x$$

$$0 = 2x \,.\, dx \,.\, p_x + x^2 .\, dp_x + 2x \,.\, dx \,.\, \sigma_x$$

or $\quad 2x \,.\, dx \,.\, \sigma_x = -2x \,.\, dx \,.\, p_x - x^2 .\, dp_x$

or $\quad 2 \,.\, \sigma_x = -2 \,.\, p_x - x \,.\, \frac{dp_x}{dx}$ (Divided both sides by $x \,.\, dx$)

or $\quad \sigma_x = -p_x - \frac{x}{2} \,.\, \frac{dp_x}{dx}$ …. (A)

Differentiating the above equation w.r.t. x., we get

$$\frac{d}{dx}(\sigma_x) = \frac{d}{dx}(-px) - \frac{1}{2}\frac{d}{dx}\left(x.\frac{dp_x}{dx}\right) = -\frac{dp_x}{dx} - \frac{1}{2}\left(x.\frac{d^2p_x}{dx^2} + \frac{dp_x}{dx}\right) \quad \ldots.(4)$$

At any point in the elementary spherical shell, there are three principle stresses:

1) The radial pressure p_x , which is compressive
2) Circumferential (or hoop stress) σ_x, which is tensile and
3) Circumferential (or hoop stress) σ_x, which is tensile of the same magnitude as of (2) and on a plane at right angles to the plane of σ_x of (2).

Now radial strain,

$$e_x = \frac{p_x}{E} + \frac{\sigma_x}{mE} + \frac{\sigma_x}{mE} \qquad \text{Here } \frac{1}{m} = \text{Poisson's ratio} = \mu$$

$$= \frac{p_x}{E} + \frac{2\sigma_x}{mE} \qquad \text{(compressive)}$$

$$= -\left(\frac{p_x}{E} + \frac{2\sigma_x}{mE}\right) \qquad \text{(tensile)} \quad \ldots.(5)$$

and circumferential strain,

$$e_y = \frac{\sigma_x}{E} - \frac{\sigma_x}{mE} + \frac{p_x}{mE} \qquad \text{(tensile)}$$

$$= \frac{1}{E}\left(\sigma_x - \frac{\sigma_x}{m} + \frac{p_x}{m}\right) = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right] \qquad \text{(tensile)} \quad \ldots.(6)$$

Substituting the values of e_x and e_y from equation (5) and (6) in equation (3),we get

$$-\left(\frac{p_x}{E} + \frac{2\sigma_x}{mE}\right) = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right] + x.\frac{d}{dx}\left[\frac{1}{E}\left\{\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right\}\right]$$

$$-\frac{1}{E}\left(p_x + \frac{2\sigma_x}{mE}\right) = \frac{1}{E}\left[\frac{\sigma_x(m-1)}{m} + \frac{p_x}{m}\right] + \frac{x}{E}\left[\left(\frac{m-1}{m}\right).\frac{d\sigma_x}{dx} + \frac{1}{m}\frac{dp_x}{dx}\right]$$

$$-\left(p_x + \frac{2\sigma_x}{mE}\right) = \left(\frac{\sigma_x(m-1)}{m} + \frac{p_x}{m}\right) + \frac{x(m-1)}{m}.\frac{d\sigma_x}{dx} + \frac{x}{m}\frac{dp_x}{dx}$$

$$-mp_x - 2\sigma_x = (m-1)\sigma_x + p_x + x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$-p_x(m+1) - \sigma_x(2+m-1) = x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$-(m+1)(p_x + \sigma_x) = x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$(m+1)(p_x + \sigma_x) + x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx} = 0.$$

