

## ME3491 THEORY OF MACHINES

Belt, Rope Drives
The belts or ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.
1 . The velocity of the belt.
2. The tension under which the belt is placed on the pulleys.
3. The arc of contact between the belt and the smaller pulley.
4. The conditions under which the belt is used. It may be noted that
(a) The shafts should be properly in line to insure uniform tension across the belt section.
(b) The pulleys should not be too close together, in order that the arc of contact on the smaller pulley may be as large as possible.
(c) The pulleys should not be so far apart as to cause the belt to weigh heavily on the shafts, thusincreasing the friction load on the bearings
The belt drives are usually classified into the following three groups :

1. Light drives. These are used to transmit small powers at belt speeds upto about $10 \mathrm{~m} / \mathrm{s}$, as in agricultural machines and small machine tools.
2. Medium drives. These are used to transmit medium power at belt speeds over $10 \mathrm{~m} / \mathrm{s}$ but up to $22 \mathrm{~m} / \mathrm{s}$, as in machine tools.
3. Heavy drives. These are used to transmit large powers at belt speeds above $22 \mathrm{~m} / \mathrm{s}$, as in compressors and generators

Types of Belts

(a) Flat belt.

(b) V-belt.

(c) Circular belt.

### 11.8. Velocity Ratio of a Compound Belt Drive

Sometimes the power is transmitted from one shaft to another, through a number of pulleys as shown in Fig. 11.7. Consider a pulley 1 driving the pulley 2 . Since the pulleys 2 and 3 are keyed to the same shaft, therefore the pulley 1 also drives the pulley 3 which, in turn, drives the pulley 4 .

Let $\quad d_{1}=$ Diameter of the pulley 1 ,
$N_{1}=$ Speed of the pulley 1 in r.p.m.,
$d_{2}, d_{3}, d_{4}$, and $N_{2}, N_{3}, N_{4}=$ Corresponding values for pulleys 2,3 and 4 .
We know that velocity ratio of pulleys 1 and 2 ,

$$
\begin{equation*}
\frac{N_{2}}{N_{1}}=\frac{d_{1}}{d_{2}} \tag{i}
\end{equation*}
$$

Similarly, velocity ratio of pulleys 3 and 4 ,

$$
\begin{equation*}
\frac{N_{4}}{N_{3}}=\frac{d_{3}}{d_{4}} \tag{ii}
\end{equation*}
$$

Multiplying equations (i) and (ii),

$$
\begin{gathered}
\frac{N_{2}}{N_{1}} \times \frac{N_{4}}{N_{3}}=\frac{d_{1}}{d_{2}} \times \frac{d_{3}}{d_{4}} \\
\frac{N_{4}}{N_{1}}=\frac{d_{1} \times d_{3}}{d_{2} \times d_{4}} \quad \ldots\left(\because N_{2}=N_{3}, \text { being keyed to the same shaft }\right)
\end{gathered}
$$

$\dagger$ little consideration will show, that if there are six pulleys, then

$$
\frac{N_{6}}{N_{1}}=\frac{d_{1} \times d_{3} \times d_{5}}{d_{2} \times d_{4} \times d_{6}}
$$

$\frac{\text { Speed of last driven }}{\text { Speed of first driver }}=\frac{\text { Product of diameters of drivers }}{\text { Product of diameters of drivens }}$

Slip
The frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This may also cause some forward motion of the belt without carrying the driven pulley with it. This is called slip of the belt and is generally expressed as a percentage

An engine, running at 150 rpm drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft being $\mathbf{4 5 0} \mathbf{~ m m}$. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft, when 1. there is no slip, and 2. there is a slip of $2 \%$ at each drive

Solution. Given : $N_{1}=150$ r.p.m. $; d_{1}=750 \mathrm{~mm} ; d_{2}=450 \mathrm{~mm} ; d_{3}=900 \mathrm{~mm} ; d_{4}=150 \mathrm{~mm}$
The arrangement of belt drive is shown in Fig. 11.10.
Let

$$
N_{4}=\text { Speed of the dynamo shaft }
$$



Fig. 11.10

## 1. When there is no slip

$$
\begin{array}{ll}
\text { We know that } & \frac{N_{4}}{N_{1}}=\frac{d_{1} \times d_{3}}{d_{2} \times d_{4}} \quad \text { or } \quad \frac{N_{4}}{150}=\frac{750 \times 900}{450 \times 150}=10 \\
\therefore & N_{4}=150 \times 10=1500 \text { r.p.m. Ans. }
\end{array}
$$

