

UNIT I

STRESS, STRAIN AND DEFORMATION OF SOLIDS

1.1 STRENGTH OF MATERIALS

Strength of materials is a subject which deals with the detailed study about the effect of external forces acts on materials and ability of material to resist deformations due to cohesion between the molecules. The study of strength of materials often refers to various methods of calculating the stresses and strains in structural members, such as beams, columns, and shafts.

1.1.2 STIFFNESS

The Stiffness may be defined as an ability of a material to withstand load without deformation.

1.1.3 STRESS

When an external force acts on a body it undergoes some deformation and at the same time the body resists deformation. The magnitude of the applied force is numerically equal to the applied force. This internal resisting force per unit area is called stress.

Mathematically

$$\text{Stress } (\sigma) = \frac{\text{Force}(P)}{\text{Area}(A)}$$

The unit of Stress is N/mm^2 or KN/m^2 , depending upon the units of Force and Area.

1.1.4. STRAIN

When a body is subjected to an external force, there is some change in dimension of the body. The ratio of change in dimension to the original dimension is known as strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

It has no unit.

1.1.5. TYPES OF STRESSES

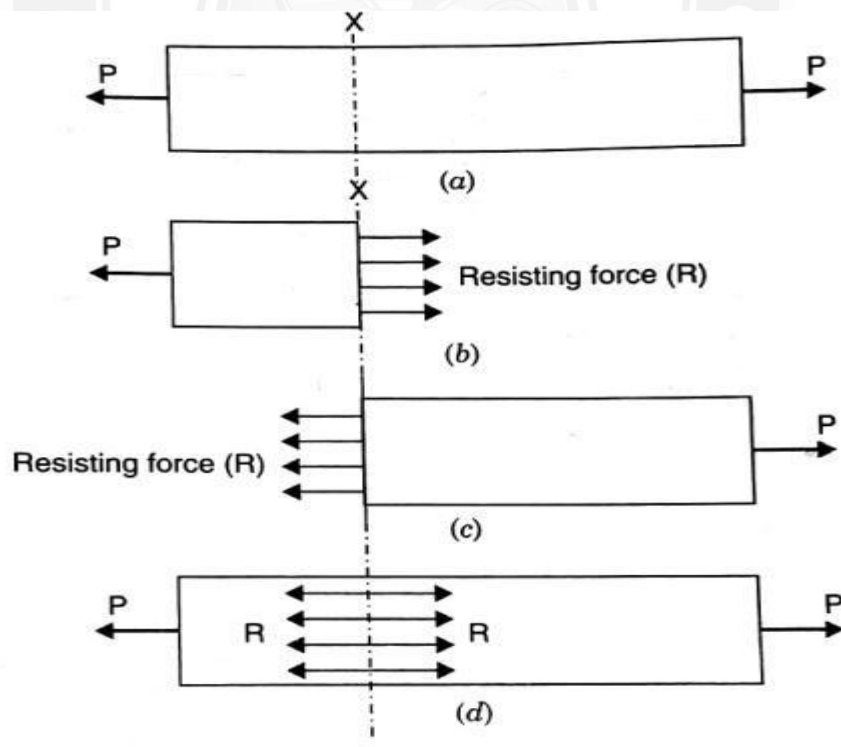
There are mainly three types of stresses. They are:

- a. Tensile stress,
- b. Compressive stress and
- c. Shear stress.

a. Tensile stress & Tensile Strain

When a member is subjected to equal and opposite pulls as shown in figure, as a result of this there is increased in length. The Stress induced at any cross section of the member is called Tensile Stress.

