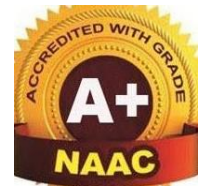




# **ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY**

## **DEPARTMENT OF MATHEMATICS**



### **LECTURE NOTES ON**

### **BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING**

### **UNIT IV : INVENTORY MODELS**

#### **INTRODUCTION**

Inventory may be defined as a stock of idle resources of any kind having an economic value kept for the purpose of future affairs. Inventories are essential for business and maintenance of inventories costs money by way of expenses on stores, personnel, equipment, insurance etc.

Various costs associated with inventory model are often classified as follows:

- **Purchase (or production) cost.** The cost of purchasing (or producing) a unit of an item is known as purchase (or production) cost. The purchase price will become important when quantity discounts are allowed for purchases above a certain quantity.
- **Ordering (or set up) cost.** If any item is purchased, an ordering cost is incurred each time an order is placed. This cost includes the following factors: administrative (paper work, telephone calls, postage), transportation of items ordered, receiving and inspection of goods etc. If a firm produces its own inventory instead of purchasing the same from an outside source, then production set-up costs are analogous to ordering costs.
- **Carrying (or holding) cost.** Holding cost represents the cost of maintaining inventory in stock. It includes the interest on capital, rent for space used for storage, insurance of stored equipment, depreciation, taxes, etc.
- **Shortage (or stock out) cost.** The penalty cost for running out of stock (i.e. when an item cannot be supplied on the customer's demand) is known as shortage cost. This cost includes the loss of potential profit through the sale of items demanded and loss of goodwill, in terms of permanent loss of customers and its associated lost profit in future sales.
- **Salvage cost (or selling price).** When the demand for an item is affected by its quantity in stock, the decision depends upon the underlying criterion and includes the revenue from sale of the item. Salvage cost is generally combined with the storage cost and hence is neglected.

- **Demand:** Demand is the number of units required per period and may be either known exactly or known in terms of probabilities. Problems in which demand is known with certainty, are called deterministic problems, whereas problems in which demand is assumed to be a random variable are called **probabilistic problems**.
- **Order cycle.** The time period between placements of two successive orders is referred to as an order cycle.
- **Time horizon.** The time period over which the inventory level will be controlled is referred to as time horizon. This can be finite or infinite depending upon the nature of demand. This is also known as the planning period over which the inventory is to be controlled.
- **Lead time.** The time between placing an order and its arrival in stock is known as lead time. The lead time can be either deterministic or probabilistic. If the lead time is zero, there is no need for placing an order in advance.
- **Economic Order Quantity (EOQ):** Economic order quantity is the size of the order representing standard quality of material and is the one for which the aggregate of the costs of procuring the inventory and costs of holding the inventory is minimum.
- **Re-order level.** The level between maximum and minimum stock, at which the purchasing (or manufacturing) activities must start for replenishment is known as reorder level.
- **Stock replenishment:** The rate at which items are added to inventory is one of the important parameters in inventory models. The actual replenishment of items may occur instantaneously or gradually. Instantaneous replacement is possible when the stock is purchased from outside sources while gradual replenishment is possible when the product is manufactured by the company.
- **Re-order quantity:** This is the quantity of replenishment order. In certain cases, it is the Economic Order Quantity.

## Notations

$Q$  = number of units ordered per order

$D$  = demand in units of inventory per time period (R)

$N$  = number of orders placed per time period

$TC$  = total inventory cost

$C_3$  = ordering cost (or setup cost per production run) per order

$C_1$  = Carrying or holding cost per unit per period of time the inventory is held

$C$  = Purchase or manufacturing price per unit inventory

$C_s$  = shortage cost per unit of inventory ( $C_2$ )

$L$  = lead time

$T$  = reorder cycle time i.e. time period between placement of two successive orders as a fractional part of standard time horizon

### **Model I: EOQ Model with Uniform Demand**

The objective of the model is to determine an optimum EOQ such that the total inventory cost is minimum. Following assumptions are made for this model:

1. Demand  $D$  is constant and known.
2. Replenishment is instantaneous i.e. the entire order quantity  $Q$  is received at one time as soon as the order is released.
3. Lead time is zero.
4. Purchase price or cost per unit is constant i.e. discounts are not allowed.
5. Carry cost and ordering cost are known and constant.
6. Shortage is not allowed.

### **Model II: EOQ with Finite Rate of Replenishment or EOQ with Uniform Replenishment**

For this model, it is assumed that the production run may take a significant time to complete. Let  $r_d$  be the demand rate in units per unit of time and  $r_p$  be the replenishment rate per unit time. Assume that each cycle time  $t$  of two parts  $t_1$  and  $t_2$  such that

- (a) production is continuous and constant until  $Q$  units are produced to stock, then it stops;
- (b) the production rate  $r_p$  is greater than demand rate  $r_d$ ;
- (c) there is no replenishment (or production) during time  $t_2$  and the inventory is decreasing at the rate  $r_d$  per unit of time.