Now substituting the value of σ_x and $\frac{d}{dx}(\sigma_x)$ from equation (A) and (4) in the above equation, we get

$$(m+1)\left(p_x + -p_x - \frac{x}{2}.\frac{dp_x}{dx}\right) + x(m-1)\times\left[-\frac{dp_x}{dx} - \frac{1}{2}\left(x\frac{d^2p_x}{dx^2} + \frac{dp_x}{dx}\right)\right] + x\frac{dp_x}{dx} = 0$$

$$(m+1)\left(-\frac{x}{2}.\frac{dp_x}{dx}\right) + x(m-1)\left(-\frac{3}{2}\frac{dp_x}{dx} - \frac{1}{2}x\frac{d^2p_x}{dx^2}\right) + x\frac{dp_x}{dx} = 0$$

$$\frac{dp_x}{dx}\left[-\frac{x}{2}(m+1) - \frac{3x}{2}(m-1) + x\right] - \frac{x^2(m-1)}{2}\frac{d^2p_x}{dx^2} = 0$$

$$x.\frac{dp_x}{dx}\left[\frac{-m-1-3m+3+2}{2}\right] - \frac{x^2}{m}(m-1)\frac{d^2p_x}{dx^2} = 0$$

$\frac{dp_x}{dx}\left(\frac{-4m+4}{2}\right) - \frac{x}{2}(m-1)\frac{d^2p_x}{dx^2} = 0$ (Cancelling x)

$$-\frac{4}{2}\frac{dp_x}{dx}(m-1) - \frac{x}{2}(m-1)\frac{d^2p_x}{dx^2} = 0$$

$\frac{4dp_x}{dx} + x\frac{d^2p_x}{dx^2} = 0$ $\left[Cancelling - \frac{(m-1)}{2}\right]$

Substituting $\frac{dp_x}{dx} = Z$ in the above equation, we get

$$4Z + x.\frac{d}{dx}\left(\frac{dp_x}{dx}\right) = 0$$

$$4Z + x.\frac{dZ}{dx} = 0$$

$$4Z = -x.\frac{dZ}{dx}$$

$$\frac{dZ}{dx} = -4\frac{dx}{x}$$

Integrating the above equation, we get

$$\log_e Z = -4\ \log_e x + \log_e C_1$$

where C_1 is the constant of integration.

The above equation can also be written as

$$\log_e Z = \log_e x^{-4} + \log_e C_1 = \log_e C_1\ \times x^{-4}$$

$$= \log_e\left(\frac{C_1}{x^4}\right) \quad \text{or} \quad Z = \frac{C_1}{x^4}$$

But $Z = \frac{dp_x}{dx}$

∴ $\frac{dp_x}{dx} = \frac{C_1}{x^4}$ or $dp_x = \frac{C_1}{x^4}dx$

Integrating the above equation, we get

$$p_x = -\frac{C_1}{3x^3} + C_2 \qquad \text{….. (7)}$$

where C_2 is another constant of integration.

Substituting this value of p_x in equation (A), we get

$$\sigma_x = -\left(-\frac{C_1}{3x^3} + c_2\right) - \frac{x}{2}\frac{dp_x}{dx}$$

$$= \frac{C_1}{3x^3} - c_2 - \frac{x}{2} \cdot \frac{C_1}{x^4} \qquad \left(\frac{dp_x}{dx} = \frac{C_1}{x^4}\right)$$

$$= \frac{C_1}{3x^3} - c_2 - \frac{C_1}{2x^3} = -\frac{C_1}{6x^3} - c_2 \qquad \text{…. (8)}$$

If we substitute $C_1 = -6b$ and $C_2 = -a$ in equation (7) and (8), we get

$$p_x = -\frac{(-6b)}{3x^3} + (-a) = \frac{2b}{x^3} - a \qquad \text{…. (9)}$$

$$\sigma_x = -\frac{(-6b)}{6x^3} - (-a) = \frac{b}{x^3} + a \qquad \text{…. (10)}$$

and

The constants a and b are obtained from initial given conditions.

For example, (i) at x = r_1, $p_x = 0$ and at x = r_2, $p_x = p$.

Substituting these values in equation (9), we get

$$0 = \frac{2b}{{r_1}^3} - a \;\text{…. (11)} \quad \text{and} \quad p = \frac{2b}{{r_2}^3} - a \;\text{…. (12)}$$

Solving equations (11) and (12), we get

$$a = \frac{pr_2^3}{r_1^3 - r_2^3} \quad \text{and} \quad b = \frac{pr_1^3 r_2^3}{2(r_1^3 - r_2^3)}.$$

Problem 5.29: A thick spherical shell of 200 mm internal diameter is subjected to an internal fluid pressure of 7 N/mm^2. If the permissible tensile stress in the shell material is 8 N/mm^2, find the thickness of the shell.

