4.3 Concentric or Composite Springs

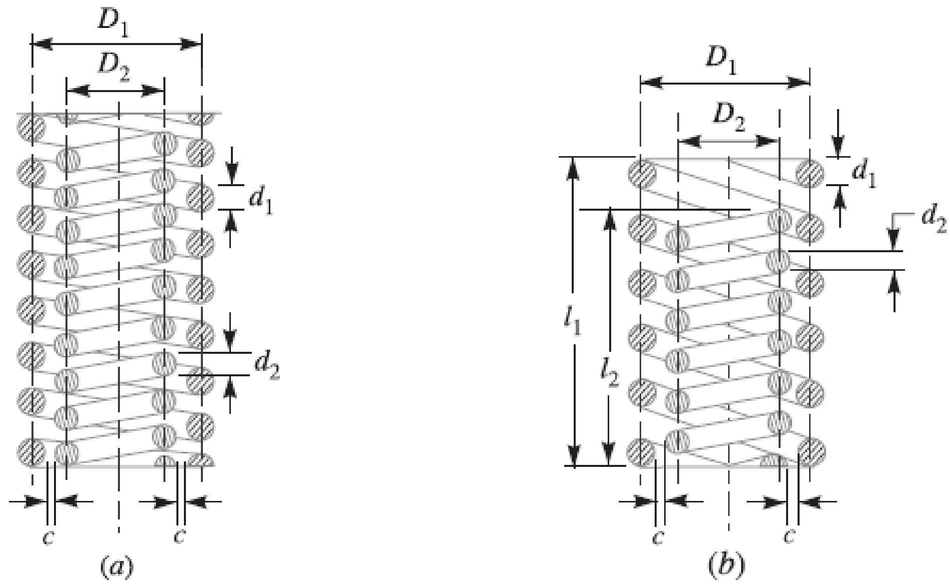

Fig. 4.15. Concentric springs.

Consider a concentric spring as shown in Fig. 4.15(*a*).

Let W = Axial load,
$W1$ = Load shared by outer spring,
$W2$ = Load shared by inner spring,
$d1$ = Diameter of spring wire of outer spring,
$d2$ = Diameter of spring wire of inner spring,
$D1$ = Mean diameter of outer spring,
$D2$ = Mean diameter of inner spring,
$\delta1$ = Deflection of outer spring,
$\delta2$ = Deflection of inner spring,
$n1$ = Number of active turns of outer spring, and
$n2$ = Number of active turns of inner spring.

Assuming that both the springs are made of same material, then the maximum shear stressinduced in both the springs is approximately same, *i.e.*

$$\tau_1 = \tau_2$$

$$\frac{8\, W_1 . D_1 . K_1}{\pi\,(d_1)^3} = \frac{8\, W_2 . D_2 . K_2}{\pi\,(d_2)^3}$$

When stress factor, $K_1 = K_2$, then

$$\frac{W_1 . D_1}{(d_1)^3} = \frac{W_2 . D_2}{(d_2)^3} \quad ...(i)$$

If both the springs are effective throughout their working range, then their free length and deflection are equal, *i.e.*

$$\delta_1 = \delta_2$$

or $$\frac{8W_1\,(D_1)^3\, n_1}{(d_1)^4\, G} = \frac{8W_2\,(D_2)^3\, n_2}{(d_2)^4\, G} \quad \text{or} \quad \frac{W_1\,(D_1)^3\, n_1}{(d_1)^4} = \frac{W_2\,(D_2)^3\, n_2}{(d_2)^4} \quad ...(ii)$$

When both the springs are compressed until the adjacent coils meet, then the solid length ofboth the springs is equal, *i.e.*

$$n1.d1 = n2.d2$$

The equation ***(ii)*** may be written as

$$\frac{W_1 (D_1)^3}{(d_1)^5} = \frac{W_2 (D_2)^3}{(d_2)^5} \quad ...(iii)$$

Now dividing equation (*iii*) by equation (*i*), we have

$$\frac{(D_1)^2}{(d_1)^2} = \frac{(D_2)^2}{(d_2)^2} \quad \text{or} \quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C, \text{ the spring index} \quad ...(iv)$$

i.e. the springs should be designed in such a way that the spring index for both the springs is same.

