LECTURE NOTES ON

BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING

UNIT I : FORMATION OF LPP & GRAPHICAL METHOD

LINEAR PROGRAMMING –GRAPHICAL METHOD

Introduction :

The linear programming method is applicable in problems characterized by the presence of decision variables. The objective function and the constraints can be expressed as **linear functions** of the **decision variables**. The decision variables represent quantities that are, in some sense, controllable inputs to the system being modeled. An objective function represents some principal objective criterion or goal that measures the effectiveness of the system such as maximizing profits or productivity, or minimizing cost or consumption. The main important feature of linear programming model is the presence of linearity in the problem.

Problem Formulation

The linear programming problem formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products. A product has a certain margin of profit per unit, and uses a common pool of limited resources. In this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints.

Procedure:

1) Graphical method in LP can be used to solve problems having only two decision variables.

2) Since a graph has two axes, select one axis each to represent one decision variable.
3) The constraints are represented as straight lines on the graph, each constraint being represented by one line.
4) Take the first constraint. Consider it only as an equation, ignoring the < or > portion in the constraint. Make y equal to zero. This shows the point where the straight line will cut the x axis. Next make x equal to zero. This shows the point where the line will cut the y axis.
5) Join the two points on the x axis and y axis by a straight line. This straight line represents the equation portion of the first constraint.
6) The ≤ or ≥ symbol in the constraint represents this line <u>and</u> a region to one side of this line. Determine this region by taking values of either x or y which are less than or greater than the point on the x or y axis. For example, if the constraint is $x+y \leq 4$, first draw the straight line $x+y = 4$. Then see on which side of this line do the values of x or y *less than 4* occur (i.e. 3, 2, 1, etc.). That side indicates the feasible region for the constraint.
7) In a similar manner, draw straight lines to represent the other constraints also. Identify the feasible region for each constraint. The feasible region common to all the constraints is the feasible region for the problem.
8) Identify all the corner points of this feasible region and name them as A, B, C, etc.
9) Find the coordinates (x, y values) of all these corners. If a corner lies on the x axis, the point will have only x value and zero y value. If a corner lies on the y axis, the point will have only y value and zero x value. If the corner lies elsewhere in the first quadrant, identify the two lines that pass through that point. Solve the equations representing these two lines as simultaneous equations to get the coordinates for that point.
10) Substitute the values of x,y at each corner in the objective function (Z). This would give the value of the objective function at each corner. Depending on whether the problem is a maximization type or minimization type, select the best (optimal) value as the solution of the problem.
11) Indicate the optimal solution of the problem by giving the values of the decision variables (x, y) and the corresponding value of the objective function (Z).

Problem : 1

Suppose an industry is manufacturing tow types of products P1 and P2. The profits per Kg of the two products are Rs.30 and Rs.40 respectively. These two products require processing in three types of machines. The following table shows the available machine hours per day and the time required on each
machine to produce one Kg of P1 and P2. Formulate the problem in the form of linear programmingmodel.

Profit/Kg	P1 Rs.30	P2 Rs.40	Total available Machine hours/day
Machine 1	3	2	600
Machine 2	3	5	800
Machine 3	5	6	1100

Solution:

The procedure for linear programming problem formulation is as

follows:Introduce the decision variable as follows:

Let x_1 = amount of P1

x_2 = amount of P2

In order to maximize profits, we establish the objective

function as$30x_1 + 40x_2$

Since one Kg of P1 requires 3 hours of processing time in machine 1 while the correspondingrequirement of P2 is 2 hours. So, the first constraint can be expressed as

$3x_1 + 2x_2 \leq 600$

Similarly, corresponding to machine 2 and 3 the constraints are

$3x_1 + 5x_2 \leq 800$

$5x_1 + 6x_2 \leq 1100$

In addition to the above there is no negative production, which may be

represented algebraically as$x_1 \geq 0$; $x_2 \geq 0$

Thus, the product mix problem in the linear programming model is as follows:

Maximize $30x_1 + 40x_2$

Subject to:

$3x_1 + 2x_2 \leq 600$

$3x_1 + 5x_2 \leq 800$

$5x_1 + 6x_2 \leq 1100$

$x_1 \geq 0$, $x_2 \geq 0$

Problem : 2

A company owns two flour mills viz. A and B, which have different production capacities for high, medium and low quality flour. The company has entered a contract to supply flour to a firm every monthwith at least 8, 12 and 24 quintals of high, medium and low quality respectively. It costs the company Rs.2000 and Rs.1500 per day to run mill A and B respectively. On a day, Mill A produces 6, 2 and 4 quintals of high, medium and low quality flour, Mill B produces 2, 4 and 12 quintals of high, medium and low quality flour respectively. How many days per month should each mill be operated in order to meet the contract order most economically.