Sol. Given :

Internal dia, = 200 mm

∴Internal radius, r_1= 100 mm

Internal fluid pressure, p = 7 N/mm^2

Permissible tensile stress, σ_x= 8 N/mm^2.

The radial pressure and hoop stress at any radius of spherical shell are given by

$$p_x = \frac{2b}{x^3} - a \;\text{…. (1)} \quad \text{and} \quad \sigma_x = \frac{b}{x^3} + a \;\text{…. (2)}$$

The hoop stress, σ_x will be maximum at the internal radius . Hence permissible tensile stress of 8 N/mm^2 is the hoop stress at the internal radius.

At x =100 mm,$p_x = 7$ N/mm^2.

Substituting these values in equation (1), we get

$$7 = \frac{2b}{100^3} - a = \frac{2b}{1000000} - a \qquad \text{.... (3)}$$

At x =100 mm,$\sigma_x = 8$ N/mm^2.

Substituting these values in equation (2), we get

$$8 = \frac{b}{100^3} + a = \frac{b}{1000000} - a \qquad \text{.... (4)}$$

Adding equations (3) and (4), we get

$$15 = \frac{3b}{1000000}$$

$$b = \frac{1000000 \times 15}{3} = 5000000.$$

Substituting the value of b in equation (4), we get

$$8 = \frac{5000000}{1000000} - a = 5 + a$$

∴ $$a = 8 - 5 = 3$$

Substituting the value of a and b in equation (1), we get

$$p_x = \frac{2 \times 5000000}{x^3} - 3$$

Let r_2 = External radius of the shell.

At outside, the pressure

$p_x = 0$ or at x = r_2, $p_x = 0$.

Substituting these values in equation (5), we get

$$0 = \frac{2 \times 5000000}{{r_2}^3} - 3 \quad \text{or}$$

$$r_2^3 = \frac{10000000}{3}$$

∴ $$r_2 = \left(\frac{10^7}{3}\right)^{1/3} = (3.333)^{1/3} \times 10^2 = 149.3 \text{ mm}$$

∴ Thickness of the shell,

$$t = r_2 - r_1 = 149.3 - 100$$

$$= 49.3\ mm.$$ **Ans.**

Problem 5.30: For the problem 5.29, find the minimum value of the hoop stress.

Sol. Given :

The data from problem 18.6 is :

$r_1 = 100$ mm , $r_2 = 149.3$ mm

p = 7 N/mm^2, σ_x at the internal radius = 8 N/mm^2

Values of constants are ;

a = 3, b = 5000000

The hoop stress at any radius of spherical shell is given by

$$\sigma_x = \frac{b}{x^3} + a = \frac{5000000}{x^3} + 3$$

The hoop stress will be minimum at the external radius i.e., at x = r_2 = 149.3 mm.

Substituting this value of x in the above equation, we get

$$\sigma_x = \frac{5000000}{(149.3)^3} + 3 = \frac{5000000}{\left(\frac{10000000}{3}\right)} + 3 = 1.5 + 3 = 4.5 \text{ N/mm}^2.$$ **Ans.**

