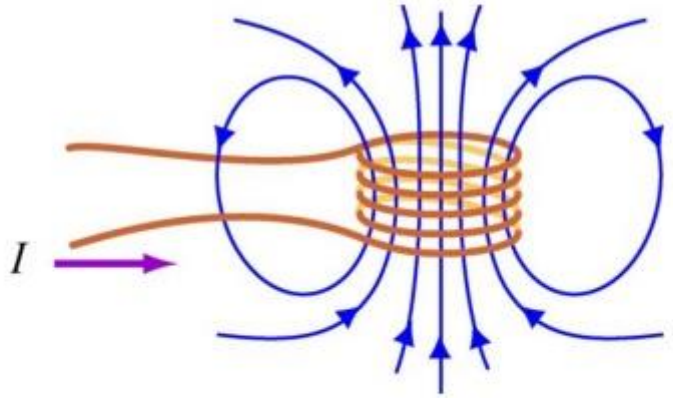
UNIT 5

SELF AND MUTUAL INDUCTANCE

Self-Inductance:

Consider again a coil consisting of N turns and carrying current I in the counterclockwise direction, as shown in Figur. If the current is steady, then the magnetic flux through the loop will remain constant. However, suppose the current I changes with time, then according t law, an induced emf will arise to oppose the change. The induced current will flow clockwise if dI/dt>0, and counterclockwise if dI/dt<0 The property of the loop in which its own magnetic field opposes any change in current is called "self-inductance," and the emf generated is called the self-induced emf or back emf, which we denote as Lε. All current-carrying loops exhibit this property. In particular, an inductor is a circuit element (symbol) which has a large self inductance.



Magnetic flux through the current loop

Mathematically, the self-induced emf can be written as

$$\varepsilon_L = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\iint \vec{\mathbf{B}}\cdot d\vec{\mathbf{A}}$$

and is related to the self-inductance L by

$$\varepsilon_L = -L\frac{dI}{dt}$$

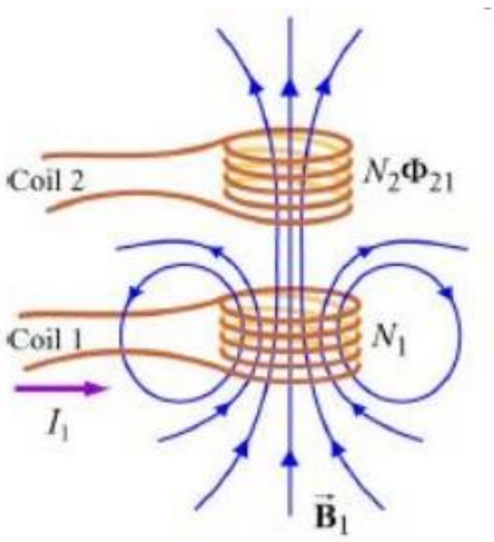
The two expressions can be combined to yield

$$L = \frac{N\Phi_B}{I}$$

Physically, the inductance L is a measure current; the larger the value of L, the lower the rate of change of current.

Mutual Inductance:

Suppose two coils are placed near each other, as shown in Figure



Changing current in coil 1 produces changing magnetic flux in coil 2.
The first coil has N1 turns and carries a current I1 which gives rise to a magnetic field B1...

Since the two coils are close to each other, some of the magnetic field lines through coil 1 will also pass through coil 2. Let F21denote the magnetic flux through one turn of coil 2 due to I1. Now, by varying I1 with time, there will be an induced emf associated with the changing magnetic flux in the second coil:

$$\varepsilon_{21} = -N_2 \frac{d\Phi_{21}}{dt} = -\frac{d}{dt}\iint_{\text{coil 2}} \vec{\mathbf{B}}_1 \cdot d\vec{\mathbf{A}}_2$$

The time rate of change21 in coil 2 is proportional of magnetic to the time rate of change flux of Φ the current in coil 1:

$$N_2 \frac{d\Phi_{21}}{dt} = M_{21}\frac{dI_1}{dt}$$

where the proportionality constantM21 is called the mutual inductance. It can also be written as

$$M_{21} = \frac{N_2\Phi_{21}}{I_1}$$

The SI unit for inductance is the henry (H):

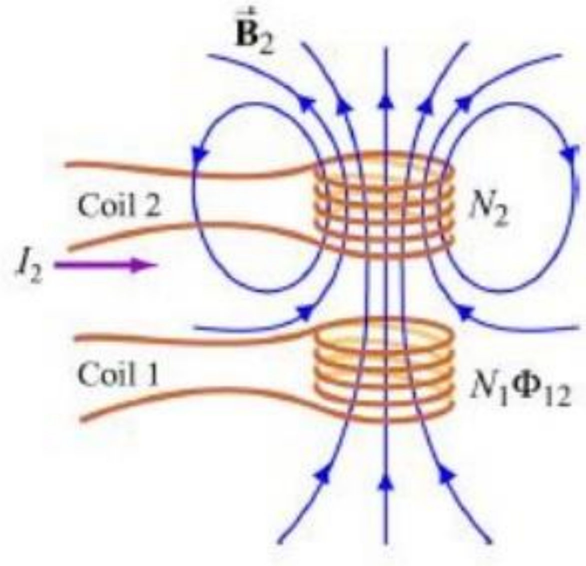
$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T}\cdot\text{m}^2/\text{A}$$

We shall see that the mutual inductanceM21 depends only on the geometrical properties of the two coils such as the number of turns and the radii of the two coils.

In a similar manner, suppose instead there is a current I2 in the second coil and it is varying with time Then the induced emf in coil 1 becomes

$$\varepsilon_{12} = -N_1 \frac{d\Phi_{12}}{dt} = -\frac{d}{dt}\iint_{\text{coil 1}} \vec{\mathbf{B}}_2 \cdot d\vec{\mathbf{A}}_1$$

and a current is induced in coil 1.



Changing current in coil 2 produces changing magnetic flux in coil 1.
This changing flux in coil 1 is proportional to the changing current in coil 2

$$N_1 \frac{d\Phi_{12}}{dt} = M_{12} \frac{dI_2}{dt}$$

where the proportionality constant M21 is another mutual inductance and can be written as

$$M_{12} = \frac{N_1\Phi_{12}}{I_2}$$

However, using the reciprocity theorem which combines-SavartAmpere's law, one may law show that the constants are equal:

$$M_{12} = M_{21} \equiv M$$

4. COEFFICIENT OF COUPLING

In coupled coils, the coefficient of coupling is defined as then fraction of the total flux produced by one coil linking another coil.
Coefficient of coupling=

$$\text{Coefficient of coupling} = K = \frac{\varphi_{12}}{\varphi_1} = \frac{\varphi_{21}}{\varphi_2}$$