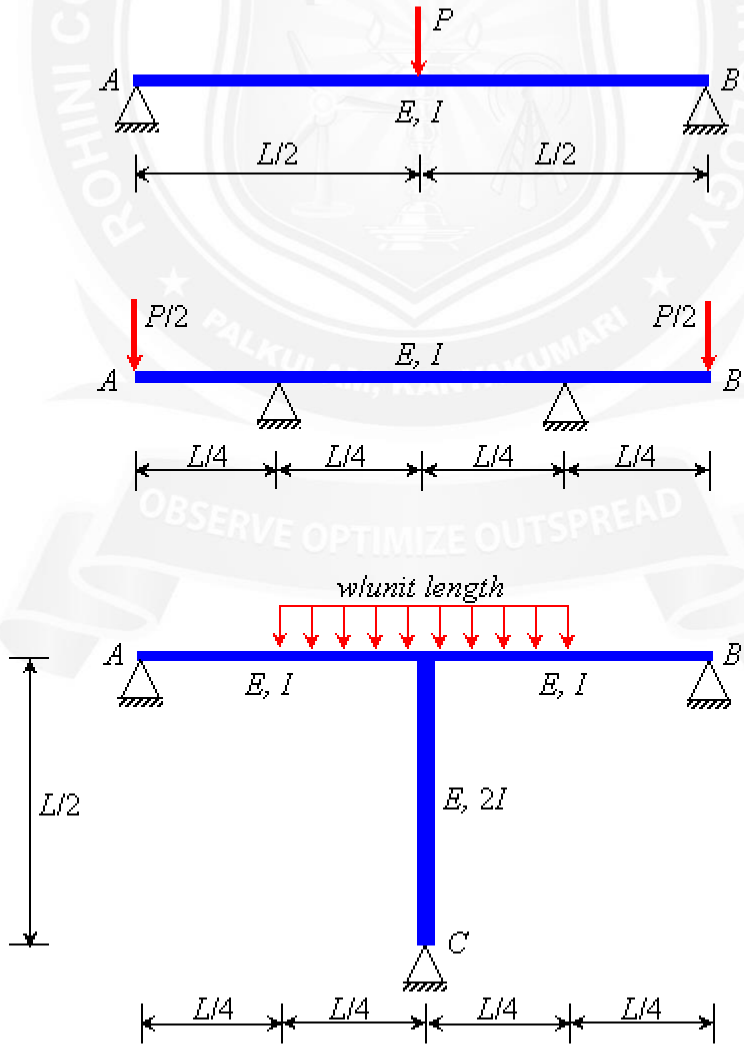
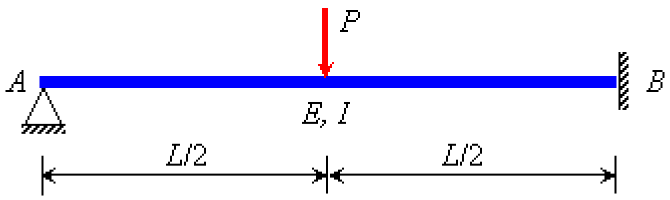
3.5 SYMMETRIC FRAMES WITH SYMMETRIC AND SKEW-SYMMETRIC LOADINGS