IMPORTANT TERMS

Circumferential Stress (OR) Hoop Stress (σ_c)	$\sigma_c = \frac{p\ x\ d}{2t}$	p = intensity of pressure inside the cylinder d = inner diameter of cylinder shell t = thickness of cylinder shell η_l= efficiency of longitudinal joint η_c= efficiency of circumferential joint
Longitudinal Stress (σ_l)	$\sigma_l = \frac{p\ x\ d}{4t}$	
Circumferential Stress with efficiency (σ_c)	$\sigma_c = \frac{p\ x\ d}{2\ t\ x\ \eta_l}$	
Longitudinal Stress with efficiency (σ_l)	$\sigma_l = \frac{p\ x\ d}{4\ t\ x\ \eta_c}$	
Circumferential Strain	$e_c = \frac{\delta d}{d} = \frac{pd}{2\ t\ E}\left(1 - \frac{1}{2}\mu\right)$	μ= poission's ratio E = young's modulus l = length of shell
Longitudinal Strain	$e_l = \frac{\delta l}{l} = \frac{pd}{2\ t\ E}\left(\frac{1}{2} - \mu\right)$	
Change in diameter	$\delta d = \frac{pd}{2\ t\ E}\left(1 - \frac{1}{2}\mu\right) d$	
Change in Length	$\delta l = \frac{pd}{2\ t\ E}\left(\frac{1}{2} - \mu\right) l$	
Volumetric Strain	$e_v = \frac{\delta V}{V} = \frac{pd}{2\ t\ E}\left(\frac{5}{2} - 2\mu\right)$ $e_v = 2e_c - e_l$	$Volume\ (V) = \frac{\pi d^2}{4} x\ l$
Change in Volume	$\delta V = \frac{pd}{2\ t\ E}\left(\frac{5}{2} - 2\mu\right) V\ (or)$ $= (2e_c - e_l)V$	

Major Principal Stress	$= \frac{\sigma_c + \sigma_l}{2} + \sqrt{\left(\frac{\sigma_c - \sigma_l}{2}\right)^2 + \tau^2}$	τ = shear stress
Minor Principal Stress	$= \frac{\sigma_c + \sigma_l}{2} - \sqrt{\left(\frac{\sigma_c - \sigma_l}{2}\right)^2 + \tau^2}$	
Maximum Shear stress	$\frac{1}{2}(Major\ Principal\ Stress - Minor\ Principal\ Stress)$	ρ = density of the material r = mean radius of the cylinder ω = angular speed of the cylinder $= \frac{2\pi N}{60}$
Rotational Stress	$\sigma_r = \rho r^2 \omega^2$	
SPHERICAL SHELLS		
Circumferential Stress (OR) Hoop Stress (σ_c)	$\sigma_c = \frac{p\ x\ d}{4t} \quad OR = \frac{p\ x\ d}{4t\ \eta}$	p = intensity of pressure inside the spherical shell d = inner diameter of spherical shell t = thickness of spherical shell E = young's modulus l = length of shell μ= poission's ratio
Circumferential Strain	$e_c = \frac{\delta d}{d} = \frac{pd}{4\ t\ E}(1 - \mu)$	
Volumetric Strain	$e_v = \frac{\delta V}{V} = \frac{3pd}{4\ t\ E}(1 - \mu)$ $Volume\ (V) = \frac{4}{3} x\ \pi r^3$	

THEORETICAL QUESTIONS

1. Define thin cylinders. Name the stresses set up in a thin cylinder subjected to int fluid pressure.
2. Prove that the circumference stress and longitudinal stress

σ_1 = pd/2t, σ_2 = pd/4t where p = int fluid pressure
D = int dia of thin cylinder
T = thickness of wall of thin cylinder

Derive an expression for circumferential stress and longitudinal stress for a thin shell subjected to an int pressure.

3. A)Derive the expression for hoop stress and longitudinal stress in a thin cylinder with ends closed by rigid flanges and subjected to an internal fluid pressure p. Take the int dia and shell thickness of the cylinder to be d and t respectively. B)Derive from the first principles of expressions for circumferential and longitudinal stresses in a thin cylinder closed at both ends and subjected to int fluid pressure.
4. Show that in thin cylinder shells subjected to int fluid pressure , the circumferential stress is twice the longitudinal stress.
5. While resighing a cylindrical vessel, which stress should be used for calculating the thickness of the cylindrical vessel.
6. Prove that max shear stress at any point in a thin cylinder, subjected to int fluid pressure is given by,

Max shear stress = pd/8t

Where p = int fluid pressure
D = int dia of thin cylinder
T = wall thickness of cylinder