From equations (*i*) and (*iv*), we have

$$\frac{W_1}{(d_1)^2} = \frac{W_2}{(d_2)^2} \quad \text{or} \quad \frac{W_1}{W_2} = \frac{(d_1)^2}{(d_2)^2} \quad ..(v)$$

From Fig. 23.22 (*a*), we find that the radial clearance between the two springs,

$$*c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$$

Usually, the radial clearance between the two springs is taken as $\frac{d_1 - d_2}{2}$.

$$\therefore \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right) = \frac{d_1 - d_2}{2}$$

or

$$\frac{D_1 - D_2}{2} = d_1 \quad ...(vi)$$

From equation (*iv*), we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation (*vi*), we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

From equation (*iv*), we find that

$$D_1 = C.d_1, \text{ and } D_2 = C.d_2$$

Substituting the values of D_1 and D_2 in equation (*vi*), we have

$$\frac{C.d_1 - C.d_2}{2} = d_1 \quad \text{or} \quad C.d_1 - 2\,d_1 = C.d_2$$

$$\therefore \quad d_1 (C - 2) = C.d_2 \quad \text{or} \quad \frac{d_1}{d_2} = \frac{C}{C - 2} \quad ...(vii)$$

Example 4.5. *A concentric spring for an aircraft engine valve is to exert a maximum force of5000 N under an axial deflection of 40 mm. Both the springs have same free length, same solidlength and are subjected to equal maximum shear stress of 850 MPa. If the spring index for both thesprings is 6, find (a) the load shared by each spring, (b) the main dimensions of both the springs, and(c) the number of active coils in each spring.*

Assume G = 80 kN/mm2 and diametral clearance to be equal to the difference between the wirediameters.

Solution. Given :

W = 5000 N

δ = 40 mm

$\tau_1 = \tau_2$ = 850 MPa = 850 N/mm^2
C = 6
G = 80 kN/mm^2 = 80 × 103 N/mm^2

The concentric spring is shown in Fig. 4.15(*a*).

(*a*) *Load shared by each spring*

Let $W1$ and $W2$ = Load shared by outer and inner spring respectively,
$d1$ and $d2$ = Diameter of spring wires for outer and inner springs respectively, and
$D1$ and $D2$ = Mean diameter of the outer and inner springs respectively.

Since the diametral clearance is equal to the difference between the wire diameters, therefore

$$(D_1 - D_2) - (d_1 + d_2) = d_1 - d_2$$

or
$$D_1 - D_2 = 2\,d_1$$

We know that $D_1 = C.d_1$, and $D_2 = C.d_2$

$$\therefore \quad C.d_1 - C.d_2 = 2\,d_1$$

or
$$\frac{d_1}{d_2} = \frac{C}{C-2} = \frac{6}{6-2} = 1.5 \quad \text{...}(i)$$

We also know that
$$\frac{W_1}{W_2} = \left(\frac{d_1}{d_2}\right)^2 = (1.5)^2 = 2.25 \quad \text{...}(ii)$$

and
$$W_1 + W_2 = W = 5000 \text{ N} \quad \text{...}(iii)$$

From equations (*ii*) and (*iii*), we find that

$$W_1 = 3462 \text{ N, and } W_2 = 1538 \text{ N} \textbf{ Ans.}$$

(*b*) *Main dimensions of both the springs*

We know that Wahl's stress factor for both the springs,

$$K_1 = K_2 = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4\times 6-1}{4\times 6-4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stress induced in the outer spring (τ_1),

$$850 = K_1 \times \frac{8\,W_1.C}{\pi(d_1)^2} = 1.2525 \times \frac{8\times 3462\times 6}{\pi(d_1)^2} = \frac{66\,243}{(d_1)^2}$$

$$\therefore \quad (d_1)^2 = 66\,243 / 850 = 78 \text{ or } d_1 = 8.83 \text{ say 10 mm } \textbf{Ans.}$$

and
$$D_1 = C.d_1 = 6\,d_1 = 6 \times 10 = 60 \text{ mm } \textbf{Ans.}$$

Similarly, maximum shear stress induced in the inner spring (τ_2),

$$850 = K_2 \times \frac{8W_2.C}{\pi(d_2)^2} = 1.2525 \times \frac{8\times 1538\times 6}{\pi(d_2)^2} = \frac{29\,428}{(d_2)^2}$$

$$\therefore \quad (d_2)^2 = 29\,428 / 850 = 34.6 \text{ or } {*d_2} = 5.88 \text{ say 6 mm } \textbf{Ans.}$$

and
$$D_2 = C.d_2 = 6 \times 6 = 36 \text{ mm } \textbf{Ans.}$$

(*c*) *Number of active coils in each spring*

Let n_1 and n_2 = Number of active coils of the outer and inner spring respectively.

