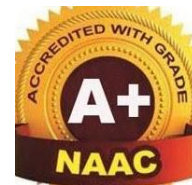




ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS



Unit III : Introduction to Game Theory

Game is defined as an activity between two or more persons according to a set of rules at the end of which each person receives some benefit or suffers loss. The set of rules defines the **game**. Going through the set of rules once by the participants defines a **play**.

Properties of a Game

1. There are finite numbers of competitors called 'players'
2. Each player has a finite number of possible courses of action called 'strategies'
3. All the strategies and their effects are known to the players but player does not know which strategy is to be chosen.
4. The game is a combination of the strategies and in certain units which determines the gain or loss.
5. The player playing the game always tries to choose the best course of action which Results in optimal pay off called 'optimal strategy'.
6. The expected pay off when all the players of the game follow their optimal strategies is known as 'value of the game'. The main objective of a problem of a game is to find the value of the game.
7. The game is said to be 'fair' game if the value of the game is zero otherwise it s known as 'unfair'.

Characteristics of Game Theory

1. Competitive game

A competitive situation is called a **competitive game** if it has the following four properties

1. There are finite number of competitors such that $n \geq 2$. In case $n = 2$, it is called a **two-person game** and in case $n > 2$, it is referred as **n-person game**.
2. Each player has a list of finite number of possible activities.
3. A play is said to occur when each player chooses one of his activities. The choices are assumed to be made simultaneously i.e. no player knows the choice of the other until he

has decided on his own.

4. Every combination of activities determines an outcome which results in a gain of payments to each player, provided each player is playing uncompromisingly to get as much as possible. Negative gain implies the loss of same amount.

2. Strategy

The strategy of a player is the predetermined rule by which player decides his course of action from his own list during the game. The two types of strategy are

1. Pure strategy
2. Mixed strategy

Pure Strategy

If a player knows exactly what the other player is going to do, a deterministic situation is obtained and objective function is to maximize the gain. Therefore, the pure strategy is a decision rule always to select a particular course of action.

Mixed Strategy

If a player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained and objective function is to maximize the expected gain. Thus the mixed strategy is a selection among pure strategies with fixed probabilities.

3. Number of persons

A game is called 'n' person game if the number of persons playing is 'n'. The person means an individual or a group aiming at a particular objective.

Two-person, zero-sum game

A game with only two players (player A and player B) is called a 'two-person, zero-sum game', if the losses of one player are equivalent to the gains of the other so that the sum of their net gains is zero.

Two-person, zero-sum games are also called rectangular games as these are usually represented by a payoff matrix in a rectangular form.

4. Number of activities

The activities may be finite or infinite.

5. Payoff

The quantitative measure of satisfaction a person gets at the end of each play is called a payoff

6. Payoff matrix

Suppose the player A has 'm' activities and the player B has 'n' activities. Then a payoff matrix Can be formed by adopting the following rules

- Row designations for each matrix are the activities available to player A
- Column designations for each matrix are the activities available to player B
- Cell entry V_{ij} is the payment to player A in A's payoff matrix when A chooses the activity i and B chooses the activity j.
- With a zero-sum, two-person game, the cell entry in the player B's payoff matrix will be negative of the corresponding cell entry V_{ij} in the player A's payoff matrix so that sum of payoff matrices for player A and player B is ultimately zero.

7. Value of the game

Value of the game is the maximum guaranteed game to player A (maximizing player) if both the players uses their best strategies. It is generally denoted by 'V' and it is unique.

Limitations of game theory

The major limitations are

- The assumption that the players have the knowledge about their own payoffs and others is rather unrealistic.
- As the number of players increase in the game, the analysis of the gaming strategies become increasingly complex and difficult.
- The assumptions of maximin and minimax show that the players are risk-averse and have complete knowledge of the strategies. It doesn't seem practical.
- Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out collusion. Then the mixed strategies are not very useful.