$$\text{Tensile Stress} = \frac{\text{Tensile load}}{\text{Area}}$$



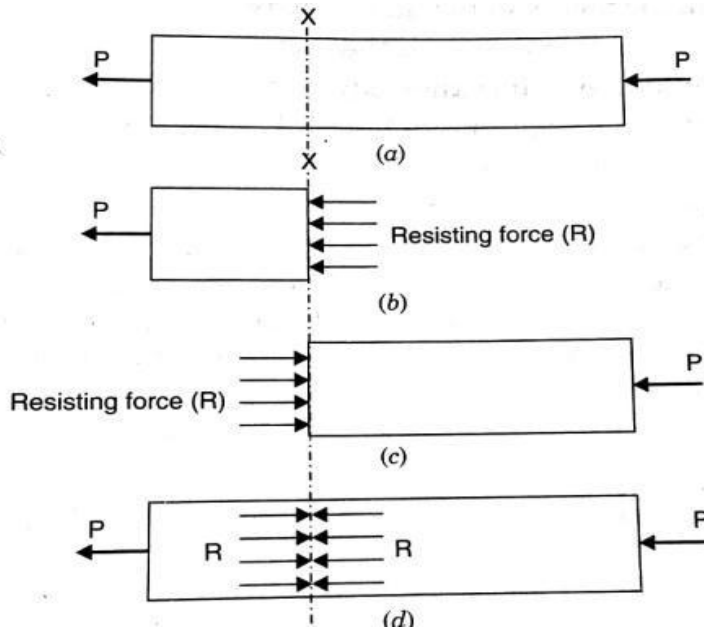
The ratio of increase in length to the original length is known as Tensile Strain

$$\text{Tensile Strain (e)} = \frac{\text{Increase in length}(\Delta l)}{\text{Original length}(l)}$$

b. Compressive Stress and Compressive Strain

When a member is subjected to equal and opposite pushes as shown in figure, as a result of this there is decreased in length. The Stress induced at any cross section of the member is called Compressive Stress.

Compressive Stress = Compressive load/Area

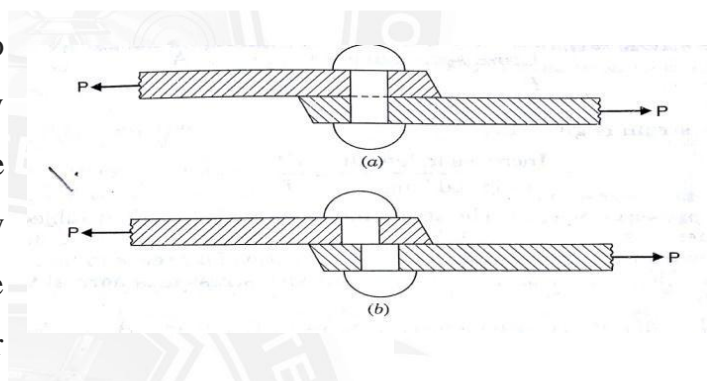


The ratio of increase in length to the original length is known as Tensile Strain

$$\text{Tensile Strain}(e) = \frac{\text{decrease in length}(\Delta l)}{\text{Original length}(l)}$$

C. Shear Stress and Shear Strain

When the member is subjected to equal and opposite forces acts tangentially at any cross sectional plane of a body the body tending to slide one part of the body over the other part as shown in figure the stress induced in that section is called shear stress and the corresponding strain is known as shear strain.



1.1.6. VOLUMETRIC STRAIN

Volumetric strain is defined as the ratio of change in volume to the original volume

$$\text{Volumetric Strain} = \frac{\text{Change in Volume}(\Delta v)}{\text{Original Volume}(v)}$$

1.1.7. HOOKE'S LAW

It States that when a material is loaded, within its elastic limit, the stress is directly proportional to the strain.

$$\text{Stress } (\sigma) \propto \text{strain}$$

1.1.8. FACTOR OF SAFETY

It is defined as the ratio of ultimate stress to the Permissible stress(working stress)

$$\text{Factor of safety} = \frac{\text{Ultimate Stress}}{\text{Permissible Stress}}$$

Problem 1.1 A mild steel rod 2m long and 3Cm diameter is subjected to an axial pull of 10KN. If E for steel is $2 \times 10^5 \text{ N/mm}^2$. Find (a) Stress, (b) Strain, (C) Elongation of the rod.

