## Properties of Fourier Transform:

Here are the properties of Fourier Transform:

### Linearity Property:

$$\text{If } x(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega)$$

$$\& \ y(t) \stackrel{\text{F.T}}{\longleftrightarrow} Y(\omega)$$

Then linearity property states that

$$ax(t) + by(t) \stackrel{\text{F.T}}{\longleftrightarrow} aX(\omega) + bY(\omega)$$

### Time Shifting Property:

$$\text{If } x(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega)$$

Then Time shifting property states that

$$x(t - t_0) \stackrel{\text{F.T}}{\longleftrightarrow} e^{-j\omega t_0} X(\omega)$$

**Frequency Shifting Property:**

$$\text{If } x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega)$$

Then frequency shifting property states that

$$e^{j\omega_0 t} . x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega - \omega_0)$$

**Time Reversal Property:**

$$\text{If } x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega)$$

Then Time reversal property states that

$$x(-t) \overset{\text{F.T}}{\longleftrightarrow} X(-\omega)$$

**Time Scaling Property:**

$$\text{If } x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega)$$

Then Time scaling property states that

$$x(at) \frac{1}{|a|} X \frac{\omega}{a}$$

**Differentiation and Integration Properties**

$$\text{If } x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega)$$

Then Differentiation property states that

$$\frac{dx(t)}{dt} \overset{\text{F.T}}{\longleftrightarrow} j\omega.\, X(\omega)$$

$$\frac{d^n x(t)}{dt^n} \overset{\text{F.T}}{\longleftrightarrow} (j\omega)^n.\, X(\omega)$$

and integration property states that

$$\int x(t)\, dt \overset{\text{F.T}}{\longleftrightarrow} \frac{1}{j\omega} X(\omega)$$

$$\iiint \ldots \int x(t)\, dt \overset{\text{F.T}}{\longleftrightarrow} \frac{1}{(j\omega)^n} X(\omega)$$

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**Multiplication and Convolution Properties:**

$$\text{If } x(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega)$$

$$\& \; y(t) \overset{\text{F.T}}{\longleftrightarrow} Y(\omega)$$

Then multiplication property states that

$$x(t).\, y(t) \overset{\text{F.T}}{\longleftrightarrow} X(\omega) * Y(\omega)$$

and convolution property states that

$$x(t) * y(t) \overset{\text{F.T}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega).\, Y(\omega)$$

## Problem

Find the Fourier transform of the signal x(t)

$$x(t) = e^{-a|t|} \qquad a > 0$$

**Sol:** Signal x(t) can be rewritten as

$$x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$$

Then

$$X(\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{1}{a - j\omega} + \frac{1}{a + j\omega} = \frac{2a}{a^2 + \omega^2}$$

**Hence, we get**

$$e^{-a|t|} \longleftrightarrow \frac{2a}{a^2 + \omega^2}$$