7. Find the expression for circumferential stress and longitudinal stress for a longitudinal joint and circumferential joint.
8. Prove that the circumferential strain and longitudinal strain produced in thin cylinder when subjected to int flud pressure are given by
d_1 = pd/2tE(1-1/2*μ)
e_2 = pd/2tE(1/2 – μ)
Where p = int fluid pressure
D = int dia of thin cylinder
T = thickness of wall of thin cylinder
μ = poisons ratio
9. A cylindrical shell is subjected to int fluid pressure find an expression for change in dia and change in length of cylinder
10. Prove that volumetric strain in case of a thin cylinder subjected to int fluid pressure is equal to two times the circumferential strain plus longitudinal strain. show that when a thin walled cylindrical vessel of dia D, length L and thickness t is subjected to an int pressure p, the change in volume = π*p*L*D^3(5-4*μ)/16tE
11. Find an expression for the change in volume of a thin cylindrical shell subjected to int fluid pressure
12. Write down expression for major principal stress, minor principle stress when a thin cylindrical shell is subjected to int fluid pressure and a torque
13. Show that when a thin walled spherical vessel of dia d and thickness t is subjected to int fluid pressure p the increase in volume equal to
$\frac{\pi}{8} * pd^4/tE(1 - \frac{1}{\mu})$
Where E = elastic modulus
μ = poisons ratio
14. Differentiate between a thin cylinder and thick cylinder. Find an expression for the radial pressure and hoop stress at any point in case of thick cylinder.
15. What do you mean by lame's equation. How will you derive these equations.
16. The hoop stress is min at the outer surface and is max at the inner surface of the thick cylinder. prove this statement. Sketch the radial pressure distribution and hoop stress distribution across the section of cylinder.
17. What do you mean by a thick compound cylinder. How will you determine the hoop stress in a thick compound cylinder.
18. What are the different methods of reducing hoop stress. Explain the terms: wire winding of thin cylinders and shrinking one cylinder over another cylinder.
19. Prove that the original difference in radii at the junction of a compound cylinder for shrinking is given by

Dr = 2r*$(a_1 - a_2)$/E

Where r* = common radius after shrinking

E = young's modulus

A = constants

Derive an expression for the radial pressure and hoop stress for a thick spherical shell.

NUMERICAL PROBLEMS

1. A cylindrical pipe of dia 2m and thickness 2cm is subjected to an int fluid pressure of 1.5 N/mm^2. determine longitudinal stress and circumferential stress. Ans = 37.5 N/mm^2, 75 N/mm^2
2. A cylinder of int dia of 3m and of thickness 6m contains a gas. If the tensile stress in the material is not to exceed 70 N/mm^2, determine the int pressure of gas. Ans = 2.8 N/mm^2
3. A cylinder of int dia 0.60m contains air at a pressure of 7.5 N/mm^2. If max permissible stress induced in the material is 75 N/mm^2 find the thickness of cylinder. Ans = 3cm
4. A thin cylinder of int dia 2m contains a fluid at an int pressure of 3 N/mm^2. Determine the max thickness of cylinder if longitudinal stress is not to exceed 303 N/mm^2 and the circumferential stress is not to exceed 403 N/mm^2. Ans = 7.5 cm
5. A water main 90cm dia contains water at a pressure head of 110m. If the weight density of water is 9810 N/mm^2, find the thickness of metal required for the water main. Given the permissible stress as 223 N/mm^2. Ans = 2.25 cm
6. A boiler is subjected to an internal steam pressure of 3 N/mm^2. The thickness of boiler plate is 2.5 cm and the permissible tensile stress is 125 N/mm^2. Find out max dia when efficiency of longitudinal joint is 90% and that of circumferential joint is 35%. Ans = 145.83cm
7. A boiler shell is to be made of 20mm thick plate having a limiting tensile stress of 125N/mm^2. If the efficient of longitudinal and circumferential joints are 80% and 30%. Determine max permissibe dia of shell for an int pressure of 2.5 N/mm^2 and permissible intensity of int pressure when the shell dia is 1.6m. ans = 120 cm, 1.875 N/mm^2
8. A cylinder of thickness 2cm has to withstand max int pressure of 2 N/mm^2. If the ultimate tensile stress in the material of cylinder is 292 N/mm^2, factor of safety 4 and joint efficiency 80%, determine the dia of cylinder. Ans = 116.8cm
9. A thin cylindrical sell of 120 cm dia, 1.5cm thick and 6m long is subjected to int fluid pressure of 2.5 N/mm^2. If poisons ratio is 0.3. find change in dia, change in length,change in volume. Ans = 0.051m, 0.06cm, 6449.7cm
10. A cylindrical shell 100cm long 20cm int dia having thickness of metal as 10mm is filled with fluid at atm pressure. If an additional 20cm of fluid is pumped into cylinder find the pressure exerted by fluid on cylinder and the hoop stress induced.poisons ratio = 0.3. ans = 10.05N/mm^2, 100.52 N/mm^2