2. When there is a slip of $2 \%$ at each drive

$$
\begin{array}{ll}
\text { We know that } & \frac{N_{4}}{N_{1}}=\frac{d_{1} \times d_{3}}{d_{2} \times d_{4}}\left(1-\frac{s_{1}}{100}\right)\left(1-\frac{s_{2}}{100}\right) \\
& \frac{N_{4}}{150}=\frac{750 \times 900}{450 \times 150}\left(1-\frac{2}{100}\right)\left(1-\frac{2}{100}\right)=9.6 \\
& N_{4}=150 \times 9.6=1440 \text { r.p.m. Ans. }
\end{array}
$$

Creep of Belt
When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as creep.

$$
\frac{N_{2}}{N_{1}}=\frac{d_{1}}{d_{2}} \times \frac{E+\sqrt{\sigma_{2}}}{E+\sqrt{\sigma_{1}}}
$$

$\sigma_{1}$ and $\sigma_{2}=$ Stress in the belt on the tight and slack side respectively, and $E=$ Young's modulus for the material of the belt.

A shaft which rotates at a constant speed of 160 r.p.m. is connected by belting to a parallel shaft 720 mm apart, which has to run at 60,80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for 1. a crossed belt, and 2. an open belt. Neglect belt thickness and slip.

1. For a crossed belt

We know that for pulleys 1 and 2,

$$
\frac{N_{2}}{N_{1}}=\frac{r_{1}}{r_{2}}
$$

or

$$
r_{2}=r_{1} \times \frac{N_{1}}{N_{2}}=40 \times \frac{160}{60}=106.7 \mathrm{~mm} \text { Ans. }
$$

and for pulleys 3 and 4,

$$
\frac{N_{4}}{N_{3}}=\frac{r_{3}}{r_{4}} \text { or } r_{4}=r_{3} \times \frac{N_{3}}{N_{4}}=r_{3} \times \frac{160}{80}=2 r_{3}
$$

We know that for a crossed belt drive,

$$
\begin{array}{lll} 
& r_{1}+r_{2}=r_{3}+r_{4}=r_{5}+r_{6}=40+106.7=146.7 \mathrm{~mm} \\
\therefore & r_{3}+2 r_{3}=146.7 \text { or } r_{3}=146.7 / 3=48.9 \mathrm{~mm} \mathrm{Ans} . \\
\text { and } & r_{4}=2 r_{3}=2 \times 48.9=97.8 \mathrm{~mm} \text { Ans. }
\end{array}
$$

Now for pulleys 5 and 6,

$$
\frac{N_{6}}{N_{5}}=\frac{r_{5}}{r_{6}} \text { or } r_{6}=r_{5} \times \frac{N_{5}}{N_{6}}=r_{5} \times \frac{160}{100}=1.6 r_{5}
$$

From equation ( $i$ ),

$$
r_{5}+1.6 r_{5}=146.7 \text { or } r_{5}=146.7 / 2.6=56.4 \mathrm{~mm} \mathrm{Ans} .
$$

and

$$
r_{6}=1.6 r_{5}=1.6 \times 56.4=90.2 \mathrm{~mm} \text { Ans. }
$$

2. For an open belt

We know that for pulleys 1 and 2,

$$
\frac{N_{2}}{N_{1}}=\frac{r_{1}}{r_{2}} \text { or } r_{2}=r_{1} \times \frac{N_{1}}{N_{2}}=40 \times \frac{160}{60}=106.7 \mathrm{~mm} \text { Ans. }
$$

and for pulleys 3 and 4,

$$
\frac{N_{4}}{N_{3}}=\frac{r_{3}}{r_{4}} \text { or } r_{4}=r_{3} \times \frac{N_{3}}{N_{4}}=r_{3} \times \frac{160}{80}=2 r_{3}
$$