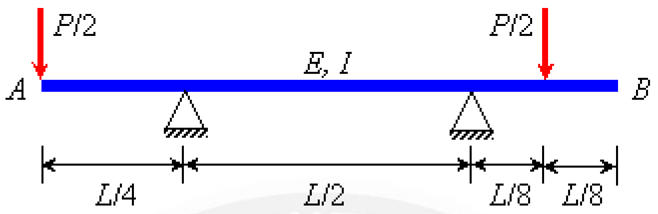
3.5.1 SYMMETRY AND ANTISYMMETRY

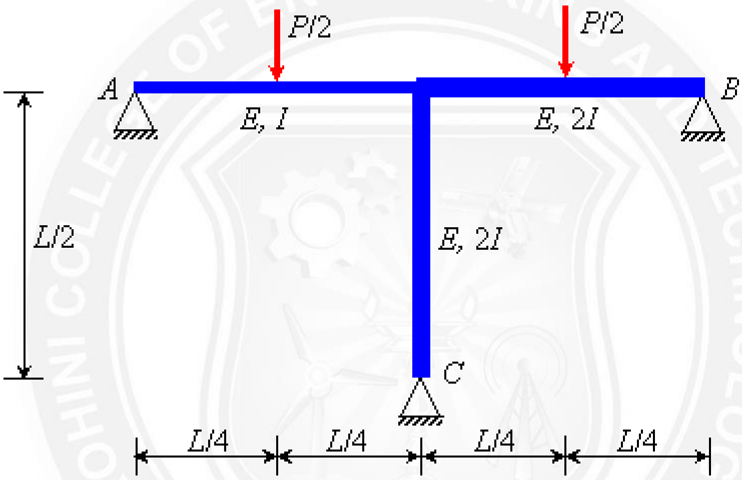
Symmetry or antisymmetry in a structural system can be effectively exploited for the purpose of analyzing structural systems. Symmetry and antisymmetry can be found in many real-life structural systems (or, in the idealized model of a real-life structural system). It is very important to remember that when we say symmetry in a structural system, it implies the existence of symmetry both in the structure itself including the support conditions and also in the loading on that structure. The systems shown in Fig. are symmetric because, for each individual case, the structure is symmetric and the loading is symmetric as well. However, the systems shown in Fig. are not symmetric because either the structure or the loading is not symmetric.



1.1. Symmetric structural systems



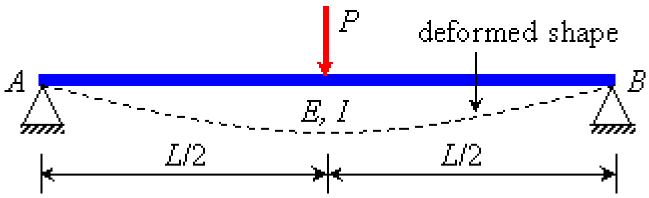


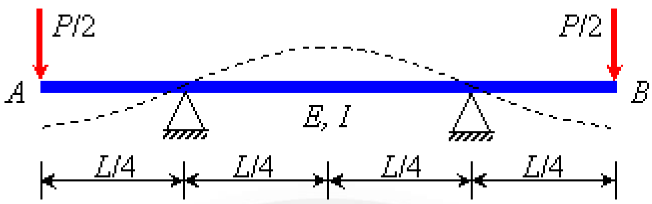


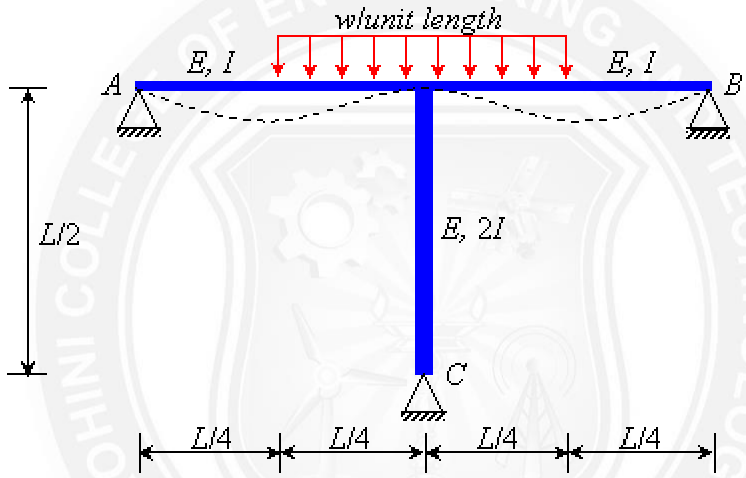
1.2. Non-symmetric (asymmetric) structural systems

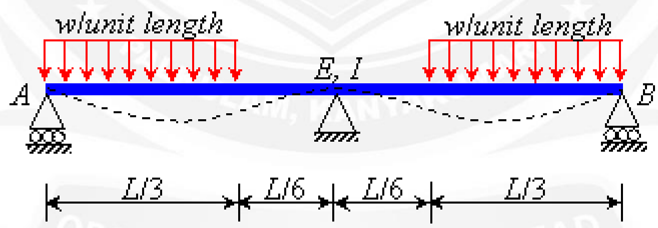
For an antisymmetric system the structure (including support conditions) remains symmetric,however, the loading is antisymmetric.The fig.1.2 ,shows the example of antisymmetric structural systems.