11. A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plates 4mm thick. The length and int dia of vessel are 100cm and 30 cm. Determine the longitudinal and hoop stress in cylindrical shell due to int fluid pressure of 2 N/mm^2. Also calculate the increase in length, dia and volume of vessel.poissons ratio = 0.3. ans = 37.53 N/mm^2, 75 N/mm^2, 0.075 cm, 0.0095cm, 50.36cm
12. A thin cylindrical tube 100mm int dia and 5mm thick is closed at the ends and is subjected to an int pressure of 5 N/mm^2. A torque of 22000Nm is also applied to the tube.find the hoop stress, longitudinal stress,max and min principle stresses and max shear stress. Ans = 50N/mm^2, 25 N/mm^2, 28N/mm^2, 68.16 N/mm^2, 6.84 N/mm^2, 30.66N/mm^2
13. A copper cylinder 100 cm long, 50 cm ext dia and wall thickness 5mm has its both ends closed by rigid blank flanges. It is initially full of oil at atm pressure. Calculate the additional volume of oil which must be pumped into it in order to raise the oil pressure to 4 N/mm^2 above the atm pressure. Ans = 486.3cm
14. A vessel in the shape of a spherical shell of 1.4dia and 4.5mm thickness is subjected to a pressure of 1.8 N/mm^2. Determine the stress induced in the material of the vessel. Ans = 140 N/mm^2
15. A thin spherical shell of 1.20mm int dia is subjected to int pressure of 1.6 N/mm^2. If the permissible stress in the plate material is 80 N/mm^2 and joint efficiency is 75% find the min thickness. Ans = 8mm
16. A thin spherical shell of int dia 1.5m and of thickness 8mm is subjected to an int pressure of 1.5 N/mm^2. Determine the increase in dia and increase in volume.poisons ratio = 0.3. ans = 0.369mm, 1304*$10^3 mm^3$
17. Determine the max hoop stress across the section of pipe of ext dia 600 mm and int dia 440 mm, when the pipe is subjected to an int fluid pressure of 0 N/mm^2. Ans = 99.9 N/mm^2
18. Find the thickness of metal necessary for a cylinder shell of int dia 150mm to withstand an int pressure of 50 N/mm^2. The max hoop stress in section is not to exceed 150 N/mm^2. Ans = 31mm
19. A compound cylinder is made by shrinking a cylinder of ext dia 200mm and int dia 160mm over another cylinder of ext dia 160mm and int dia 120mm. The radial pressure at the junction after shrinking is 8 N/mm^2. Find the final stress set up across the section when the compound cylinder is subjected to an int fluid pressure of 60 N/mm^2. Ans = inner F_{60} = 90.9 and F_{80} = 57.9 N/mm^2, outer F_{80} = 122.9 and F_{100} = 25.9 N/mm^2
20. A steel cylinder of 200mm ext dia is to be shrunk to another steel cylinder of 100mm int dia. After shrinking the dia at junction is 150 mm and radial pressure

at the junction is 12.5N/mm. Find the original difference in radii at the junction. Ans = 0.02025mm

21. A steel tube of 240mm ext dia is to be shrunk on another steel tube of 80mm int dia. After shrinking the dia at junction is 160mm. Before shrinking on the difference of dia at the junction was 0.08mm. calculate the radii pressure at the junction and hoop stress developed in the two tubes after shrinking.
22. A thick spherical shell of 400mm int dia is subjected to an int fluid pressure of 1.5 N/mm^2. If the permissible tensile stress in the shell material is 3 N/mm^2. Find the necessary thickness of shell. Ans = 52mm