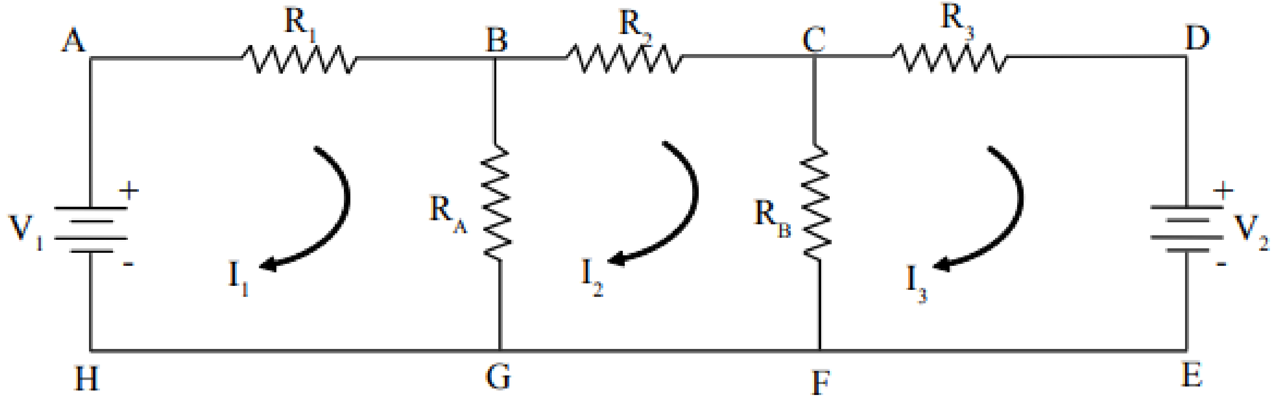
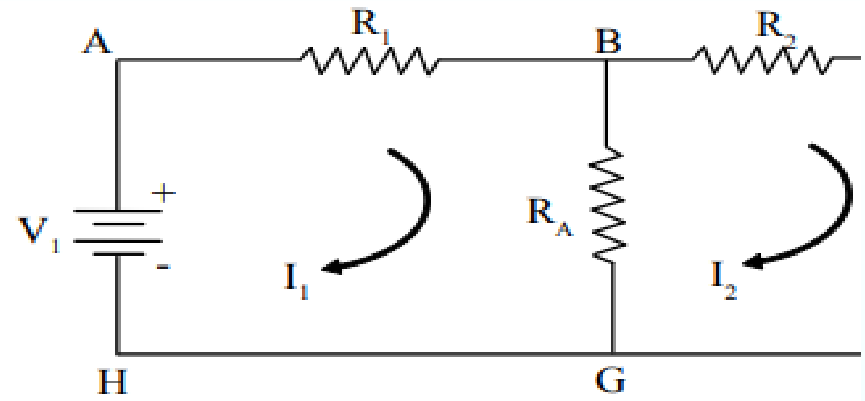
MESH METHOD (Loop Method)

Mesh method was first propsed by maxwell, which simplifies the solution for several networks. Mesh method is also called as loop method or loop current method. In this method loop currents are considered instead of branch currents.

Consider a circuit shown in figure in which two batteries are connected in a five resistor network. A loop is a closed path for current flow. In each loop, a loop currents is assumed as I_1, I_2 and I_3. The current flowing through R_1 is I_1. Both currents I_1 and I_2 flow through R_A but in opposite directions.