We know that the axial deflection for the outer spring (δ),

$$40 = \frac{8\, W_1 . C^3 . n_1}{G . d_1} = \frac{8 \times 3462 \times 6^3 \times n_1}{80 \times 10^3 \times 10} = 7.48\, n_1$$

$\therefore \quad n_1 = 40 / 7.48 = 5.35$ say 6 **Ans.**

Assuming square and ground ends for the spring, the total number of turns of the outer spring,

$$n_1' = 6 + 2 = 8$$

$\therefore$ Solid length of the outer spring,

$$L_{S1} = n_1' . d_1 = 8 \times 10 = 80 \text{ mm}$$

Let n_2' be the total number of turns of the inner spring. Since both the springs have the same solid length, therefore,

$$n_2' . d_2 = n_1' . d_1$$

or $$n_2' = \frac{n_1' . d_1}{d_2} = \frac{8 \times 10}{6} = 13.3 \text{ say } 14$$

and $n_2 = 14 - 2 = 12$ **Ans.** ... ($\because n_2' = n_2 + 2$)

Since both the springs have the same free length, therefore

Free length of outer spring

= Free length of inner spring

$= L_{S1} + \delta + 0.15\,\delta = 80 + 40 + 0.15 \times 40 = 126$ mm **Ans.**

Other dimensions of the springs are as follows:

Outer diameter of the outer spring

$= D_1 + d_1 = 60 + 10 = 70$ mm **Ans.**

Inner diameter of the outer spring

$= D_1 - d_1 = 60 - 10 = 50$ mm **Ans.**

Outer diameter of the inner spring

$= D_2 + d_2 = 36 + 6 = 42$ mm **Ans.**

Inner diameter of the inner spring

$= D_2 - d_2 = 36 - 6 = 30$ mm **Ans.**

4.1.12 Helical Torsion Springs

The helical torsion springs as shown in Fig. 4.16, may be made from round, rectangular orsquare wire. These are wound in a similar manner as helical compression or tension springs but theends are shaped to transmit torque. The primarystress in helical torsion springs is bending stresswhereas in compression or tension springs, thestresses are torsional shear stresses. The helicaltorsion springs are widely used for transmittingsmall torques as in door hinges, brush holders inelectric motors, automobile starters etc.

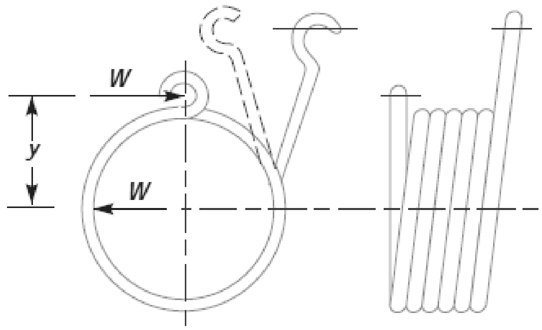

Fig. 4.16. Helical torsion spring.

A little consideration will show that theradius of curvature of the coils changes when thetwisting moment is applied to the spring. Thus,the wire is under pure bending. According to A.M.Wahl, the bending stress in a helical torsion springmade of round wire is

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = K \times \frac{32\,W.y}{\pi.d^3}$$

where K = Wahl's stress factor = $\frac{4C^2 - C - 1}{4C^2 - 4C}$,

C = Spring index,

M = Bending moment = $W \times y$,

W = Load acting on the spring,

y = Distance of load from the spring axis, and

d = Diameter of spring wire.

and total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi\,d^4/64} = \frac{64\,M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\frac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi d^4 / 64} = \frac{64 M.D.n}{E.d^4}$$

where l = Length of the wire = $\pi.D.n$,

E = Young's modulus,

I = Moment of inertia = $\frac{\pi}{64} \times d^4$,

D = Diameter of the spring, and

n = Number of turns.

and deflection, $$\delta = \theta \times y = \frac{64 M.D.n}{E.d^4} \times y$$

When the spring is made of rectangular wire having width b and thickness t, then

$$\sigma_b = K \times \frac{6 M}{t.b^2} = K \times \frac{6 W \times y}{t.b^2}$$

where $$K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$$

Angular deflection, $$\theta = \frac{12 \pi M.D.n}{E.t.b^3}; \quad \text{and} \quad \delta = \theta.y = \frac{12 \pi M.D.n}{E.t.b^3} \times y$$

In case the spring is made of square wire with each side equal to b, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6 M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12 \pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12 \pi M.D.n}{E.b^4} \times y$$

Example 4.6. *A helical torsion spring of mean diameter 60 mm is made of a round wire of6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and theangular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for thespring material is 200 kN/mm^2. The number of effective turns may be taken as 5.5.*

Solution.

Given :
D = 60 mm ;
d = 6 mm
M = 6 N-m = 6000 N-mm
C = 10 ;E = 200 kN/mm^2 = 200 × 103 N/mm^2
n = 5.5

Bending stress induced

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa } \textbf{Ans.}$$

Angular deflection of the spring

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28° \textbf{ Ans.}$$