UNIT I

1.5 THIN CYLINDERS

1.5.1 INTRODUCTION

The vessels such as boilers, compressed air receivers etc., are of cylindrical and spherical forms. These vessels are generally used for storing fluids (liquid or gas) under pressure. The walls of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylindrical vessels is less than 1/15 to 1/20 of its internal diameter, the cylindrical vessel is known as thin cylinder. In case of thin cylinders, the stress distribution is assumed uniform over the thickness of the wall.

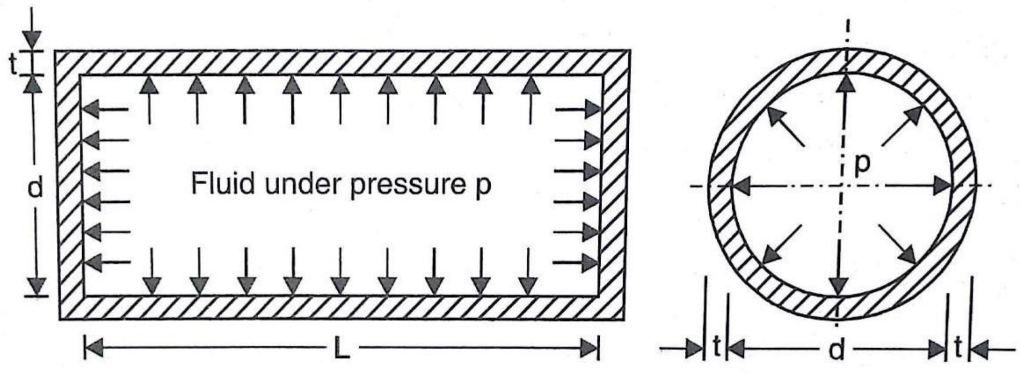
1.5.2 THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

Let d = Internal diameter of the thin cylinder

t =Thickness of the wall of the cylinder

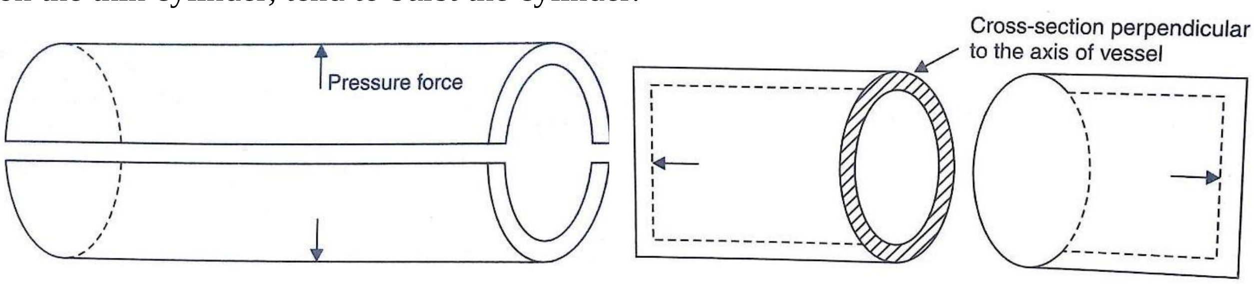
p =Internal pressure of the fluid

L =Length of the cylinder



One of the internal pressure p, the cylindrical vessel may fail by splitting up in any one of the two ways.

The forces, due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder.



The forces, due to pressure of the fluid, acting at the thin cylinder, tend to burst the thin cylinder.

1.5.3. STRESSES IN A THIN CYLINDRICAL VESSEL SUBJECTED TOINTERNAL PRESSURE

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the cross section along the axis and on the cross section perpendicular to the axis are set up. These stresses are tensile and are known as:

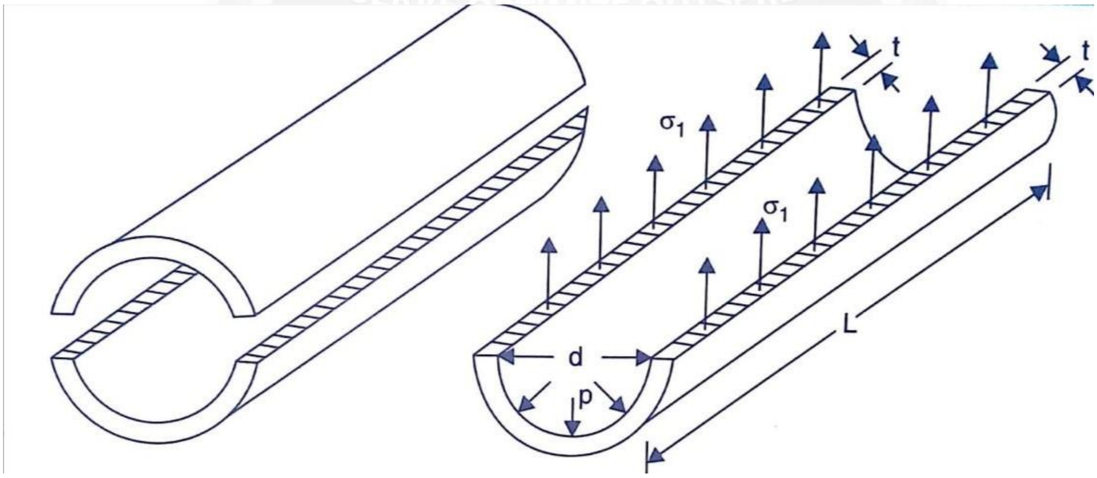
1. Circumferential stress (or hoop stress) and
2. Longitudinal stress