We know that length of belt for an open belt drive,

$$
\begin{aligned}
L & =\pi\left(r_{1}+r_{2}\right)+\frac{\left(r_{2}-r_{1}\right)^{2}}{x}+2 x \\
& =\pi(40+106.7)+\frac{(106.7-40)^{2}}{720}+2 \times 720=1907 \mathrm{~mm}
\end{aligned}
$$

Since the length of the belt in an open belt drive is constant, therefore for pulleys 3 and 4, length of the belt ( $L$ ),

$$
\begin{aligned}
&=\pi\left(r_{3}+2 r_{3}\right)+\frac{\left(2 r_{3}-r_{3}\right)^{2}}{720}+2 \times 720 \\
&=9.426 r_{3}+0.0014\left(r_{3}\right)^{2}+1440 \\
& 0.0014\left(r_{3}\right)^{2}+9.426 r_{3}-467=0 \\
& \therefore \quad r_{3}=\frac{-9.426 \pm \sqrt{(9.426)^{2}+4 \times 0.0014 \times 46}}{2 \times 0.0014} \\
&=\frac{-9.426 \pm 9.564}{0.0028}=49.3 \mathrm{~mm} \text { Ans. } \\
& r_{4}=2 r_{3}=2 \times 49.3=98.6 \mathrm{~mm} \text { Ans. }
\end{aligned}
$$

or
and
Now for pulleys 5 and 6,

$$
\begin{aligned}
\frac{N_{6}}{N_{5}} & =\frac{r_{5}}{r_{6}} \text { or } \\
r_{6} & =\frac{N_{5}}{N_{6}} \times r_{5}=\frac{160}{100} \times r_{5}=1.6 r_{5}
\end{aligned}
$$

and length of the belt $(L)$,

$$
\begin{aligned}
1907 & =\pi\left(r_{5}+r_{6}\right)+\frac{\left(r_{6}-r_{5}\right)^{2}}{x}+2 x \\
& =\pi\left(r_{5}+1.6 r_{5}\right)+\frac{\left(1.6 r_{5}-r_{5}\right)^{2}}{720}+2 \times 720 \\
& =8.17 r_{5}+0.0005\left(r_{5}\right)^{2}+1440
\end{aligned}
$$

or

$$
\begin{aligned}
& 0.0005\left(r_{5}\right)^{2}+8.17 r_{5}-467=0 \\
& \therefore \quad r_{5}= \\
& \quad=\frac{-8.17 \pm \sqrt{(8.17)^{2}+4 \times 0.0005 \times 467}}{2 \times 0.0005} \\
& \\
& =\frac{-8.17 \pm 8.23}{0.001}=60 \mathrm{~mm} \text { Ans. }
\end{aligned}
$$

Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN , and the coefficient of friction between the belt and pulley is 0.25 ?

Solution. Given : $d_{1}=450 \mathrm{~mm}=0.45 \mathrm{~m}$ or $r_{1}=0.225 \mathrm{~m} ; d_{2}=200 \mathrm{~mm}=0.2 \mathrm{~m}$ or $r_{2}=0.1 \mathrm{~m} ; x=1.95 \mathrm{~m} ; N_{1}=200$ r.p.m. $; T_{1}=1 \mathrm{kN}=1000 \mathrm{~N} ; \mu=0.25$

We know that speed of the belt,

$$
v=\frac{\pi d_{1} \cdot N_{1}}{60}=\frac{\pi \times 0.45 \times 200}{60}=4.714 \mathrm{~m} / \mathrm{s}
$$

Length of the belt
We know that length of the crossed belt,

$$
\begin{aligned}
L & =\pi\left(r_{1}+r_{2}\right)+2 x+\frac{\left(r_{1}+r_{2}\right)^{2}}{x} \\
& =\pi(0.225+0.1)+2 \times 1.95+\frac{(0.225+0.1)^{2}}{1.95}=4.975 \mathrm{~m} \text { Ans. }
\end{aligned}
$$