Three loop network

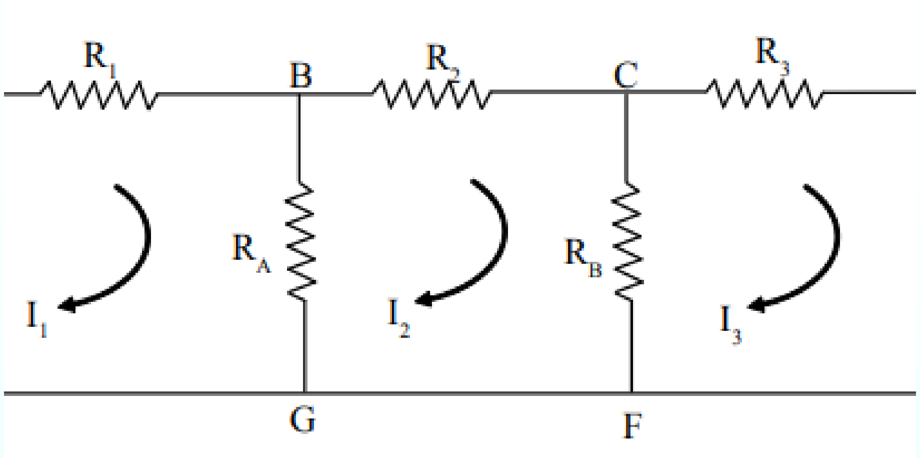


By applying kirchhoff's voltage law in the loop 1 [ABGHA], we get

$$V_1 - I_1R_1 - (I_1 - I_2)R_A = 0$$

$$\Rightarrow I_1R_1 + (I_1 - I_2)R_A = V_1$$

$$\Rightarrow I_1(R_1 + R_A) - I_2R_A = V_1$$

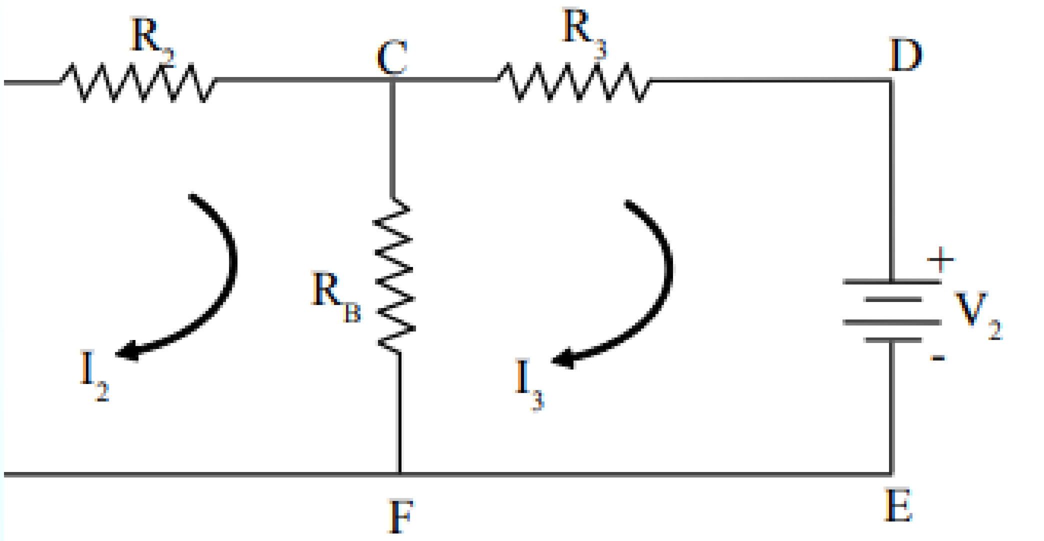


By applying KVL in the loop2 [BCFGB], we get

$$-I_2R_2 - (I_2 - I_3)\,R_B - (I_2 - I_1)\,R_A = 0$$

$$\Rightarrow I_2R_2 + (I_2 - I_3)R_B + (I_2 - I_1)R_A = 0$$

$$\Rightarrow -I_1R_A + (R_A + R_B + R_2)I_2 - I_3R_B = 0$$



By applying KVL in the loop 3 [CDEFC], we get

$$-(I_3 - I_2)R_B - I_3R_3 - V_2 = 0$$

$$\Rightarrow (I_3 - I_2)R_B + I_3R_3 = -V_2$$

$$\Rightarrow -I_2R_B + I_3(R_B + R_3) = -V_2$$

$$\Rightarrow I_1(R_1 + R_A) - I_2 R_A = V_1$$

$$\Rightarrow -I_1 R_A + (R_A + R_B + R_2) I_2 - I_3 R_B = 0$$

$$\Rightarrow -I_2 R_B + I_3 (R_B + R_3) = -V_2$$

These equations can be written in the matrix form as,

$$\begin{bmatrix} (R_1 + R_A) & -R_A & 0 \\ -R_A & (R_A + R_2 + R_B) & -R_B \\ 0 & -R_B & (R_B + R_3) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ 0 \\ -V_2 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} \qquad I_2 = \frac{\Delta_2}{\Delta} \qquad I_3 = \frac{\Delta_3}{\Delta}$$