The name of the stress is given according to the direction in which the stress is acting. The stress acting along the circumference of the cylinder is called circumferential stress whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stress. The circumferential stress is also known as hoop stress. The stress set up in is circumferential stress whereas the stress set up in is longitudinal stress.

1.5.4 EXPRESSION FOR CIRCUMFERENTIAL STRESS (OR) HOOP STRESS

Consider a thin cylinder vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place.

The expression for hoop stress or circumferential stress is obtained as given below.



Let $\quad p$ =Internal pressure of the fluid d =Internal diameter of the cylinder

t =Thickness of the wall of the cylinder

σ_1 =Circumferential or hoop stress in the material

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material. In the limiting case, the two forces should be equal.

Forces due to fluid pressure $= p \times$Area on which p is acting

$= p \times (d \times L)$…................(i)

Forces due to circumferential stress

$= \sigma_1 \times$ Area on which σ_1 is acting

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t$…................(ii)

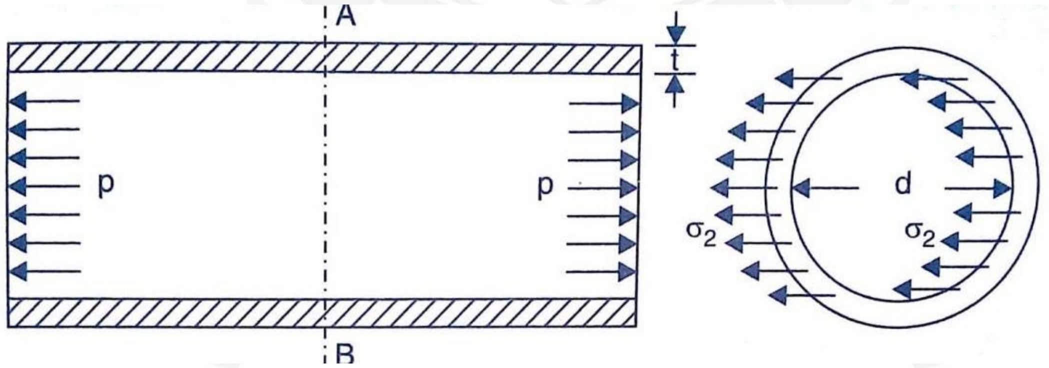
Equating (i) and (ii) we get

$P \times d \times L = 2\sigma_1 \times L \times t$

$\sigma_1 = \frac{Pd}{2t}$ This stresses is tensile.

1.5.5 EXPRESSION FOR LONGITUDINAL STRESS

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB.



The longitudinal stress (σ_2) developed in the material is obtained as:

Let p =Internal pressure of fluid stored in thin cylinder

d =Internal diameter of cylinder

t =Thickness of the cylinder

σ_2 =Longitudinal stress in the material

Thus bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress developed in the material. In the limiting case, both the forces should be equal.

Forces due to fluid pressure = p × Area on which p is acting

$= p \times \frac{\pi}{4} d^2$

Resisting force = σ_2 × area on which σ_2 is acting

$= \sigma_2 \times \pi d \times t$

Hence in the limiting case

Force due to fluid pressure = resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\sigma_2 = \frac{pd}{4t}$$

The stress σ_2 is also tensile equation can be written as

$$\sigma_2 = \frac{pd}{2 \times 2t}$$

$$\sigma_2 = \frac{1}{2} \times \sigma_1 \qquad \left(\because \quad \sigma_1 = \frac{pd}{2t}\right)$$

or Longitudinal stress=Half of circumferential stress

This also means that circumferential stress is two times the longitudinal stress Hence in the material of the cylinder the permissible stress should be less than the circumferential stress should not be greater than the permissible stress.

Maximum shear stress At any point in the material of the cylindrical shell, there are two principle stresses, namely a circumferential stress of magnitude $\sigma_1 = pd/2t$ acting circumferentially and a longitudinal stress of magnitude $\sigma_2 = pd/4t$ acting parallel to the axis of the shell. These two stresses are tensile and perpendicular to each other.