Let $\quad \theta=$ Angle of contact between the belt and each pulley.
We know that for a crossed belt drive,

$$
\begin{aligned}
\sin \alpha & =\frac{r_{1}+r_{2}}{x}=\frac{0.225+0.1}{1.95}=0.1667 \text { or } \alpha=9.6^{\circ} \\
\therefore \quad \theta & =180^{\circ}+2 \alpha=180^{\circ}+2 \times 9.6^{\circ}=199.2^{\circ} \\
& =199.2 \times \frac{\pi}{180}=3.477 \mathrm{rad} \text { Ans. }
\end{aligned}
$$

Power transmitted
Let

$$
T_{2}=\text { Tension in the slack side of the belt. }
$$

We know that

$$
\begin{aligned}
2.3 \log \left(\frac{T_{1}}{T_{2}}\right) & =\mu . \theta=0.25 \times 3.477=0.8692 \\
\log \left(\frac{T_{1}}{T_{2}}\right) & =\frac{0.8692}{2.3}=0.378 \text { or } \frac{T_{1}}{T_{2}}=2.3 \\
\therefore \quad T_{2} & =\frac{T_{1}}{2.387}=\frac{1000}{2.387}=419 \mathrm{~N}
\end{aligned}
$$

$$
\left.\log \left(\frac{T_{1}}{T_{2}}\right)=\frac{0.8692}{2.3}=0.378 \text { or } \frac{T_{1}}{T_{2}}=2.387 \quad \ldots \text { (Taking antilog of } 0.378\right)
$$

We know that power transmitted,

$$
P=\left(T_{1}-T_{2}\right) v=(1000-419) 4.714=2740 \mathrm{~W}=2.74 \mathrm{~kW} \text { Ans. }
$$

Example 11.15. An open belt running over two pulleys 240 mm and 600 mm diameter connects two parallel shafts 3 metres apart and transmits 4 kW from the smaller pulley that rotates at 300 r.p.m. Coefficient of friction between the belt and the pulley is 0.3 and the safe working tension is 10 N per mm width. Determine : 1. minimum width of the belt, 2. initial belt tension, and 3. length of the belt required.

Solution. Given : $d_{2}=240 \mathrm{~mm}=0.24 \mathrm{~m} ; d_{1}=600 \mathrm{~mm}=0.6 \mathrm{~m} ; x=3 \mathrm{~m} ; P=4 \mathrm{~kW}=4000 \mathrm{~W}$; $N_{2}=300$ r.p.m. ; $\mu=0.3 ; T_{1}=10 \mathrm{~N} / \mathrm{mm}$ width

## 1. Minimum width of belt

We know that velocity of the belt,

$$
v=\frac{\pi d_{2} \cdot N_{2}}{60}=\frac{\pi \times 0.24 \times 300}{60}=3.77 \mathrm{~m} / \mathrm{s}
$$

Let

$$
T_{1}=\text { Tension in the tight side of the belt, and }
$$

$$
T_{2}=\text { Tension in the slack side of the belt. }
$$

$\therefore$ Power transmitted $(P)$,

$$
4000=\left(T_{1}-T_{2}\right) v=\left(T_{1}-T_{2}\right) 3.77
$$

or

$$
\begin{equation*}
T_{1}-T_{2}=4000 / 3.77=1061 \mathrm{~N} \tag{i}
\end{equation*}
$$

We know that for an open belt drive,

$$
\sin \alpha=\frac{r_{1}-r_{2}}{x}=\frac{d_{1}-d_{2}}{2 x}=\frac{0.6-0.24}{2 \times 3}=0.06 \text { or } \alpha=3.44^{\circ}
$$

and angle of lap on the smaller pulley,

$$
\begin{aligned}
\theta & =180^{\circ}-2 \alpha=180^{\circ}-2 \times 3.44^{\circ}=173.12^{\circ} \\
& =173.12 \times \pi / 180=3.022 \mathrm{rad}
\end{aligned}
$$

We know that

$$
\begin{align*}
2.3 \log \left(\frac{T_{1}}{T_{2}}\right) & =\mu \cdot \theta=0.3 \times 3.022=0.9066 \\
& \log \left(\frac{T_{1}}{T_{2}}\right)=\frac{0.9066}{2.3}=0.3942 \text { or } \frac{T_{1}}{T_{2}}=2.478 \tag{ii}
\end{align*}
$$