Given:

$$\begin{aligned} \text{Length of the rod, } L &= 2\text{m} = 2000\text{mm}; \\ \text{Diameter of the rod, } D &= 3\text{cm} = 30\text{mm}; \\ \text{Axial load, } P &= 10\text{KN} = 10 \times 10^3 \text{N}; \\ \text{Young's Modulus, } E &= 2 \times 10^5 \text{ N/mm}^2 \end{aligned}$$

To find:

(a) Stress, (b) Strain, (C) Elongation of the rod

Solution:

We know that,

$$\text{Stress}(\sigma) = \frac{\text{Load}(P)}{\text{Area}(A)} = \frac{10 \times 10^3}{\frac{\pi}{4} \times D^2} = \frac{10 \times 10^3}{\frac{\pi}{4} \times 30^2}$$

$$(\sigma) = 14.14 \text{ N/mm}^2.$$

$$\text{Young's modulus, (E)} = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \text{Strain}(e) = \frac{\text{Stress}}{\text{Young's Modulus}}$$

$$= \frac{14.14}{2 \times 10^5}$$

$$= 7.07 \times 10^{-5}$$

$$\text{Strain (e)} = \frac{\delta l}{l}$$

$$\text{or } 7.07 \times 10^{-5} = \frac{\delta l}{2000}$$

$$\delta l = 2000 \times 7.07 \times 10^{-5}$$

$$= 0.141 \text{ mm}$$

Problem:1.2 A hollow Cylinder 2m long has an outside diameter of 50mm and inside diameter of 30mm. If the cylinder is carrying a load of 25kN. Find the stress in the cylinder. Also find the deformation of the cylinder. Take $E=100\text{GPa}$.

Given Data:

Length,	$L = 2\text{m} = 2000\text{mm},$
Outside diameter,	$D = 50\text{ mm},$
Inside diameter,	$d = 30\text{ mm},$
Load,	$P = 25\text{ kN} = 25 \times 10^3\text{N}$
Young's modulus,	$E = 100\text{ GPa} = 100 \times 10^9\text{Pa}$ $= 100 \times 10^9\text{N/m}^2 = 100 \times 10^9 \frac{\text{N}}{\text{mm}^2}$ $= 100 \times 10^3\text{N/mm}^2$

To find: Stress(σ) and Deformation(Δl)

Solution:

$$\begin{aligned}
 \text{Stress}(\sigma) &= \frac{\text{Load}(P)}{\text{Area}(A)} \\
 &= \frac{25 \times 10^3}{\frac{\pi}{4} (D^2 - d^2)} \times 100 \times 10^3 \\
 &= \frac{25 \times 10^3}{\frac{\pi}{4} (50^2 - 30^2)} \\
 &= 19.89\text{N/mm}^2.
 \end{aligned}$$

$$\begin{aligned}
 \text{Deformation}(\delta l) &= \frac{Pl}{AE} \\
 &= \frac{(25 \times 10^3 \times 2000)}{\frac{\pi}{4} (50^2 - 30^2) \times 100 \times 10^3} \\
 &= 0.398\text{mm}
 \end{aligned}$$

Problem:1.3 A short hollow cast iron cylinder of external diameter 200mm is to carry a compressive load of 1.9MN. Determine the inner diameter of the cylinder, if the ultimate crushing stress for the material is 480MN/m^2 . Use the factor of safety of 4.

Given Data:

External Diameter,	$D = 200\text{mm},$
Load,	$P = 1.9\text{MN} = 1.9 \times 10^6\text{N},$
Ultimate Stress,	$\sigma_u = 480\text{MN/m}^2 = 480 \frac{10^6}{10^6} \text{N/mm}^2$ $= 480\text{N/mm}^2,$
Factor of Safety	$= 4$

To Find: Internal diameter(d)

Solution:

$$\begin{aligned} \text{Working Stress } (\sigma) &= \frac{\text{Ultimate Stress}}{\text{Factor of Safety}} \\ &= \frac{480}{4} \\ &= 120\text{N/mm}^2 \end{aligned}$$

$$\text{Stress}(\sigma) = \frac{\text{Load}(P)}{\text{Area}(A)}$$

$$\text{Or } 120 = \frac{1.9 \times 10^6}{\frac{\pi}{4} \times (200^2 - d^2)}$$

$$\text{Or } 40000 - d^2 = \frac{1.9 \times 10^6}{\frac{\pi}{4} \times 120^2}$$

$$\begin{aligned} \text{Or } d^2 &= 40000 - 20159.58 \\ &= 19840.4\text{mm}^2 \end{aligned}$$

$$\begin{aligned} \therefore d &= \sqrt{19840.4} \\ &= 140.856\text{mm} \\ &= \text{say } \mathbf{141\text{mm}} \end{aligned}$$