ROHINI cOLLEGE OF ENGINEERING \& TECHNOLOGY DEPARTMENT OF MATHEMATICS

## UNIT I - FOURIER SERIES

## PERIODIC FUNCTION

## Periodic Functions

A function $f(x)$ is periodic if there is a positive number $T$ such that for every value of $x$ :

$$
f(x+T)=f(x)
$$

The smallest such value of $T>0$ is called the fundamental period or simply the period.
$>$ If we know what the graph looks like in an interval of length $T$, then we can use replication to sketch the entire graph.


Problem 1: What is the fundamental period of the following function?

(a)

(b)

(c)

Solution:
a) The period is 3
b) The period is 6
c) The period is 4

## Period of Trigonometric Functions

## Period $\pi$ :

$$
\begin{aligned}
\tan (x+\pi) & =\tan x \\
\cot (x+\pi) & =\cot x
\end{aligned}
$$

## Period 2 $\pi$ :

$$
\begin{array}{ll}
\tan x=\frac{\sin x}{\cos x} & , \quad \cot x=\frac{1}{\tan x} \\
\sec x=\frac{1}{\cos x} & , \quad \csc x=\frac{1}{\sin x}
\end{array}
$$

$$
\sin (x+2 \pi)=\sin x
$$

$$
\cos (x+2 \pi)=\cos x
$$

$$
\sec (x+2 \pi)=\sec x
$$

$$
\csc (x+2 \pi)=\csc x
$$



Domain: $-\infty<x<\infty$
Range: $-1 \leq y \leq 1$
Period: $2 \pi$


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Domain: $x \neq \pm \frac{\pi}{2}, \pm \frac{3 \pi}{2}, \ldots$
Range: $y \leq-1$ or $y \geq 1$ Period: $2 \pi$


Domain: $x \neq 0, \pm \pi, \pm 2 \pi, \ldots$
Range: $-\infty<y<\infty$
Period: $\pi$
c) $y=\sin (2 x)$,

Problem 2: Sketch
$\begin{array}{ll}\text { a) } y=\sin x, & \text { b) } y=2 \sin x \text { and }\end{array}$ What is the period of each function?

## Solution:

a) The period is $2 \pi$

$-2 \dagger$
$y=\sin x$
b) The period is $2 \pi$


c) The period is $\pi$


$$
\begin{aligned}
& -2 \nmid \\
& y=\sin (2 x)
\end{aligned}
$$