...(Taking antilog of 0.3942)
From equations $(i)$ and $(i i)$,

$$
T_{1}=1779 \mathrm{~N}, \text { and } T_{2}=718 \mathrm{~N}
$$

Since the safe working tension is 10 N per mm width, therefore minimum width of the belt,

$$
b=\frac{T_{1}}{10}=\frac{1779}{10}=177.9 \mathrm{~mm} \text { Ans }
$$

2. Initial belt tension

We know that initial belt tension,

$$
T_{0}=\frac{T_{1}+T_{2}}{2}=\frac{1779+718}{2}=1248.5 \mathrm{~N} \text { Ans. }
$$

3. Length of the belt required

We know that length of the belt required,

$$
\begin{aligned}
L & =\frac{\pi}{2}\left(d_{1}-d_{2}\right)+2 x+\frac{\left(d_{1}-d_{2}\right)^{2}}{4 x} \\
& =\frac{\pi}{2}(0.6+0.24)+2 \times 3+\frac{(0.6-0.24)^{2}}{4 \times 3} \\
& =1.32+6+0.01=7.33 \mathrm{~m} \text { Ans. }
\end{aligned}
$$

Friction in brakes- Band and Block brakes.
A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc.

## Simple Band Brake

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Fig. 19.11, is called a simple band brake in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.

(a) Clockwise rotation of drum.

(b) Anticlockwise rotation of drum.

Example 19.6. A band brake acts on the 3/4th of circumference of a drum of 450 mm diameter which is keyed to the shaft. The band brake provides a braking torque of $225 \mathrm{~N}-\mathrm{m}$. One end of the band is attached to a fulcrum pin of the lever and the other end to a pin 100 mm from the fulcrum. If the operating force is applied at 500 mm from the fulcrum and the coefficient of friction is 0.25 , find the operating force when the drum rotates in the (a) anticlockwise direction, and (b) clockwise direction.

Solution. Given : $d=450 \mathrm{~mm}$ or $r=225 \mathrm{~mm}=0.225 \mathrm{~m} ; T_{\mathrm{B}}=225 \mathrm{~N}-\mathrm{m} ; b=O B=100 \mathrm{~mm}$ $=0.1 \mathrm{~m} ; l=500 \mathrm{~mm}=0.5 \mathrm{~m} ; \mu=0.25$

$$
\text { Let } \quad P=\text { Operating force. }
$$

(a) Operating force when drum rotates in anticlockwise direction

The band brake is shown in Fig. 19.11. Since one end of the band is attached to the fulcrum at $O$, therefore the operating force $P$ will act upward and when the drum rotates anticlockwise, as shown in Fig. 19.11 (b), the end of the band attached to $O$ will be tight with tension $T_{1}$ and the end of the band attached to $B$ will be slack with tension $T_{2}$. First of all, let us find the tensions $T_{1}$ and $T_{2}$.

We know that angle of wrap,


Drums for band brakes.

$$
\begin{aligned}
\theta & =\frac{3}{4} \text { th of circumference }=\frac{3}{4} \times 360^{\circ}=270^{\circ} \\
& =270 \times \pi / 180=4.713 \mathrm{rad}
\end{aligned}
$$

and

$$
\begin{align*}
& 2.3 \log \left(\frac{T_{1}}{T_{1}}\right)=\mu . \theta=0.25 \times 4.713=1.178 \\
\therefore & \log \left(\frac{T_{1}}{T_{2}}\right)=\frac{1.178}{2.3}=0.5123 \text { or } \frac{T_{1}}{T_{2}}=3.253 \tag{i}
\end{align*}
$$

... (Taking antilog of 0.5123)
We know that braking torque ( $T_{\mathrm{B}}$ ),

$$
\begin{array}{rlrl} 
& 225 & =\left(T_{1}-T_{2}\right) r=\left(T_{1}-T_{2}\right) 0.225 \\
& \therefore & T_{1}-T_{2} & =225 / 0.225=1000 \mathrm{~N} \tag{ii}
\end{array}
$$