### Model III: Economic Order Quantity with Different Rates of Demands in Different Cycles

Here the stock will vanish at different time periods with a policy of ordering same quantity for replenishment of inventory. Here replenishment rate is infinite, replenishment is instantaneous and shortage is not allowed.

#### 8.3.4. Model IV: Economic Order Quantity when Shortages are Allowed

The assumptions of this model are same as that of model I except that shortages are allowed and shortages may occur regularly.

#### Problem : 1

An ice cream company sells one of its types of ice creams by weight. If the product is not sold on the day it is prepared, it can be sold for a loss of 50 paise per pound but there is an unlimited market for one day old ice creams. On the other hand, the company makes a profit of Rs 3.20 on every pound of ice cream sold on the day it is prepared. If daily orders form a distribution with  $f(x) = 0.02 - 0.0002x$ ,  $0 \leq x \leq 100$ , how many pounds of ice cream should the company prepare everyday ?

Solution : given  $C_1 = \text{Rs. } 0.50$   $C_2 = \text{Rs. } 3.20$

Let  $Q$  be the amount of ice cream prepared every day

$$\int_0^Q f(x) dx = \frac{C_2}{C_1 + C_2}$$
$$\text{Now } \int_0^Q (0.02 - 0.0002x) dx = \frac{3.20}{0.50 + 3.20} \Rightarrow 0.02Q - \frac{0.0002Q^2}{2} = 0.865$$
$$\Rightarrow 0.0001Q^2 - 0.02Q + 0.865 = 0$$

It is Quadratic equation, to solve this equation, use  $ax^2 + bx + c = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Therefore  $Q = 136.7$  (or)  $Q = 63.5$

136.7 is not admissible, so  $Q = 63.5$

**Problem : 2**

A commodity is to be supplied at a constant rate of 200 units per day. Supplies for any amounts can be had at any required time, but each ordering costs Rs 50.00. Cost of holding the commodity in inventory is Rs 2.00 per unit per day while the delay in the supply of the items induces a penalty of Rs 10.00 per unit per delay of one day. Formulate the average cost function of this situation and find the optimal policy (q,t) where t is the reorder cycle period and q is the inventory level after re-order. What should be the best policy if the penalty cost becomes infinite?

**Solution:**

$R = 200 \text{ units}$  ,  $C_1 = \text{Rs.}2$  ,  $C_3 = \text{Rs.}50$ ,  $C_2 = \text{Rs.}10 \text{ per unit per day}$

$$EOQ = q^* = \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{2C_3 R}{C_1}} = 109.5 \text{ units}$$

$$t_0 = \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{2C_3}{C_1 R}} = 0.547$$

When  $C_2 \rightarrow \infty$ , then  $q^* = \sqrt{\frac{2C_3 R}{C_1}} = 100 \text{ units}$ .

**Problem : 3**

A manufacturer has to supply his customer with 600 units of his products per year. Shortage are not allowed and storage cost amounts to 60 paise per unit year. The set up cost is Rs. 80. find (1) the economic order quantity (2) the minimum average yearly cost (3) the optimum number of orders per year (4) the optimum period of supply per optimum order.

**Soln.**

$R = 600 \text{ units}$  ,  $C_1 = 0.60 \text{ per unit year}$  ,  $C_3 = \text{Rs.}80$

$$(1) \text{ Economic Order Quantity} = q^* = \sqrt{\frac{2C_3 R}{C_1}} = 400 \text{ units / year}$$

$$(2) C_0 = \sqrt{2C_3 C_1 R} = \text{Rs.}240$$

$$(3) N^* = \frac{\text{Demand}}{EOQ} = \frac{600}{400} = \frac{3}{2}$$

$$(4) \text{ Time between two consecutive orders} = t_0 = \sqrt{\frac{2C_3}{C_1 R}} = 0.667 = \frac{2}{3} \text{ of a year}$$

**Problem : 4**

A company uses to order a new machine after a certain fixed time. It is observed that one of the parts of the machine is very expensive if it is ordered with out the machine. The cost of spare part when ordered with out the machine. The cost of spare part when ordered with the machine is Rs.500. The cost of down time of the machine and the cost of arranging the new part is Rs. 10,000.00. From the past records it is observed that spare part is required with the probabilities mentioned below.

Demand (r )	0	1	2	3	4	5	6
Probability P( r)	0.90	0.05	0.02	0.01	0.01	0.01	0.00

Find the optimal number of spare parts which should be ordered with the order of the machine?

**Soln.**

Demand (r )	0	1	2	3	4	5
Probability P( r)	0.90	0.05	0.02	0.01	0.01	0.01
Cumulative probability	0.90	0.95	0.97	0.98	0.99	1.00

$$\text{Now } \frac{C_2}{C_1 + C_2} = \frac{10,000}{10500} = 0.952 \quad [C_1 = \text{Rs.}500, C_2 = \text{Rs.}10000]$$

Here 0.952 lies between 1 and 2.

Therefore the optimum number of spare parts to be ordered is 2.

**Problem : 5**

The demand for an item in a company is 18000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one set up is Rs.500.00 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages.