Maximum shear stress $\tau_{max} = \sigma_1 - \frac{\sigma_2}{2}$

$$= \frac{pd}{4t} - \frac{\frac{pd}{4t}}{2}$$

$$\tau_{max} = \frac{pd}{8t}$$

Problem 1.5.1: A cylindrical pipe of diameter 1.5m and the thickness 1.5 cm is subjected to an internal fluid pressure of 1.2N/mm^2 Determine (i) Longitudinal stress developed in the pipe, and (II)circumferential stress developed in the pipe.

Given data:

Diameter of pipe	$d=1.5\ m = 1.5\times10^3\ mm$
Thickness	$t=1.5cm = 15\ mm$
Internal fluid pressure	$p=1.2\ N/mm^2$

To find:

Longitudinal stress	$\sigma_2 = ?$
Circumferential stress	$\sigma_1 = ?$

Solution:

As the ratio $\frac{t}{d} = \frac{1.5\times10-2}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$ hence this is a case of thin cylinder.

Here unit of pressure (p) in N/mm^2 Hence the unit of σ_1 and σ_2 will also be in N/mm^2

(i) The longitudinal stress(σ_2) is given by equation

$$\sigma_2 = \frac{pd}{4t} = \frac{1.2\times1.5\times10^3}{4\times15} = 30\ N/mm^2$$

(ii) The circumferential stress (σ_1) is given by equation

$$\sigma 1 = \frac{pd}{2t} = \frac{1.2\times1.5\times10^3}{2\times15} = 60\ N/mm^2$$

Result:

Longitudinal stress	$\sigma_2 = \mathbf{30\ N/mm^2}$
Circumferential stress	$\sigma_1 = \mathbf{60\ N/mm^2}$

Problem1.5.2: A cylinder of internal diameter 2.5m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm^2 determine the internal pressure of the gas.

Given data:

Internal diameter of cylinder	$d =2.5m = 2.5\times10^3\ mm$
Thickness of the cylinder	$t =5cm=50\ mm$
Maximum permissible stress	$=80\ N/mm^2$

To find:

Internal pressure of the gas p =?

Solution:

Maximum permissible stress is available in the circumferential stress (σ_1)

$$\therefore \quad \text{circumferential stress } (\sigma_1) = \frac{pd}{2t}$$

$$80 = \frac{p \times 2.5 \times 10^3}{2 \times 50}$$

$$\gg \quad p = 3.2 N/mm^2$$

Result:

Internal pressure of the gas p =**3.2N/mm²**

Problem 1.5.3: A cylinder of internal diameter 0.50 m contains air at a pressure of 7N/mm²(gauge). If the maximum permissible stress induced in the material is 80N/mm², find the thickness of the cylinder.

Given data:

Internal dia of cylinder d=0.50m = 500 mm

Internal pressure of air, p=7 N/mm²

Circumferential stress, σ_1=80N/mm² ($\because$ maximum permissible stress)

To find:

Thickness of cylinder t =?

Solution:

$$\text{Wkt} \quad \text{Circumferential stress } (\sigma_1) = \frac{pd}{2t}$$

$$80 = \frac{7 \times 500}{2 \times t}$$

$$\gg \quad t = 21.88mm$$

If the value t is taken more than 21.875 mm(sat t=21.88 mm), the stress induced will be less than 80N/mm².

Hence take t=21.88 mm or say 22 mm

Result:

Thickness of cylinder t = **22 mm**

Problem 1.5.4: A thin cylinder of internal diameter 1.25m contains a fluid at an internal pressure of $2N/mm^2$. Determine the maximum thickness of the cylinder if (i) The longitudinal stress is not to exceed $30N/mm^2$ and (ii)The circumferential stress is not to exceed $45N/mm^2$

Given data:

Internal dia of cylinder, $d = 1.25\ m = 1.25\times10^3\ mm$

Internal pressure of fluid, $p = 2N/mm^2$

Longitudinal stress $\sigma_2 = 30n/mm^2$

Circumferential stress, $\sigma_1 = 45N/mm^2$

To find:

Thickness of cylinder $t = ?$

Solution:

Wkt Circumferential stress $(\sigma_1) = \dfrac{pd}{2t}$

$$45 = \frac{2 \times 1.25\times10^3}{2\times t}$$

$\gg$ $t = 27.7\ mm$

Wkt, longitudinal stress $\sigma_2 = \dfrac{pd}{4t}$

$$30 = \frac{2 \times 1.25\times10^3}{4 \times t}$$

$\gg$ $t = 28.0\ mm$

from the above two thickness value it is clear that t should not be less than 27.7mm. Hence take t=28. mm.

Result:

Thickness of cylinder $t = 28\ mm$

Problem 1.5.5: A water main 80 cm diameter contains water at a pressure head of 100m. If the weight density of water is $9810N/m^3$, find the thickness of the metal required forthe water main given the permission stress as $20N/mm^2$.