From equations (i) and (ii), we have

$$
T_{1}=1444 \mathrm{~N} ; \text { and } T_{2}=444 \mathrm{~N}
$$

Now taking moments about the fulcrum $O$, we have

$$
\begin{aligned}
& P \times l & =T_{2} \cdot b \quad \text { or } \quad P \times 0.5=444 \times 0.1=44.4 \\
\therefore & P & =44.4 / 0.5=88.8 \mathrm{~N} \text { Ans. }
\end{aligned}
$$

Example 19.7. The simple band brake, as shown in Fig. 19.12, is applied to a shaft carrying a flywheel of mass 400 kg . The radius of gyration of the flywheel is 450 mm and runs at $300 \mathrm{r} . \mathrm{p} . \mathrm{m}$.

If the coefficient of friction is 0.2 and the brake drum diameter is 240 mm , find:

1. the torque applied due to a hand load of 100 N ,
2. the number of turns of the wheel before it is brought to rest, and
3. the time required to bring it to rest, from the moment of the application of the brake.

Solution. Given : $m=400 \mathrm{~kg} ; k=450 \mathrm{~mm}=0.45 \mathrm{~m}$; $N=300$ r.p.m. or $\omega=2 \pi \times 300 / 60=31.42 \mathrm{rad} / \mathrm{s} ; \mu=0.2$; $d=240 \mathrm{~mm}=0.24 \mathrm{~m}$ or $r=0.12 \mathrm{~m}$

## 1. Torque applied due to hand load



All dimensions in mm .
Fig. 19.12

First of all, let us find the tensions in the tight and slack sides of the band i.e. $T_{1}$ and $T_{2}$ respectively.

From the geometry of the Fig. 19.12, angle of lap of the band on the drum,

$$
\theta=360^{\circ}-150^{\circ}=210^{\circ}=210 \times \frac{\pi}{180}=3.666 \mathrm{rad}
$$

We know that

$$
\begin{align*}
& 2.3 \log \left(\frac{T_{1}}{T_{2}}\right)=\mu \cdot \theta=0.2 \times 3.666=0.7332 \\
& \log \left(\frac{T_{1}}{T_{2}}\right)=\frac{0.7332}{2.3}=0.3188 \text { or } \frac{T_{1}}{T_{2}}=2.08 \tag{i}
\end{align*}
$$

... (Taking antilog of 0.3188 )
Taking moments about the fulcrum $O$,

$$
\begin{array}{rlrrr} 
& & T_{2} \times 120 & =100 \times 300=30000 & \text { or }
\end{array} T_{2}=30000 / 120=250 \mathrm{~N}, ~ \ldots[\text { From equation }(i)]
$$

We know that torque applied,

$$
T_{\mathrm{B}}=\left(T_{1}-T_{2}\right) r=(520-250) 0.12=32.4 \mathrm{~N}-\mathrm{m} \text { Ans. }
$$

2. Number of turns of the wheel before it is brought to rest

Let $\quad n=$ Number of turns of the wheel before it is brought to rest.
We know that kinetic energy of rotation of the drum

$$
=\frac{1}{2} \times I \cdot \omega^{2}=\frac{1}{2} \times m \cdot k^{2} \cdot \omega^{2}=\frac{1}{2} \times 400(0.45)^{2}(31.42)^{2}=40000 \mathrm{~N}-\mathrm{m}
$$

This energy is used to overcome the work done due to the braking torque ( $T_{\mathrm{B}}$ ).
or

$$
\begin{aligned}
\therefore \quad 40000 & =T_{\mathrm{B}} \times 2 \pi n=32.4 \times 2 \pi n=203.6 n \\
n & =40000 / 203.6=196.5 \text { Ans. }
\end{aligned}
$$

Time required to bring the wheel to rest
We know that the time required to bring the wheel to rest $=\mathrm{n} / \mathrm{N}=196.5 / 300=0.655 \mathrm{~min}=39.3 \mathrm{~s}$ Ans

Example 19.13. The arrangement of an internal expanding friction brake, in which the brake shoe is pivoted at ' C ' is shown in Fig. 19.26. The distance ' CO ' is 75 mm , $O$ being the centre of the drum. The internal radius of the brake drum is 100 mm . The friction lining extends over an arc $A B$, such that the angle $A O C$ is $135^{\circ}$ and angle BOC is $45^{\circ}$. The brake is applied by means of a force at $Q$ perpendicular to the line CQ, the distance CQ being 150 mm .