**Soln.**

$R = 18000 \text{ units per year} = 3000 \text{ units per month}$ ,  $C_1 = \text{Rs.}0.15 \text{ per month}$ ,  
 $C_3 = \text{Rs.}500$ ,  $C_2 = \text{Rs.}20$ ,  $K = 3000 \text{ units per month}$ .

$$EOQ = q^* = \sqrt{\frac{C_1 + C_2}{C_2}} \times \sqrt{\frac{K}{K - R}} \times \sqrt{\frac{2C_3 R}{C_1}} = 4490 \text{ units}$$

$$\text{Number of Shortages} = S = \frac{C_1}{C_1 + C_2} q^* \left(1 - \frac{R}{K}\right) = 16.71 \text{ units.}$$

### Problem : 6

For an item production is instantaneous. The storage cost of one item is Rs. 1 per month and the set up cost is Rs.25 per run. If the demand is 200 units per month, find the optimum quantity to be produced per set-up and hence determine the total cost of storage and set-up per month.

**Soln.**

$R = 200 \text{ units per month}$ ,  $C_1 = \text{Rs.}25 \text{ per run}$ ,  $C_3 = \text{Rs.}1 \text{ per unit per month}$

$$\text{Given } EOQ = q^* = \sqrt{\frac{2C_3 R}{C_1}} = 100 \text{ units}$$

$$C_{\min} = C^* = \sqrt{2C_1 C_3 R} = \text{Rs.}100$$

Total costs of storage and set up =  $25 + (1 * 100) = \text{Rs.} 125$ .

### Problem : 7

If the demand for a certain product has a rectangular distribution between 4000 and 5000, find the optimal order quantity if storage cost is Rs.1 per unit and shortage cost is Rs. 7 per unit.

**Soln.**

Here  $C_2 = \text{Rs.} 7$ ,  $C_1 = \text{Rs.}1$

$$\int_0^Q f(x) dx = \frac{C_2}{C_1 + C_2} \Rightarrow \int_{4000}^Q \frac{1}{5000 - 4000} dx = \frac{7}{8} \Rightarrow \int_{4000}^Q \frac{1}{1000} dx = \frac{7}{8} \Rightarrow Q = 4875$$

### Problem : 8

A milk company sells one of its types of ice creams by weight. If the product is not sold on the day it is prepared, it can be sold for a loss of 50 paise per pound but there is an unlimited market for one day old milk. On the other hand, the company makes a profit of Rs 3.20 on every pound of milk sold on the day it is prepared. If daily orders form a distribution with  $f(x) = 0.02 - 0.0002x$ ,  $0 \leq x \leq 100$ , how many pounds of ice cream should the company prepare everyday ?

**Solution :**

given  $C_1 = \text{Rs. } 0.50$   $C_2 = \text{Rs. } 3.20$

Let  $Q$  be the amount of milk prepared every day

$$\int_0^Q f(x) dx = \frac{C_2}{C_1 + C_2}$$

$$\text{Now } \int_0^Q (0.02 - 0.0002x) dx = \frac{3.20}{0.50 + 3.20} \Rightarrow 0.02Q - \frac{0.0002Q^2}{2} = 0.865$$

$$\Rightarrow 0.0001Q^2 - 0.02Q + 0.865 = 0$$

It is Quadratic equation, to solve this equation, use  $ax^2 + bx + c = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Therefore  $Q = 136.7$  (or)  $Q = 63.5$ , 136.7 is not admissible, so  $Q = 63.5$ .

**Problem : 9**

A commodity is to be supplied at a constant rate of 200 units per day. Supplies for any amounts can be had at any required time, but each ordering costs Rs 50.00. Cost of holding the commodity in inventory is Rs 2.00 per unit per day while the delay in the supply of the items induces a penalty of Rs 10.00 per unit per delay of one day. Formulate the average cost function of this situation and find the optimal policy  $(q, t)$  where  $t$  is the reorder cycle period and  $q$  is the inventory level after re-order. What should be the best policy if the penalty cost becomes infinite?

**Solution:**

$R = 200 \text{ units}$ ,  $C_1 = \text{Rs. } 2$ ,  $C_3 = \text{Rs. } 50$ ,  $C_2 = \text{Rs. } 10 \text{ per unit per day}$

$$EOQ = q^* = \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{2C_3R}{C_1}} = 109.5 \text{ units}$$

$$t_0 = \sqrt{\frac{C_1 + C_2}{C_2}} \sqrt{\frac{2C_3}{C_1R}} = 0.547$$

When  $C_2 \rightarrow \infty$ , then  $q^* = \sqrt{\frac{2C_3R}{C_1}} = 100 \text{ units}$ .

**Problem : 10**

A manufacturer has to supply his customer with 600 units of his products per year. Shortage are not allowed and storage cost amounts to 60 paise per unit year. The set up cost is Rs. 80. find (1)



the economic order quantity (2) the minimum average yearly cost (3) the optimum number of orders per year (4) the optimum period of supply per optimum order.

**Soln.**

$$R = 600 \text{ units}, C_1 = 0.60 \text{ per unit year}, C_3 = \text{Rs.}80$$

$$(4) \text{ Economic Order Quantity} = q^* = \sqrt{\frac{2C_3R}{C_1}} = 400 \text{ units / year}$$

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