4.2 Stress and Deflection in Helical Springs of Non-circular Wire

This expression is applicable when the longer side (*i.e.* $t > b$) is parallelto the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to theaxis of the spring, then maximum shear stress,

$$\delta = \frac{2.45\, W.D^3.n}{G.b^3\ (t - 0.56\, b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\, W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\, W.D^3.n}{G.b^4} = \frac{5.568\, W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

Note : In the above expressions,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C}, \text{ and } C = \frac{D}{b}$$

Example 4.4. *A loaded narrow-gauge car of mass 1800 kg and moving at a velocity72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section.The mean diameter of the coil is six times the side of the square section. In bringing the car to rest,the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa andspring index of 6, find :*

1. *Maximum load on each spring, 2. Side of the square section of the wire, 3. Mean diameter ofcoils, and 4. Number of active coils.Take modulus of rigidity as 80 kN/mm^2.*

Solution.

Given :
m = 1800 kg
v = 72 m/min = 1.2 m/s
δ = 200 mm ;
τ = 365 MPa = 365 N/mm^2
C = 6
G = 80 kN/mm^2 = 80 × 10^3 N/mm^2

To Find
1.Maximum load on each spring,
2. Side of the square section of the wire,
3. Mean diameter of coils, and
4. Number of active coils

1. ***Maximum load on each spring,***

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800\ (1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0 + W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$$\therefore \quad W = 1296 \times 10^3 / 200 = 6480 \text{ N } \textbf{Ans.}$$

2. *Side of the square section of the wire*

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

and maximum shear stres (τ),

$$365 = K \times \frac{2.4\ W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\ b}{b^3} = \frac{116\ 870}{b^2}$$

$$\therefore \quad b^2 = 116\ 870 / 365 = 320 \quad \text{or} \quad b = 17.89 \text{ say } 18 \text{ mm } \textbf{Ans.}$$

3. *Mean diameter of the coil*

We know that mean diameter of the coil,

$$D = 6\ b = 6 \times 18 = 108 \text{ mm } \textbf{Ans.}$$

4. *Number of active coils*

Let n = Number of active coils.

We know that the deflection of the spring (δ),

$$200 = \frac{5.568\ W.C^3.n}{G.b} = \frac{5.568 \times 6480 \times 6^3 \times n}{80 \times 10^3 \times 18} = 5.4\ n$$

$$\therefore \quad n = 200 / 5.4 = 37 \textbf{ Ans.}$$