The local rate of wear on the lining may be taken as proportional to the normal pressure on an element at an angle of ' $\theta$ ' with $O C$ and may be taken as equal to $p_{1} \sin \theta$, where $p_{1}$ is the maximum intensity of normal pressure.

The coefficient of friction may be taken as 0.4 and the braking torque required is $21 \mathrm{~N}-\mathrm{m}$. Calculate the force $Q$ required to operate the brake when 1. The drum rotates clockwise, and 2. The drum rotates anticlockwise.

Solution. Given : $O C=75 \mathrm{~mm} ; r=100 \mathrm{~mm}$;


All dimensions in mm
Fig. 19.26
$\theta_{2}=135^{\circ}=135 \times \pi / 180=2.356 \mathrm{rad} ; \theta_{1}=45^{\circ}=45 \times \pi / 180=0.786 \mathrm{rad} ; l=150 \mathrm{~mm}$;
$\mu=0.4 ; T_{\mathrm{B}}=21 \mathrm{~N}-\mathrm{m}=21 \times 10^{3} \mathrm{~N}-\mathrm{mm}$

## 1. Force ' $Q$ ' required to operate the brake when drum rotates clockwise

We know that total braking torque due to shoe ( $T_{\mathrm{B}}$ ),

$$
\begin{aligned}
21 \times 10^{3} & =\mu \cdot p_{1} \cdot b \cdot r^{2}\left(\cos \theta_{1}-\cos \theta_{2}\right) \\
& =0.4 \times p_{1} \times b(100)^{2}\left(\cos 45^{\circ}-\cos 135^{\circ}\right)=5656 p_{1} \cdot b \\
\therefore \quad p_{1} \cdot b & =21 \times 10^{3} / 5656=3.7
\end{aligned}
$$

Total moment of normal forces about the fulcrum $C$,

$$
\begin{aligned}
M_{\mathrm{N}} & =\frac{1}{2} p_{1} b . r . O C\left[\left(\theta_{2}-\theta_{1}\right)+\frac{1}{2}\left(\sin 2 \theta_{1}-\sin 2 \theta_{2}\right)\right] \\
& =\frac{1}{2} \times 3.7 \times 100 \times 75\left[(2.356-0.786)+\frac{1}{2}\left(\sin 90^{\circ}-\sin 270^{\circ}\right)\right] \\
& =13875(1.57+1)=35660 \mathrm{~N}-\mathrm{mm}
\end{aligned}
$$

and total moment of friction force about the fulcrum $C$,

$$
\begin{aligned}
M_{\mathrm{F}} & =\mu \cdot p_{1} \cdot b \cdot r\left[r\left(\cos \theta_{1}-\cos \theta_{2}\right)+\frac{O C}{4}\left(\cos 2 \theta_{2}-\cos 2 \theta_{1}\right)\right] \\
& =0.4 \times 3.7 \times 100\left[100\left(\cos 45^{\circ}-\cos 135^{\circ}\right)+\frac{75}{4}\left(\cos 270^{\circ}-\cos 90^{\circ}\right)\right] \\
& =148 \times 141.4=20930 \mathrm{~N}-\mathrm{mm}
\end{aligned}
$$

Taking moments about the fulcrum $C$, we have

$$
\begin{array}{ll} 
& Q \times 150=M_{\mathrm{N}}+M_{\mathrm{F}}=35660+20930=56590 \\
\therefore & Q=56590 / 150=377 \mathrm{~N} \text { Ans. }
\end{array}
$$

2. Force ' $Q$ ' required to operate the brake when drum rotates anticlockwise Taking moments about the fulcrum $C$, we have

$$
\begin{aligned}
& Q \times 150=M_{\mathrm{N}}-M_{\mathrm{F}}=35660-20930=14730 \\
\therefore & Q=14730 / 150=98.2 \mathrm{~N} \text { Ans. }
\end{aligned}
$$