Solution:

Let us define x1 and x2 are the mills A and B. Here the objective is to minimize the cost of the machineruns and to satisfy the contract order. The linear programming problem is given by

Minimize $2000x_1 + 1500x_2$

Subject to:

$6x_1 + 2x_2 \geq 8$

$2x_1 + 4x_2 \geq 12$

$4x_1 + 12x_2 \geq 24$

$x_1 \geq 0$, $x_2 \geq 0$

Problem : 3

A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires one minute of processing time on G and two machines on H; type B requires one minute on G and one minute on H. The machine G is available for not more than 6 hour 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as a linear programming problem.

Soln.:

Let x_1 and x_2 be the number of products of type A and type B.

Find x_1 and x_2 such that the profit $Z = 2x_1+3x_2$

is maximum, subject to the conditions

$x_1 + x_2 \le 400$

$2x_1 + x_2 \le 600$, x_1 , $x_2 \ge 0$

Problem : 4

Solve the following LPP by Graphical method

Maximize $Z = 5X_1 + 3X_2$

Subject to constraints

$2X_1 + X_2 \le 1000$, $X_1 \le 400$, $X_1 \le 700$ and $X_1, X_2 \ge 0$

Solution:

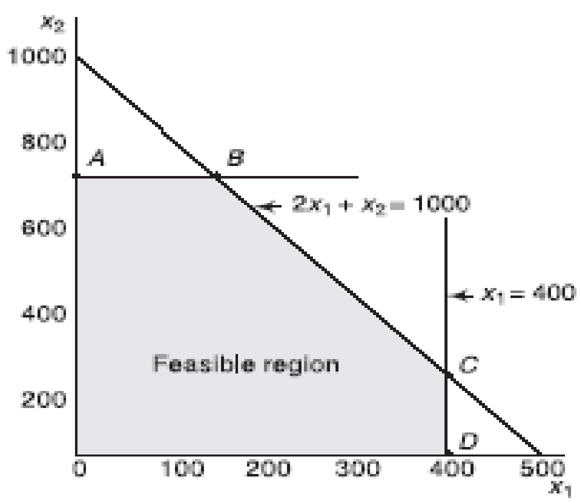
The first constraint $2X_1 + X_2 \le 1000$ can be represented as follows. We set $2X_1 + X_2 = 1000$

When $X_1 = 0$ in the above constraint, we get, $2 \times 0 + X_2 = 1000 = X_2 = 1000$

Similarly when $X_2 = 0$ in the above constraint, we get, $2X_1 + 0 = 1000 = X_1 = 1000/2 = 500$

The second constraint $X_1 \le 400$ can be represented as follows, We set $X_1 = 400$

The third constraint $X_2 \le 700$ can be represented as follows, We set $X_2 = 700$



The constraints are shown plotted in the above figure

Point	X_1	X_2	$Z = 5X_1 + 3X_2$
0	0	0	0
A	0	700	Z = 5 x 0 + 3 x 700 = 2,100
B	150	700	Z = 5 x 150 + 3 x 700 = 2,850* Maximum
C	400	200	Z = 5 x 400 + 3 x 200 = 2,600
D	400	0	Z = 5 x 400 + 3 x 0 = 2,000

The Maximum profit is at point B

When $X_1 = 150$ and $X_2 = 700$ then Z = <u>2850</u>

Problem : 5

A company purchasing scrap material has two types of scarp materials available. The first type has 30% of material X, 20% of material Y and 50% of material Z by weight. The second type has 40% of material X, 10% of material Y and 30% of material Z. The costs of the two scraps are Rs.120 and Rs.160 per kg respectively. The company requires at least 240 kg of material X, 100 kg of material Y and 290 kg of material Z. Find the optimum quantities of the two scraps to be purchased so that the company requirements of the three materials are satisfied at a minimum cost.

Solution

First we have to formulate the linear programming model. Let us introduce the decision variables x1 andx2 denoting the amount of scrap material to be purchased. Here the objective is to minimize the purchasing cost. So, the objective function here is

Minimize $120x_1 + 160x_2$

Subject to:

