Brayton cycle

Brayton cycle is a constant pressure cycle for a perfect gas. It is also called Joule cycle. The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle. The cycle is shown in the Fig. 1.8 (a) and it is represented on p-v and T-s diagrams as shown in Figs. 1.8 (b) and (c).

The various operations are as follows:

Operation 1-2. The air is compressed isentropic ally from the lower pressure p1 to the upper pressure p2, the temperature rising from T1 to T2. No heat flow occurs.

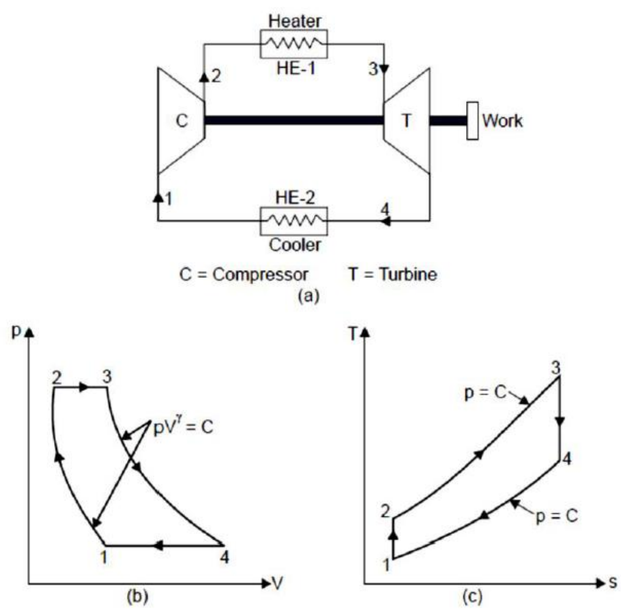
Operation 2-3. Heat flows into the system increasing the volume from V2 to V3 and temperature from T2 to T3 whilst the pressure remains constant at p2.

Heat received = mcp (T3 – T2).

Operation 3-4. The air is expanded isentropic ally from p2 to p1, the temperature falling from T3 to T4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V4 to V1 and the temperature from T4 to T1 whilst the pressure remains constant at p1.

Heat rejected = mcp (T4 – T1).



$$\eta_{\text{air-standard}} = \frac{\text{Work done}}{\text{Heat received}}$$

$$= \frac{\text{Heat received/cycle} - \text{Heat rejected/cycle}}{\text{Heat received/cycle}}$$

$$= \frac{mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)}{mc_p\,(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

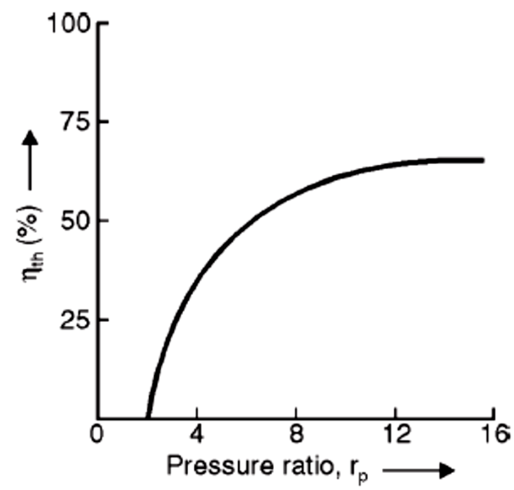
Now, from isentropic expansion,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1\,(r_p)^{\frac{\gamma-1}{\gamma}}, \text{ where } r_p = \text{pressure ratio.}$$

Similarly $\frac{T_3}{T_4} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$ or $T_3 = T_4\,(r_p)^{\frac{\gamma-1}{\gamma}}$

$$\therefore \quad \eta_{\text{air-standard}} = 1 - \frac{T_4 - T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}} - T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}} \qquad \text{...(13.16)}$$



The eqn. (13.16) shows that the efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$

$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)$$

$$= mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$

$$= mc_p\ T_3\ \left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

Work output during the cycle

$$= \text{Heat received/cycle} - \text{heat rejected/cycle}$$

$$= mc_p\,(T_3 - T_2) - mc_p\,(T_4 - T_1)$$

$$= mc_p\,(T_3 - T_4) - mc_p\,(T_2 - T_1)$$

$$= mc_p\ T_3\ \left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

Since, $$\frac{T_3}{T_4} = (r_p)^{\frac{\gamma - 1}{\gamma}} = \frac{T_2}{T_1}$$

Using the constant $$\text{'}z\text{'} = \frac{\gamma - 1}{\gamma},$$

we have, work output/cycle

$$W = K\left[T_3\left(1 - \frac{1}{r_p{}^z}\right) - T_1\,(r_p{}^z - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3 \times \frac{z}{r_p(z+1)} - T_1\,zr_p{}^{(z-1)}\right] = 0 \text{ for a maximum}$$

$$\therefore \qquad \frac{zT_3}{r_p{}^{(z+1)}} = T_1 z\,(r_p)^{(z-1)}$$

$$\therefore \qquad r_p{}^{2z} = \frac{T_3}{T_1}$$

$$\therefore \qquad r_p = (T_3/T_1)^{1/2z} \quad i.e., \quad r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma - 1)}} \qquad ...(13.17)$$

Thus, the *pressure ratio for maximum work is a function of the limiting temperature ratio.*

13.10.3. Work Ratio

Work ratio is defined as the *ratio of net work output to the work done by the turbine.*

$$\therefore \qquad \text{Work ratio} = \frac{W_T - W_C}{W_T}$$

$$\left[\begin{array}{ll}\text{where,} & W_T = \text{Work obtained from this turbine,} \\ \text{and} & W_C = \text{Work supplied to the compressor.}\end{array}\right]$$

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p\,(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4}$$

$$= 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}} \qquad ...(13.18)$$

1.In a Brayton cycle, the air enters the compressor at 1 bar and 25ºC. the pressure of air leaving the compressor is 3 bar and temperature at turbine inlet is 650ºC. determine per kgof air, i) cycle efficiency ii) heat supplied to air iii) work input iv) heat rejected in the cooler and v) temperature of air leaving the turbine.

Given data:

$P_1 = 1$ bar

$T_1 = 25^oC$

$T_3 = 650^oC$

$P_2 = 3$ bar

Solution:

Consider the process 1-2 adiabatic compression:

$T_2/T_1 = (P_2/P_1)^{y-1/y}$

$T_2 = (P_2/P_1)^{y-1/y} \times T1$

$T_2 = (3/1)^{1.4-1/1.4} \times 298$

3-4 adiabatic expansion:

$T_4/T_3 = (P_4/P_3)^{y-1/y}$

$T_4 = (P_4/P_3)^{y-1/y} \times 923 = 674.3k$

Air standard efficiency:

$\eta = 1 - 1/(R_p)^{y-1/y} = 1 - 1/(3)^{1.4-1/1.4} = 0.2694$

$= 26.94\%$

Heat supplied $Q_s = C_p (T_3 - T_2) = 1.005 (923 - 408) = 517.575$ KJ/kg

Heat rejected $Q_R = C_p (T_4 - T_1) = 1.008 (673.4 - 298) = 377.277$KJ/kg

Compressor work $W_C = C_p (T_2 - T_1) = 1.005 \times (408 - 298) = 110.55$Kj/kg

Similarly, for expander,

$W_e = C_p \times (T_3 - T_4) = 1.005 (923 - 6734)$

$W_e = 250.848 - 110.55 = 140.288$KJ/kg

Temperature of air leaving the turbine = 673.4K

2.In an air standard Brayton cycle, the air enters the compressor at 1 bar and 15ºC. The pressure leaving the compressor is 5 bar the maximum temperature in the cycle 900ºC. Find the following.

a) Compressor and expander work per kg of air. b) the cycle efficiency.
If an ideal regenerator is incorporated into the cycle, determine the percentage change in efficiency.

Given data:

$P_1 = P4 = 1$ bar $= 100KN/m^2$

$T_1 = 15^oC = 288k$

$P_2 = P3 = 5$ bar $= 500Kn/m^2$

$T_3 = 900^oC = 1173k$

Solution:

1-2 isentropic compression:

$T_2/T_1 = (p_2/P_1)^{y-1/y}$; $T_2 = (P_2/P_1)^{y-1/y} \times T_1 = 456k$Consider the process 3-4 isentropic expansion:

$T_4/T_3 = (P_4/P_3)^{y-1/y}$: $T_4 = (P_4/P_3)^{y-1/y} \times T_3 = 740.6k$

Work done by the compressor when it operates isentropic ally is given by

Compressor work $W_c = C_p (T_2 – T_1) = 1.005(456 – 288) = 168.756KJ$ for

expander $W_e = C_p (T_3 –T_4) = 1.005(1173 – 740.6) = 434.34KJ$

Air standard efficiency:

$\eta = 1- 1/(R_p)^{y-1/y} = 1- 1/ (5)^{1.4-1/1.4} = 36.86\%$

When ideal regenerator is incorporated:

$T_3 = T_5 \times T_2 = T_6$

Heat supplied $Q_s = C_p (T_4 –T_3)$

Heat rejected $Q_R = C_p (T_6 –T_1)$

$T_1 = 288k$

$T_2 =T_6 = 456k$

$T_3 = T_5 =740.6K$

$T_4 = 1173k$

$Q_s = 1.005 (1173 -790.6)$
$= 434.56$ KJ/kg

$Q_R = 1.005 (456 -288)$

$=186.84$ KJ/kg

Efficiency:

$\eta = 1- Q_R/Q_s = 168.84/434.56 =0.6114 = 61.14\%$

% change in efficiency: = 61.14 – 36.86/61.14 = 39.71%

3. A closed cycle ideal gas plant operates temperature limited of 800°C and 30°C and produces a power of 100Kw.The plant is designed such that there is no need for a regenerator. A fuel of calorific value 45000KJ/kg is used. Calculate the mass flow rate of air through the plant and the

rate of fuel combustion take C_p = 1 KJ/kgk and = 1.4

Given data:

T1 = 30ºC = 303k

T3 = 800ºC

P = 100KW,

Cp = 1 KJ /kgK KJ/kg,

y = 1.4

Solution:

For maximum net work done:

$T_4 = T_2 = \sqrt{T1\ x T3} = \sqrt{1073\ x\ 303}$
= 570.2k

Net work done

$W_{net} = C_p [(T_3 - T_4) - (T_2 - T_1)$

= 235.6 KJ/kg

Total power development

$P = m_a \times W_{net}$ = 100/ 235.6

= 0.4244kg/sec

Heat supply to the system:

$m_f \times C_v = m_a \times C_p \times (T_3 - T_2)$

$m_f = m_a \times C_p \times (T_3 - T_2)$ $/C_v$ 0.4244 x 1 (1073 – 570.2)/45.000

$= 4.742 \times 10^{-3}$ kg/s