2.2 FOURIER TRANSFORM

The Fourier representation of periodic signals has been extended to non-periodic signals by letting the fundamental period T tend to infinity and this Fourier method of representing non- periodic signals as a function of frequency is called Fourier transform.

Definition of Continuous time Fourier Transform

The Fourier transform (FT) of Continuous time signals is called Continuous Time Fourier Transform

Let $x(t) = continuous\ time\ signal$

$$X(j\omega) = F\{x(t)\}$$

The Fourier transform of continuous time signal, x(t) is defined as

$$X(j\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

Conditions for existence of Fourier transform

The Fourier transform $x(t)$ exist if it satisfies the following Dirichlet's condition

1. $x(t)$ should be absolutely integrable

$$ie\ , \quad \int_{-\infty}^{\infty} x(t)dt < \infty$$

2. $x(t)$should have a finite number of maxima and minima with in any finite interval.
3. $x(t)$ should have a finite number of discontinuities with in any interval.

Definition of Inverse Fourier Transform

The inverse Fourier Transform of $X(j\omega)$ is defined as,

$$x(t) = F^{-1}\{X(j\omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{jt} d\Omega$$

EXAMPLE 1: Find Fourier transform of impulse signal.

Solution:

$$\mathrm{F}\{x(t)\} = X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt$$

$$\left[\because \textit{Impulse signal } \delta(t) = \begin{cases} 1 \textit{ for } t = 0 \\ 0 \textit{ for } t \neq 0 \end{cases}\right]$$

$$F[\delta(t)] = \delta(0)e^{-j\omega(0)} = 1$$

EXAMPLE 2: Find Fourier transform of double sided exponential signal.

Solution:

$$F\left[e^{-a|t|}\right] = \begin{Bmatrix} e^{-at} & : t \geq 0 \\ e^{at} & : t \leq 0 \end{Bmatrix}$$

$$F\left[e^{-a|t|}\right] = \int_{-\infty}^{0} e^{at}e^{-j\omega t}dt + \int_{0}^{\infty} e^{-at}e^{-j\omega t}dt$$

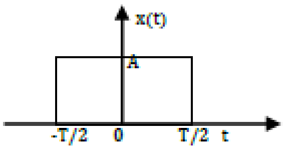
$$= \int_{-\infty}^{0} e^{(a-j\omega)t}dt + \int_{0}^{\infty} e^{-(a+j\omega)t}dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{(a-j\omega)}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_{0}^{\infty}$$

$$= \frac{1}{(a-j\omega)} + \frac{1}{(a+j\omega)} = \frac{a+j\omega+a-j\omega}{a^2+\omega^2}$$

$$= \frac{2a}{a^2+\omega^2}$$

EXAMPLE 3: Find Fourier transform of rectangular pulse function shown in figure



Solution:

$$x(t) = \pi(t) = A\,; \frac{-T}{2} \le t \le \frac{T}{2}$$

$$F[\pi(t)] = \int_{-\frac{T}{2}}^{\frac{T}{2}} Ae^{-j\omega t}dt = A\left[\frac{e^{-j\omega t}}{-j\omega}\right]_{\frac{-T}{2}}^{\frac{T}{2}}$$

$$= \frac{A}{-j\omega}\left[e^{-j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}\right] = \frac{2A}{j\omega}\left[\frac{e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}}{2}\right] = \frac{2A}{\omega}\sin\omega\frac{T}{2}$$

$$= \frac{2A}{\omega T}T\, sin\omega\frac{T}{2} = AT\frac{\sin\omega\frac{T}{2}}{\omega\frac{T}{2}} = ATsinc\omega\frac{T}{2}$$

EXAMPLE 4: Find inverse Fourier transform $X(j\,\omega) = \delta(\omega)$.

Solution:

$$F^{-1}[X(j\omega)] = F^{-1}[\delta(\omega)]$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta(\omega)\, e^{j\omega t}d\omega = \frac{1}{2\pi}[1]$$

$$\delta(\omega) = \begin{cases} 1 \;\; for\ \omega = 0 \\ 0 \; for\ \omega \ne 0 \end{cases}$$

$$F^{-1}[\delta(\omega)] = \frac{1}{2\pi}$$