$0.3x_1 + 0.4x_2 \geq 240$

$0.2x_1 + 0.1x_2 \geq 100$

$0.5x_1 + 0.3x_2 \geq 290$

$x_1 \geq 0; x_2 \geq 0$

Multiply by 10 both sides of the inequalities, then the

problem becomes

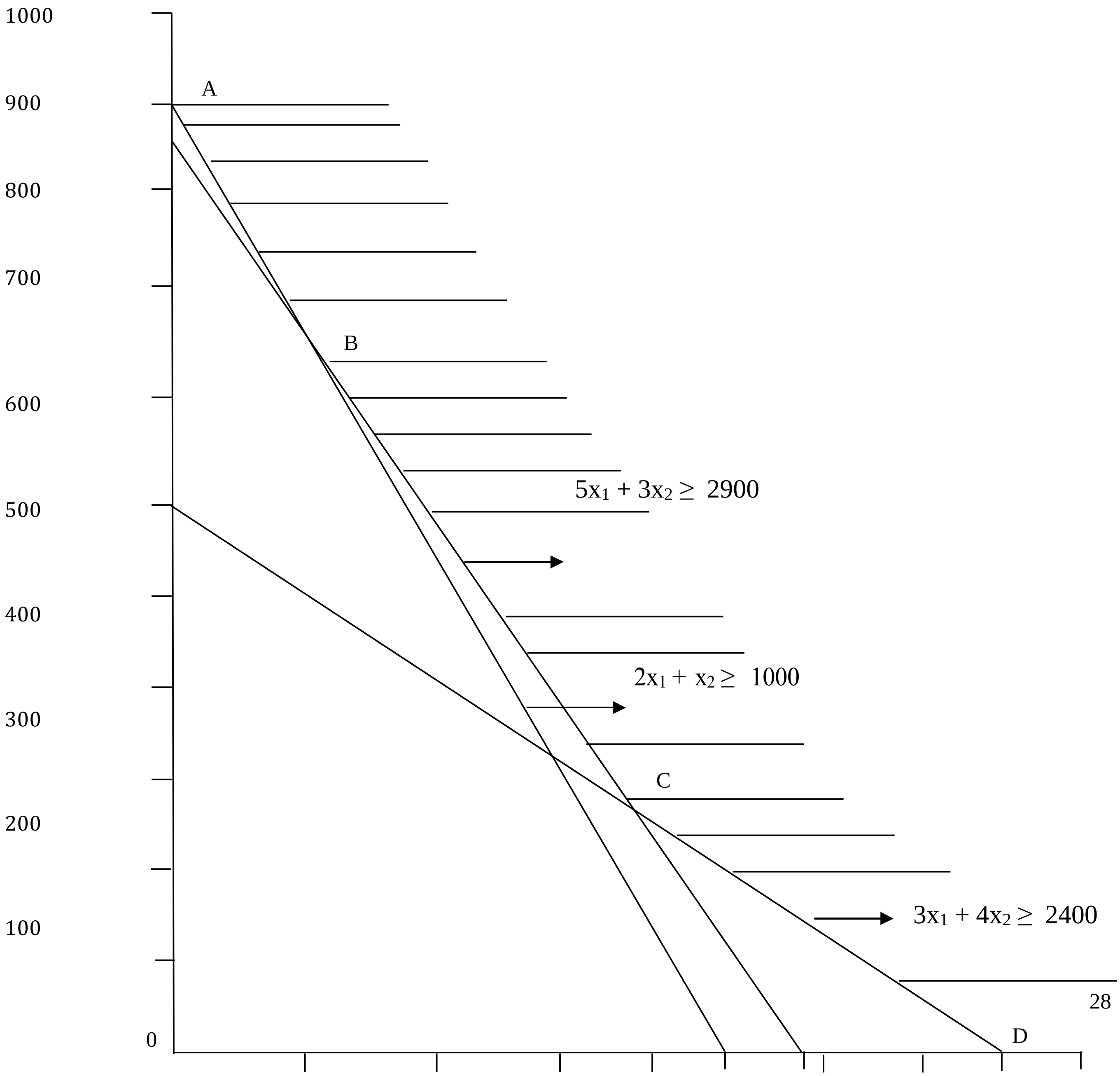
Minimize $120x_1 + 160x_2$

Subject to: $3x_1 + 4x_2 \geq 2400$ $2x_1 + x_2 \geq 1000$ $5x_1 + 3x_2 \geq 2900$

$x_1 \geq 0; x_2 \geq 0$

Let us introduce parameter M in the objective function i.e. $120x_1 + 160x_2 = M$. Then we have todetermine the different values for M, which is shown in the following Table

Extreme Point	Coordinates X_1	X_2	Objective Function $120x_1 + 160x_2$
A	$X_1 = 0$	$X_2 = 1000$	160000
B	$X_1 = 150$	$X_2 = 740$	136400
C	$X_1 = 400$	$X_2 = 300$	96000
D	$X_1 = 800$	$X_2 = 0$	96000



The extreme points are A, B, C, and D. One of the objective functions $120x_1 + 160x_2 = M$ familycoincides with the line CD at the point C with value M=96000, and the optimum value variables are $x_1 = 400$, and $x_2 = 300$. And at the point D with value M=96000, and the optimum value variables are $x_1 = 800$, and $x_2 = 0$. Thus, every point on the line CD minimizes objective function value and the problem contains multiple optimal solutions.