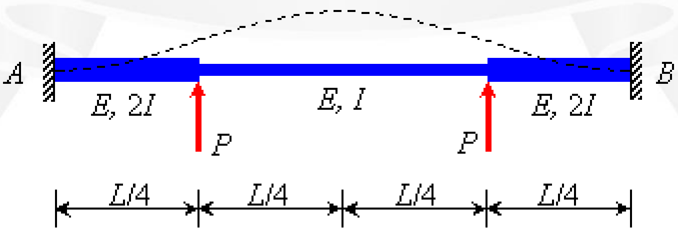
It is not difficult to see that the deformation for a symmetric structure will be symmetric about the same line of symmetry. This fact is illustrated in Fig. 1.3, where we can see that every symmetric structure undergoes symmetric deformation. It can be proved using the rules of structural mechanics (namely, equilibrium conditions, compatibility conditions and constitutive relations), that deformation for a symmetric system is always symmetric. Similarly, we always get antisymmetric deformation for antisymmetric structural systems



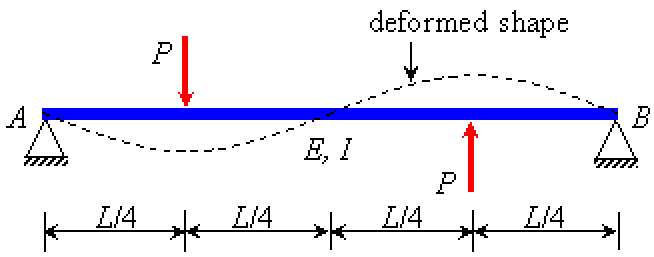


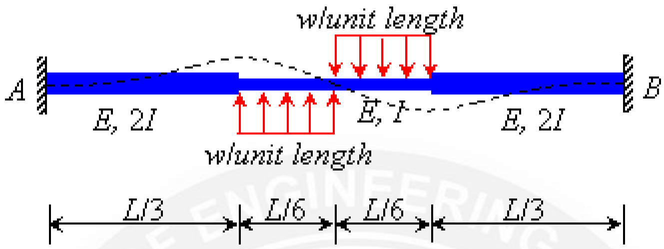


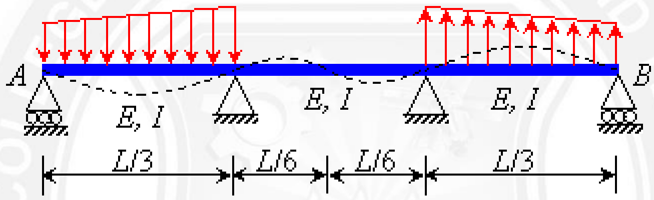




1.3. Deformation in symmetric systems

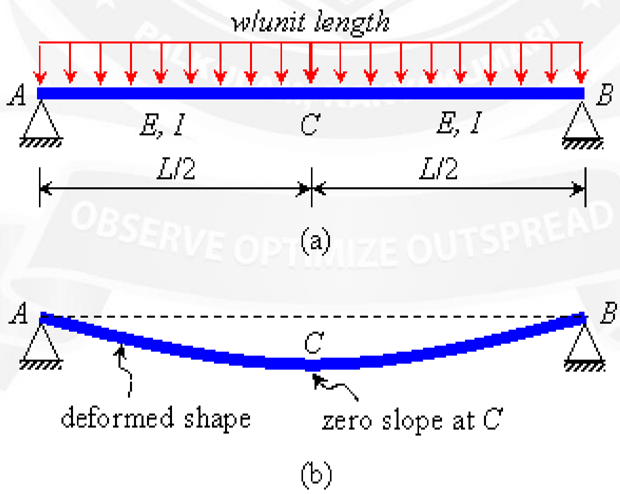






1.4. Deformation in antisymmetric systems

Let us look at beam *AB* in Fig. 1.20(a), which is symmetric about point C. The deformed shape of the structure will be symmetric as well (Fig. 1.20(b)). So, if we

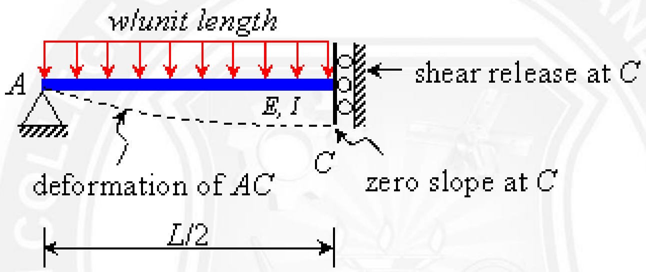


solve for the forces and deformations in part *AC* of the beam, we do not need to solve for part *CB* separately. The symmetry (or antisymmetry) in deformation gives us additional information prior to analyzing the structure and these information can be used to reduce the size of the structure that needs to be considered for analysis.

