3.5 ANALYSIS OF FIXED ARCHES SETTLEMENT AND TEMPERATURE EFFECTS

Fixed arches

It is a structure which is statically indeterminate to third degree, due to the presence of three reactions at each support. The fixed arch has three independent static equilibrium equations and the degree if indeterminacy is three. The construction of fixed arch is easy, but the analysis is more complex.

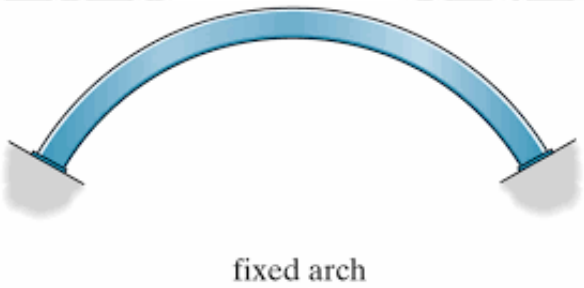

Fig. 3.5.1 Fixed arches

Advantages of fixed arches

a. These kind of arches are taken in application for longer spans, where the rigid foundations are available.
b. The fixed arches can be provided with temporary hinges at the springs, to avoid of shrinkage in reinforcing concrete. This makes the structure statically determinate.
c. The fixed arc are cheap and economical.
d. The fixed arch permits accurate analysis of stresses and therefore help in saving of material.
e. The deflection of the fixed arch is quite lesser than two hinged arches.
f. The positive moment at the centre off the span in minute, when compared with two hinge arches.

Disadvantages of fixed arches

a. Absolute fixity at the ends of the ends of the arches which increases the bending moment at the centre.
b. Fixed arches are not stable, durable compared to arches with hinges.

Settlement in arches

Fixed arches are generally made up of reinforced concrete. These are statically indeterminate to third and therefore require strong abutments. These are affected b he settlement of supports.

Example:

Find the reaction components at the supports of a symmetrical parabolic fixed arch 20m span 3m central rise when it is subjected to a uniformly distributed load of 2KN/m over the left half span .

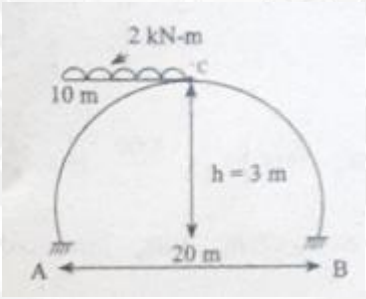

Fig. 3.5.2 Fixed arches

Solution :

Given span of arch (l) = 20m

central rise of the arch (h) = 3m

The equation of the parabolic arch is given by

$$Y = 4hx/l^2 \ (l-x)$$

$= 4\times3/20^2 \times x\times(20-x)$

$=0.03\times x\times(20-x)$

$=0.06x-0.03x^2$

The strain energy U due to bending is given by

$U = {}^1\int_0 M^2_x /2EI\, dx$

M_x for BC is equal to (V_{Bx}-H_B y- M_B) and the limits between 0-10m

M_x for CA is equal to (V_{Bx}-H_B y- H_B -2(x-10).(x-10)/2 and the limits between 10-20m

Now,

$\partial M/\partial V_B = x$:

$\partial M/\partial H_B = -y$:

$\partial M/\partial M_B = -1$

calculation of reaction components

$U = M^2_x /2EI\, dx$

$\partial U/\partial V_B = 2M_x /2EI \times \partial M_x /\partial V_B$

$\int \partial U/V_B = 1/EI \int Mx.\, \partial Mx /\partial VB$

$= 1/EI\, [\ {}^{10}\int_0 (V_B\, x- H_B\ y-M_B\ (x).dx + {}^{20}\int_{10} (V_B\, x -H_B\ y -M_B -2(x-2)^2/2\ (x)\, dx]$

$=1/EI\ [{}^{10}\int_0\ [V_Bx^2-(H_B\ \times(0.6x-0.03x^2)(x)]-M_B\ x]dx + {}^{20}\int_{10} V_B\ x^2\ -H_B\ (0.6x - 0.03x^2\ )(x)-M_B\ x -2x\ (x-2)^2/2$

$=1/EI\ [{}^{10}\int_0[V_Bx^2-(H_B\ \times(0.6x^2-0.03x^3)]-M_B\ x]dx + [{}^{20}\int_{10}\{V_Bx^2\ -H_B\ (0.6x^2 - 0.03x^3\ )-M_B\ x -x\ (x-2)^2\}dx)]$

$=1/EI \{[V_B x^3/3-(H_B (0.6 \times x^3/3 \times x^4/4))-M_B .X^2/2]_0^{10}+[V_B \times X^3/3-$
$H_B (0.6 \times x^3/3 \times 0.03x^4/4)-M_B X^2/2-x^4/4-4 \times x^2/2+4.x^3/3]_{10}^{20}$

$=1/EI \{(V_B.1000/3 - 200H_B +75H_B - 50M_B)+(V_B.8000/3 - 1600H_B + 1200H_B - 200M_B -40000-800+10666.66)-(V_B.1000/3-200H_B +75H_B -50M_B - 2500-200+1333.33)\}$

$=1/EI \{(V_B.1000/3 - 200H_B +75H_B - 50M_B +V_B.8000/3 - 1600H_B + 1200H_B - 200M_B -4000-800+10666.66)-V_B.1000/3+200H_B -75H_B +50M_B +2500+200-1333.33\}$

$=1/EI (8000/3 V_B- 400H_B -200M_B -28766.67)$_____(1)

$\partial U/\partial H_B$

$= 1/EI \int_0^1 Mx. \partial Mx/\partial H_B .dx$

$\partial U/\partial H_B$

$=1/EI[\int_0^{10} (V_B x - H_B y -M_B) (-y) dx +\int_{10}^{20} (V_B x-H_B y-M_B -(x-2)^2 /2)](-y)dx]$

$y = 0.6x - 0.03x^2$

$=1/EI [\int_0^{10} (V_B x - H_B y -M_B) (-0.6x - 0.03x^2) dx +\int_{10}^{20} (V_B x-H_B y-M_B -(x-2)^2 /2)](-0.6x - 0.03x^2)dx]$

$=1/EI [\int_0^{10} (-0.6 V_B x^2 + 0.03V_B x^3 + HB (0.6x-0.03x^2)^2 +0.6M_B x -0.03M_B x^2)dx +\int_{10}^{20} (0.6V_B x^2 + 0.03V_B x^3+H_B (0.6x-0.03x^2)^2 + 0.6M_B x -0.03M_B x^2 +0.6x^3-0.03x^4-2.4x+0.12x^2+2.4x^2-0.12x^3]$

$=1/EI [\int_0^{10} (-0.6 V_B x^2 +0.03V_B x^3 + H_B (0.36x^2-0.0009x^4-0.036x^3) +0.6M_B x -0.03M_B x^2)dx +\int_{10}^{20} (-0.6V_B x^2 + 0.03V_B x^3+ H_B (0.36x^2+0.0009x^4-0.036x^3) + 0.6M_B x -0.03M_B x^2 +0.48x^3-0.03x^4-2.52x^2-2.4x)dx]$

$=1/EI\ [\ (-0.6\ V_B\ x^3\ /3+0.03V_Bx^4\ /4+\ 0.36H_B\ \ x^3\ /3+\ 0.0009HBx^5/5-0.036HBx^4/4)$
$+0.6M_B\ x^2\ /2\ -0.03M_B\ x^3/3)]_0^{10}\ +\ (-0.6V_B\ x^3\ /3+\ 0.03V_Bx^4\ /4+\ H_B$
$0.36x^3/3+0.0009x^5-5-0.036HBx^4/4)+\ 0.6M_B\ x^2\ /2\ -0.03M_B\ x^3\ /3\ +0.48x^4\ /4$
$-\quad 0.03x^5/5\ -2.52x^3/3-\ 2.4x^2/2)]_{10}^{20}$

$=1/EI\ [(-125V_B+48H_B+20M_B\ )+(-275V_B+48H_B+20M_B\ +4920)]$

$=1/EI\ (-400V_B+96H_B+40M_B\ +4920)$_____(2)

$=\eth\ U/M_B=\ 1/EI\int\ mx.\eth\ M/\eth\ M_B.dx\ \&\ \eth\ M/\eth\ M_B\ =\ -1$

$=1/EI\ {}_0[\int^{10}\ (V_B\ x\ -\ H_B\ \ y\ -M_B\ )\ (-1)\ dx\ +\ {}_{10}\int^{20}(V_Bx-H_B\ \ y-M_B\ -(x-2)^2\ /2)](-1)dx]$

$=1/EI[{}_0\int^{10}\ (-V_B\ x\ +\ H_B\ \ \ (0.6x-0.03x^2\ )+M_B\ )\ \ dx\ +{}_{10}\int^{20}\ (-V_Bx+H_B\ \ \ (0.6x-0.03x^2$
$)+M_B\ +(x-2)^2dx]$

$=1/EI[\ (-V_B\ x^2\ /2\ +\ H_B\ 0.6x^2\ /2\ -0.03x^3\ /3)+M_B\ x]_0^{10}\ +\ (-V_Bx^2\ /2\ +H_B\ 0.6x^2\ /2\ -$
$0.03x^3/3)+M_B\ x+x^3\ /3+4x\ -4x^2\ /2\ ]_{10}^{20}$

$=1/EI\ [(-50V_B+30H_B-\ 10H_B\ +10M_B\ )+(-150V_B+90H_B-70H_B+10M_B$
$+2333.33+40-600)]$

$=1/EI\ [(-200V_B+40H_B\ +20M_B\ +\ 1773.33)$_____(3)

by equating eqn

$$\partial U/\partial V_B = 0 ,$$

$$\partial U/\partial H_B = 0 ,$$

$$\partial U/\partial M_B = 0$$

we get from equation 1,2&3

$2666.66V_B - 400H_B - 200M_B = 28766.67)$

$400V_B + 96H_B + 40M_B = -4920$

$-200V_B + 40H_B + 20M_B = -1773.33$

solving these three equation ,we get

$V_B = 16.55KN$

$H_B = -85.83KN$

$M_B = 248.50KN\text{-}m$