Given data:

Diameter of main, $d = 80\ cm = 800\ mm$

Pressure head of water, $h=100$ m $= 100 \times 10^3$ mm

Weight density of water $\omega = \rho \times g = 1000\times9.81 = 9810$ N/m^3

Permissible stress $=20$N/mm^2

To find:

Thickness of the metal $t = ?$

Solution:

Permissible stress is equal to circumferential stress($\sigma1$)

Pressure of water inside the water main,

$$p = \rho \times g \times h = \omega.h = 9810\times100 \text{ N/m}^2$$

Here σ_1 is in N/mm^2 hence pressure (p) should be N/mm^2. The value of p in N/mm^2 is given as

$$P = 9810\times100/1000^2$$

$$= 0.981 \text{N/mm}^2$$

Wkt Circumferential stress $\sigma_1 = \frac{pd}{2t}$

$$20 = \frac{0.981 \times 800}{2\times t}$$

$\gg$ $t = 20$ mm

Result:

Thickness of the metal $t = 20$ mm

EFFICIENCY OF A JOINT

The cylindrical shells such as boilers are having two types of joints namely longitudinal joint and circumferential joint. In case of a joint, holes are made in the material of the shell for the rivets. Due to the holes, the area offering resistance decreases. Due to the decreases in area, the stress developed in the material of the shell will be more.

Hence in case of riveted shell the circumferential and longitudinal stresses are greater than what are given by eqn. If the efficiency of a longitudinal joint and circumferential joint are given then the circumferential and longitudinal stresses are obtained as:

Let η_l = Efficiency of a longitudinal joint, and

η_c = Efficiency of the circumferential joint.

Then the circumferential stress (σ_1) is given as

$$\sigma_1 = \frac{pd}{2t\times\eta_l}$$

and the longitudinal stress (σ_2) is given as

$$\sigma_2 = \frac{pd}{4t\times\eta_c}$$

Problem 1.5.6: A boiler is subjected to an internal steam pressure of $2N/mm^2$. The thickness of boiler plate is 2.0cm and permissible tensile stress is $120N/mm^2$. Find out the maximum diameter when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%

Given data:

Internal steam pressure p = $2N/mm^2$

Thickness of boiler plate t = 2.0cm = 20mm

Permissible tensile stress $=120N/mm^2$

Efficiency of Longitudinal joint, η_l=90%=0.90

Efficiency of circumferential joint, η_c=40%=0.40

To find:

Find the maximum diameter = ?

Solution:

For Circumferential stress σ_1= 120 N/ mm^2

Wkt Circumferential stress $\sigma_1 = \frac{pd}{2t\times\eta_l}$

$$120 = \frac{2\times d}{2\times 20\times 0.90}$$

$\gg$ d = 2160 mm .... (i)

For longitudinal stress σ_2 = 120 N/ mm^2

Wkt, longitudinal stress $\sigma_2 = \frac{pd}{4t\times\eta_c}$

$$120 = \frac{2\times d}{4\times 20\times 0.40}$$

$\gg$ d = 1920 mm ....(ii) hence suitable maximum diameter d=1920 mm.

Note: If d is taken as equal to216cm the longitudinal stress will be more than the given permissible value as shown below.

$$\sigma_2 = \frac{pd}{4t\times\eta_c}$$

$$\sigma_2 = \frac{2 \times 216}{4 \times 20 \times 0.40} = 135 \text{ N/ mm}^2$$

Problem 1.5.7: A boiler shell is to be mad of 15mm thick plate having a limiting tensile stress of 120 N/mm^2. If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine;

(i) The max permissible diameter of the shell for an internal pressure of 2N/mm^2

(ii) permissible intensity of internal pressure when the shell diameter is 1.5m

Given data:

Thickness of boiler shell, t =15mm

Limiting tensile stress =120N/mm^2

Efficiency of longitudinal joint η_l=70%=0.70

Efficiency of circumferential joint η_c=30%=0.30

To find:

Maximum permissible diameter d =?

Internal pressure p =?

Solution:

i) Maximum permissible diameter for an internal pressure p = 2 N/mm^2

The boiler shell should be designed for the limiting tensile stress of 120N/mm^2. First consider the limiting tensile stress as circumferential stress and then as longitudinal stress. The minimum diameter of the two case will satisfy the condition.

(a) Taking limiting tensile stress =circumferential stress σ_1 =120N/mm^2

Wkt the circumferential stress σ_1

$$\sigma_1 = \frac{pd}{2t\times\eta_l}$$

$$120 = \frac{2 \times d}{2\times15\times0.70}$$

$$\gg \qquad d = 2160 \text{ mm} \ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

(b) Taking limiting tensile stress = longitudinal stress σ_2=120N/mm^2

Wkt the longitudinal stress σ_2

$$\sigma_2 = \frac{pd}{4t\times\eta_c}$$

$$120 = \frac{2\times d}{4\times15\times0.30}$$

≫ $d = 1080$ mm

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m

d =1.5m=1500mm

(a) Taking limiting tensile stress=circumferential stress(σ_1) =120 N/mm^2

Wkt the circumferential stress σ_1

$$\sigma_1 = \frac{pd}{2t\times\eta_l}$$

$$120 = \frac{p\times1500}{2\times15\times0.70}$$

≫ $p = 1.68$ N/mm^2……………………………… (i)

(b) Taking limiting tensile stress = longitudinal stress σ_2=120 N/mm^2

Wkt the longitudinal stress σ_2

$$\sigma_2 = \frac{pd}{4t\times\eta_c}$$

$$120 = \frac{p\times1500}{4\times15\times0.30}$$

≫ $p = 1.44\ N/mm^2$ ………………………………… (ii)

value of pressure given by (i) & (ii)

Max permissible internal pressure is taken as the minimum value of (i) & (ii)

$p = 1.44 N/mm^2$

$$\sigma_2 = \frac{pd}{4t\times\eta_c}$$

$$= \frac{1.44\times1500}{4\times15\times0.30} = 140 N/mm^2.$$

Problem 1.5.8: A cylinder of thickness 1.5cm, has to withstand maximum internal pressure of 1.5N/mm2. If the ultimate tensile stress in the material of the cylinder is 300N/mm2 factor of safety 3.0 and joint efficiency 80% determine the diameter of the cylinder.

Given Data:

Thickness of cylinder $t = 1.5cm = 15$ mm

internal pressure $p = 1.5 N/mm^2$

ultimate tensile stress $= 300 N/mm^2$

FOS $= 3.0$

joint efficiency $= 80\%$

To find:

Diameter of the cylinder $d = ?$

Solution:

Working stress, σ_1 = Ultimate tensile stress/FOS

$= 300/3$

$= 100 N/mm^2$

Joint efficiency, $\eta = 80\% = 0.80$

Joint efficiency means the efficiency of longitudinal joint η_l

The stress corresponding to longitudinal joint is given by equation

$$\sigma_1 = \frac{pd}{2t\times\eta_l}$$

$$100 = \frac{\times}{d\ 2\times 15\times 0.80}$$

≫ d =1600 mm=1.6m

1.5.7. EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL

When a fluid having internal pressure (p) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are:

(i) Hoop circumferential stress(σ_1),acting on longitudinal section.

(ii)Longitudinal stress(σ_2) acting on the circumferetial section.

These stresses are principal stresses, as they are acting on principal planes. The stress in the third principal plane is zero as the thickness (t) of the cylinder is very small.

Actually the stress in the third principal plane is radial stress which is very small for thin cylinders and can be neglected.

Let p = Internal pressure of fluid

L= Length of cylindrical shell

d = Diameter of the cylindrical shell

t = Thickness of the cylindrical shell

E = Modulus of Elasticity for the material of the shell

σ_1 = Hoop stress in the material

σ_2 = Longitudinal stress in the material

δd = change in diameter due to stresses set up in the material

δL = change in length

δv = change in volume

μ =poison ratio

The value of σ_1 and σ_2 are given by eqn and as

$$\sigma_1=\frac{pd}{2t}$$

$$\sigma_2=\frac{pd}{4t}$$

Let e_1 = circumferential strain,

e_2 = Longitudinal strain,

Then circumferential strain,

$$e = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{pd}{2tE} - \mu\frac{pd}{4tE}$$

$$= \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

and longitudinal strain,

$$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$= \frac{pd}{4tE} - \mu\frac{pd}{2tE}$$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

But circumferential strain is also given as,

$$e_1 = \frac{\text{Change in circumferential due to pressure}}{\text{original circumference}}$$

$$= \frac{\text{Final circumference} - \text{original circumference}}{\text{original circumference}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$= \frac{\pi d + \pi\delta d - \pi d}{\pi d}$$

$$= \frac{\delta d}{d}$$

Equating the two values of e_1 given by equations and we get

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

Change in diameter

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$$

similarly longitudinal strain is also given as,

e_2=change in length due to pressure/original length

$= \delta L / L$

Equating the two values of e_2 given by equation

$$\delta L / L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

Change in length

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

Volumetric strain.

It is defined as change in volume divided by original volume

Volumetric strain = $\frac{\delta V}{V}$

But change in volume (δV) = Final volume - Original volume

Original volume (V) = Area of cylindrical shell × Length

$$= \frac{\pi}{4} \times d^2 \times L$$

Final volume = (Final area of cross section)×Final length

$$= \frac{\pi}{4}(d+\delta d)^2 \times (L+\delta L)$$

$$= \frac{\pi}{4}\left[d^2 + (\delta d)^2 + 2d\delta d\right] \times [L+\delta L]$$

$$= \frac{\pi}{4}\left[d^2L + (\delta d)^2L + 2dL\delta d + d^2\delta L + (\delta d)^2\delta L + 2d\delta d\delta L\right]$$

Neglecting the smaller quantities such as $(\delta d)^2L$, $(\delta d)^2\delta L$ and $2d\delta d\delta L$, we get

Final volume $= \frac{\pi}{4}\left[d^2L + 2dL\delta d + d^2\delta L\right]$

Change in volume (δV)

$$= \frac{\pi}{4}\left[d^2L + 2dL\delta d + d^2\delta L\right] - \frac{\pi}{4}\times d^2\times L$$

$$= \frac{\pi}{4}\left[2dL\delta d + d^2\delta L\right]$$

Then volumetric strain = $\delta V/V$

$$= \frac{\frac{\pi}{4}\left[2dL\delta d + d^2\delta L\right]}{\frac{\pi}{4}\times d \times L}$$

$$= 2\frac{\delta d}{d} + \frac{\delta L}{L} \quad = 2e_1 + e_2 \qquad \left[\because \frac{\delta d}{d} = e_1,\ \frac{\delta L}{L} = e_2\right]$$

$$= 2 \times \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) + \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

Substitutes the value of e_1 and e_2

$$= \frac{pd}{2tE}\left(2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right)$$

$$= \frac{pd}{2tE}\left(2 + \frac{1}{2} - \mu - \mu\right)$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

Also change in volume (δV)=V($2e_1+e_2$)

Problem 1.5.9 Calculate (i) the change in diameter, (ii) change in length (iii)change in volume of a thin cylindrical shell 100 cm diameter, 1cm thick and 5m long when subjected to internal pressure of 3N/mm2. Take the value of E=2×105N/mm^2 and poisons ratio μ = 0.3

Given data:

Diameter of shell d =100cm = 1000mm

Thickness of shell	t =1cm = 10 mm
Length of shell	L= 5m=5×10^3 mm
Internal pressure	p = 3N/mm^2
Young's modulus	E = 2×10^5
Poisson's ratio	μ = 0.30

To find:

(i)	change in diameter	δd =?
(ii)	change in length	δL =?
(iii)	change in volume	δV =?

Solution:

(i) Change in diameter (δd) is given by equation

$$\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$$

$$= \frac{3\times 1000^2}{2\times10\times2\times10^5}\left(1-\frac{0.30}{2}\right)$$

$$= 0.6375 \text{ mm}$$

(ii) Change in length (δL) is given by equation

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$= \frac{3\times 1000\times5\times10^3}{2\times10\times2\times10^5}\left(\frac{1}{2}-0.30\right)$$

$$= 0.75 \text{ mm}$$

(iii) change in volume (δV) is given by equation

$$\delta V = V[2e_1+e_2]$$

$$= V\left[2\times\frac{\delta d}{d}+\frac{\delta L}{L}\right]$$

substituting the values of δd, δL, d and L, we get

$$\delta V = V\left[2\times\frac{0.06375}{1000}+\frac{0.075}{5\times10^3}\right]$$

Where V=original volume = $\frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times1000^2\times 5\times10^3 = 3.92\times10^9$ mm^3

$$\delta V = 3.92\times10^9\left[2\times\frac{0.06375}{1000}+\frac{0.075}{5\times10^3}\right]$$

$$= 5.595\times10^6 \text{ mm}^3$$

Problem 1.5.10: A cylindrical shell 90cm long20cm internal diameter having thickness of metal as 8mm is filled with fluid at atmospheric pressure. If an additional $20cm^3$ of fluid is pumped into the cylinder find (i)the pressure exerted by the fluid on the cylinderand (ii) the hoop stress induced. Take $E=2\times10^5$ N/mm^2 and μ=0.3

Given data:

Length of cylinder L=90cm = 900mm

Diameter od cylinder d=20cm = 200 mm

Thickness of cylinder t=8mm

Increase in volume δV=Volume of additional fluid $=20\times10^3\ mm^3$

$E = 2\times10^5 N/mm^2$

μ = 0.3

To find:

(i) pressure exerted by the fluid

(ii) hoop stress induced

Solution:

volume of cylinder $V=\frac{\pi}{4}d^2\times L=\frac{\pi}{4}\times200^2\times900=2.827\times10^7\ mm^3$

(i) Let p=pressure of exerted by fluid on the cylinder

Now using eqn volumetric strain is given as

$$\frac{\delta V}{V}=2e_1+e_2$$

$$\frac{20\times10^3}{2.827\times10^7}=2e_1+e_2 \quad \text{.......(i)}$$

But e_1 and e_2 are circumferential and longitudinal strains and given by equation and respectively as

$$e_1=\frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)$$

$$e2=\frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

substitute these values in eqn (i) we get

$$\frac{20\times10^3}{2.827\times10^7}=2\frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)+\frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$\frac{20\times10^3}{2.827\times10^7} = 2\frac{p\times200}{2\times8\times2\times10^5}\left(1-\frac{0.3}{2}\right)+\frac{p\times200}{2\times8\times2\times10^5}\left(\frac{1}{2}-0.3\right)$$

$$p = 5.386\ N/mm^2$$

(ii) Hoop stress (σ_1) is given by equation

$$\sigma_1 = \frac{pd}{2t} = \frac{5.386\times200}{2\times8}$$

$$= 67.33 N/mm^2$$

Result:

(i) pressure exerted by the fluid (p) = 5.386 N/mm^2

(ii) hoop stress induced (σ_1) = 67.33 N/mm^2

Problem 1.5.11: A cylindrical vessel whose ends are closed by means of rigid flanges plates, is made of steel plate 3mm thick. The length and the internal diameter of the vessel are 50cm and 25cm respectively. Determine the longitudinal and hoop stress in the cylindrical shell due to an internal fluid pressure of 3N/mm2. Also calculate the increase in length, diameter and volume of the vessel. Take $E=2\times10^7$N/mm2 and μ=0.3

Given data:

Thickness	t = 3mm
Length of the cylindrical vessel	L= 50cm = 500 mm
Internal diameter	d = 25cm = 250 mm
Internal fluid pressure	p = 3N/mm^2
Young's modulus	E = 2×10^5 N/mm^2
Poisson's ratio	μ = 0.3

To find:

Longitudinal stress and hoop stress =?

Increase in length, diameter and volume =?

Solution:

Using equation for hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{3\times250}{2\times3}$

$= 125 N/mm^2$

Using equation for longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{3\times250}{4\times3}$

$= 62.5\ N/mm^2$

Using equation for circumferential strain $e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = = \frac{1}{E}(\sigma_1 - \frac{\sigma_2}{2})$

$= \frac{1}{2\times10^5}(125 - \frac{62.5}{2})$

$= 53.125\times10^{-5}$

And longitudinal strain, $e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$

But circumferential strain is also given by equation

$e_1 = \delta d/d$

Equating the two values of circumferential strain e_1 we get

$\delta d/d = 53.125\times10^{-5}$

$\delta d = 53.125\times10^{-5}\times d$

$= 53.125\times10^{-5}\times250$

$= 0.133$ mm

Increase in diameter δd=0.133mm

Longitudinal strain is given by equation, $e_2 = \delta L/L$, But $e_2 = \frac{\sigma_2}{E} - m\frac{\sigma_1}{E}$

Then

$\delta L/L = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$

$\delta L/L = \frac{1}{E}(\sigma_2 - \mu\sigma_1)$

$= \frac{1}{2\times10^5}(62.5\sigma_2 - \mu125)$

$=12.5\times10^{-5}$

Increase in length $\delta L = 12.5\times10^{-5}\times L$

$= 12.5\times10^{-5}\times500$

$= 0.0625$ mm.

Volumetric strain is given as $\delta V/V = 2\times e_1 + e_2$

$= 2\times53.125\times10^{-5} + 12.5\times10\text{-}5$

$= 118.75\times10^{-5}$

Increase in volume $\delta V = 118.75\times10^{-5}\times V$

$= 118.75\times10^{-5}\times\frac{\pi}{4}\times250^2 \times500$

$= 29.13 \times10^3\ mm^3$

Result :

Hoop stress and Longitudinal stress **$\sigma_1 = 125\ N/mm^2$; $\sigma_2 = 62.5\ N/mm^2$**

Increase in length, diameter and volume **$\delta L = 0.0625$ mm**

$\delta d = 0.133$ mm ;

$\delta V = 29.13 \times10^3\ mm^3$

Problem 1.5.12: A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of $3N/mm^2$. If the maximum principal stress is not to exceed $150N/mm^2$, find the thickness of the shell. Assume $E=2\times10^5N/mm^2$ and poison's ratio=0.25 Find the changes in diameter, Length and volume of the shell.

Given Data:

Diameter	d = 1.5m=1500mm
Length	L = 4m=4000mm
Internal pressure	$p = 3N/mm^2$
Max principal stress is as	$\sigma_1 = 150N/mm^2$
	$E = 2\times10^5N/mm^2$
poison's ratio	$\mu = 0.25$

To find:

Thickness of cylinder t =?

Change in length, diameter and volume =?

(i) Using hoop stress equations $\sigma_1 = \frac{pd}{2t}$

$$t = \frac{pd}{2\times\sigma_1} = \frac{3\times1500}{2\times150}$$

$=15mm$

(ii) Change in diameter $\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$

$$= \frac{3\times1500^2}{2\times15\times2\times10^5}\left(1-\frac{0.30}{2}\right)$$

$= 0.984mm$

(iii) Change in length $\delta L = \frac{pdL}{2tE}(\frac{1}{2} - \mu)$

$= \frac{3\times 1500\times 4\times 10^3}{2\times 15\times 2\times 10^5}(\frac{1}{2} - 0.30)$

$= 0.75mm$

(iv) Change in volume $\delta V/V = \frac{pd}{2tE}(\frac{5}{2} - 2\mu)$

$\delta V = \frac{pd}{2tE}(\frac{5}{2} - 2\mu)\, V$

$= \frac{3\times 1500}{2\times 15\times 2\times 10^5}(\frac{5}{2} - 2\times 0.30)\times[\frac{\pi}{4}\times 250^2 \times 500]$

$=10602875mm^3$

Result:

Thickness of cylinder **t = 15 mm**

Change in length, diameter and volume **δL = 0.984 mm**

δd = 0.75 mm ;

δV =10602875 mm³

Problem 1.5.13: A cylindrical shell 3m long which is closed as the ends has an internaldiameter of 1m and a wall thickness of 15mm. Calculate the circumferential andlongitudinal stresses induced and also changes in the dimensions of the shell, if it issubjected to an internal pressure of $1.5N/mm^2$. Take $E=2\times10^5$ N/mm^2 and $\mu=0.3$ **Given data:**

Length of shell	L=3m=3000 mm
Internal diameter	d=1m=1000 mm
Wall thickness	t=15mm
Internal pressure	$p=1.5N/mm^2$
Young’s modulus	$E= 2\times10^5\ N/mm^2$
Poison’s ratio	μ =0.3

To find:

Longitudinal stress and hoop stress =?

Increase in length, diameter and volume =?

soln:

Using equation for hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{1.5 \times 1000}{2 \times 15}$

$= 50 N/mm^2$

Using equation for longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{1.5 \times 1000}{4 \times 15}$

$= 25 N/mm^2$

Change in dimensions

Using equation for the change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{1}{2} \times \mu\right]$$

$$= \frac{1.5 \times 1000^2}{2 \times 1.5 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.3\right]$$

$$= 0.2125 \times 10^{-2} \text{ mm}$$

Using equation for change in length we get

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{1.5 \times 1000^2}{2 \times 1.5 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

$$= 0.15 \text{ mm}$$

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right) V$$

$$= \frac{1.5 \times 1000}{2 \times 1.5 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.30\right) \times \left[\frac{\pi}{4} \times 1000^2 \times 3000\right]$$

$$= 1119190.85 \text{ mm}^3$$

Result:

Hoop stress and Longitudinal stress **$\sigma_1 = 50 N/mm^2$; $\sigma_2 = 25 N/mm^2$**

Change in length, diameter and volume **δL = 0. 0.002125mm**

δd = 0.15 mm ;

$\delta V = 1119190.85 mm^3$

Problem 1.5.14: A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure Length=1.2m external diameter =20cm, thickness of metal=8mm.

Find the value of the pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of $25cm^3$ of liquid is pumped into the cylinder. Take $E=2.1\times10^5 N/mm^2$ and poison ratio=0.33

Given data:

Length L=1.2m=1200mm

External diameter D = 20cm=200 mm

Thickness t = 8mm

Internal diameter d = D-(2×t) = 200-(2×8) = 184mm

Additional Volume $\delta V=25cm^3 = 25\times10^3 mm^3$

To find:

Pressure exerted by the liquid on the walls p =?

Hoop stress induced σ_1 =?

solution:

Volume of liquid or inside volume of cylinder

$$V = \frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times184^2\times 1200$$

$$= 31908528\ mm^3$$

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\frac{25000}{31908528} = \frac{p\times184}{2\times8\times2.1\times10^5}\left(\frac{5}{2} - 2\times0.33\right)$$

$$p = \frac{2500\times2\times8\times2.1\times10^5}{31908528\times184\times(2.5-0.66)} = 7.77\ N/mm^2$$

Using Circumferential stress equations

$$\sigma_1 = \frac{pd}{2t} = \frac{7.77\times184}{2\times8} = 89.42\ N/mm^2$$

Result:

Pressure exerted by the liquid on the walls p = **7.77 N/mm²**

Hoop stress induced σ_1 = **89.42 N/mm²**

Problem 1.5.15: A hallow cylindrical drum 600mm in diameter and 3m long, has a shell thickness of 10mm. If the drum is subjected to an internal air pressure of 3 N/mm^2, determine the increases in its volumes. Take $E=2\times10^5$ N/mm^2 and poisons ratio =0.3 for the material.

Given data:

External diameter D = 600mm

Length of drum L = 3m=3000mm

Thickness of drum t = 10mm

Internal pressure $p = 3N/mm^2$

Young's modulus $E = 2\times10^5$ N/mm^2

Poison's ratio $\mu = 0.3$

To find:

Increases in volumes δV =?

Solution:

Internal diameter $d = D - (2\times t) = 600 - (2\times10)$

$= 580mm$

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)V \qquad \left[\because V = \frac{\pi}{4} \times d^2 \times L\right]$$

$$= \frac{3\times580}{2\times10\times2\times10^5}\left(\frac{5}{2} - 2\times0.3\right)\times\left[\frac{\pi}{4}\times580^2\times3000\right]$$

$= 792623000\ mm^3$

Result:

Increases in volumes **δV** = **792623000 mm^3**