To elaborate on this fact, we need to look at the deformation condition at the point/line of symmetry (or antisymmetry) in a system. The following general rules about deformation can be deduced looking at the examples in Fig. 1.3 and Fig. 1.4:

- For a symmetric structure: slope at the point/line of symmetry is zero.
- For an antisymmetric structure: deflection at the point/line of symmetry is zero.

These information have to be incorporated when we reduce a symmetric (or antisymmetric) structure to a smaller one. If we want to reduce the symmetric beam in Fig. 1.20 to its one symmetric half *AC* , we have to integrate the fact the slope at point



1.6.Reduced system AC is adopted for analysis for beam AB

C for the reduced system AC will have to be zero. This will be a necessary boundary condition for the reduced system AC . We can achieve this by providing a support at C, which restricts any rotation, but allows vertical displacement, as shown in Fig. 1.6 (Note: this specific type of support is known as a “shear-release” or “shear-hinge”). Everything else (loading, other support conditions) remains unchanged in the reduced system. We can use this system AC for our analysis instead of the whole beam AB .

3.5.2 INTERNAL FORCE DIAGRAMS FOR A) A SYMMETRIC SYSTEM, AND B) AN ANTISYMMETRIC SYSTEM

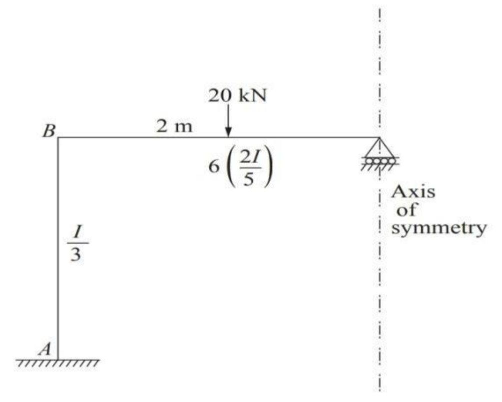
Having a priory knowledge about symmetry/antisymmetry in the structural system and in its deformed shape helps us know about symmetry/antisymmetry in internal forces in that system. (Symmetry in the system implies symmetry in equilibrium and constitutive relations, while symmetry in deformed shape implies symmetry in geometric compatibility.) Internal forces in a symmetric system are also symmetric about the same axis and similarly antisymmetric systems have antisymmetric internal forces. Detailed discussion on different types of internal forces in various structural systems and on internal force diagrams are provided in the next module (Module 2: Analysis of Statically Determinate Structures). Once we know about these diagrams we can easily see the following:

- A symmetric beam-column system has a symmetric bending moment diagram.
- A symmetric beam-column system has an antisymmetric shear force diagram.
- An antisymmetric beam-column system has an antisymmetric bending moment diagram.
- An antisymmetric beam-column system has a symmetric shear force diagram.

3.5.3 NUMERICAL EXAMPLES ON(SYMMETRIC AND SKEW-SYMMETRIC FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- **Fixed end moments in skew symmetric case**

  MFBC = $-Wab^2/l^2 = -20x2x3^2/5^2 = -4.8$ kNm;

  MFCB = $Wa^2b/l^2 = 20x2^2x3/5^2 = 9.6$ kNm;

- **Distribution Factor Table for skews symmetric case**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	I/3 = 0.33I	2.73I	0.12
	BC	6(2I)/5 = 2.4I		0.88

- **Moment Distribution Table**:

Joint	A	B	
Members	AB	BA	BC
DF	0	0.12	0.88
FEM			– 4.80
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

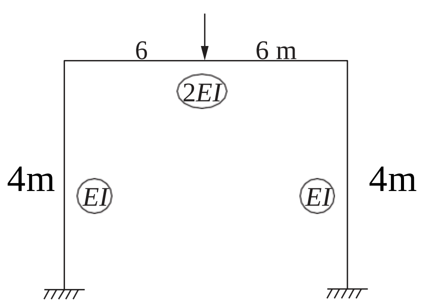
- **Result:**

MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm

PROBLEM NO:02

For the portal rigid frame compute the bending moments and draw the BMD

60KN



- **Fixed end moments in skew symmetric case**

  MFAB = MFBA = 0

  MFBC = -Wl/8 = -60X12/8 = -90 kNm

  MFCB = Wl/8 = 60X12/8 = 90 kNm

  MFCD = MFDC = 0

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3E(2I)/l = 6EI/12	EI	0.43
	BC	3E(2I)/l = 6EI/12		0.57

- **Moment Distribution Table**:

Joint	A	B	
Members	AB	BA	BC
	0	0.12	0.88
FEM		-90	90
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

- **Result:**

  MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm