### 2.3 RECIPROCAL LEVELLING

It is the operation of levelling in which the difference in elevation between two points is accurately determined by two sets of reciprocal observations. This method is very useful when the instrument cannot be set up between the two points due to an obstruction such as a valley, river, etc., and if the sights are much longer than are ordinarily permissible. For such long sights the errors of reading the staff, the curvature of earth, and the imperfect adjustments of the instrument become prominent. Special methods like reciprocal levelling should be used to minimize these errors.

In this method the instrument is set up near one point say A, on one side on the valley, and a reading is takenon the staff held at A (Fig. (a)) near the instrument and on the staff at $B$ on the other side of the valley. Let these readings be a and $b$, respectively. The near reading a is without error, whereas the reading b would have an error e due to curvature, refraction and collimation.

The instrument is then shifted near to $B$ on the other side of the valley and the reading is taken on the staff held at B and that on A. Let these readings be c and d (Fig. (b)). The near reading $c$ is without error, whereas reading $d$ would contain an error e due to the reasons discussed above. Let $h$ be the true difference of elevationbetween $A$ and $B$.


## Reciprocal levelling

In the $1^{\text {st }}$ case (Fig. (a)), $\mathrm{h}=(\mathrm{b}-$
e) -a

In the $2^{\text {nd }}$ case (Fig. (b)), $h=c-(d-e)$
$2 \mathrm{~h}=(\mathrm{b}-\mathrm{a})+(\mathrm{c}-\mathrm{d})$ or $\mathrm{h}=1 / 2[(\mathrm{~b}-\mathrm{a})+$
$(c-d)]$ and $e=1 / 2[(b-a)-(c-d)]$
In the above derivations it is assumed that the effect of refraction is the same while making observations fromboth the stations. However, if only one level is used, there will be a time lag in transferring the instrument to the opposite bank, during which time the value of refraction may change. Therefore, to ensure better results, some surveyors recommend the use of two levels, one at each bank, so that sights are taken simultaneously. Although this will give better results but each level may have a different collimation error. The instruments should therefore be interchanged and the entire procedure repeated. The mean of the four values will be the most probable difference in the level between the two points.

Example:1 The following notes refer to the reciprocal levels taken with one level:

| Instrument <br> station | Staff readings on |  | Remarks |
| :---: | :---: | :---: | :---: |
|  | $A$ | $B$ |  |
| $A$ | 1.03 | 1.630 | Distance $A B=800 \mathrm{~m}$ |
| $B$ | 0.95 | 1.540 | R.L. of $A=450 \mathrm{~m}$ |

Find:
(i) True R.L. of B
(ii) Combined correction for curvature and refraction
(iii) The error in collimation adjustment of the instrument.

## Solution

(i) True R.L. of B

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Instrument at A
Incorrect level difference between A and $\mathrm{B}=1.630-1.03$
$=0.600$ mInstrument at B
Incorrect level difference between $A$ and $B=1.540-0.95=0.59 \mathrm{~m}$
True difference of level between $A$ and $B=$ mean of the two incorrect differences
$=0.6+0.59 / 2=0.595 \mathrm{~m}$ (fall from A to B )
The results can also be obtained by using the expression

$$
\begin{aligned}
\mathrm{h} & =(\mathrm{b}-\mathrm{a})+(\mathrm{c}-\mathrm{d}) / 2 \\
& =(1.630-1.03)+(1.540-0.95) / 2=0.595 \mathrm{~m}
\end{aligned}
$$

(ii) Combined correction for curvature and refraction
$=0.0673 \mathrm{D} 2$
$=0.0673(800 / 1000)^{2}=0.043$

## (iii) Error in collimation adjustment

Reading of $\mathrm{A}=1.03 \mathrm{~m}$
Fall from $A$ to $B=$
0.595 m

Required reading of level lime $=1.03+0.595=$
1.625 m The actual staff reading at B (touching horizontal line)
$=1.625+0.043=1.668 \mathrm{~m}$
But the observed reading at $\mathrm{B}=1.630 \mathrm{~m}$
Error in collimation adjustment $=1.668-1.630=0.038 \mathrm{~m}$
Error of collimation is negative since the observed reading is less than the actual.

