## 2.2 Pigeon Hole Principle and Generalized Pigeon Hole Principle

**Pigeonhole Principle:**

The pigeonhole principle in its simplest incarnation, states the following

If you have more pigeons than pigeonholes, and you try to stuff the pigeons into the holes, then Atleast one hole must contain at least two pigeons.

**Basic Pigeonhole Principle:**

If $k + 1$ or more objects are placed into $k$ boxes, then there is Atleast one box containing two or more of the objects.

**Pigeonhole Principle:**

**If $(\boldsymbol{n+1})$ Pigeon occupies "$\boldsymbol{n}$" holes then atleast one hole has more than one pigeon.**

**Proof:**

Assume $(n + 1)$ pigeon occupies "$n$" holes.

**Claim:** Atleast one hole has more than one pigeon.

Suppose not,

Atleast one hole has not more than one pigeon.

Therefore each and every hole has exactly one pigeon.

Since, there are "$n$" hole, which implies, we have totally "$n$" pigeon.

Which is a contradiction to our assumption that there are $(n+1)$ pigeon.

Therefore atleast one hole has more than one pigeon.

Hence the proof.

**Generalized Pigeon Hole Principle**

**If $m$ pigeon occupies "$n$" holes $(m > n)$ then atleast one hole has more than $\left[\frac{m-1}{n}\right]+1$ pigeon.Here $[x]$ denotes the greatest integer less than or equal to $x$, which is a real number.**

**Proof:**

Assume "$m$" pigeon occupy "$n$" holes $(m > n)$

**Claim:** Atleast on hole has more than $\left[\frac{m-1}{n}\right]+1$ pigeon.

Suppose not, i.e., Atleast one hole has not more than $\left[\frac{m-1}{n}\right]+1$ pigeon.

Each and every hole has exactly $\left[\frac{m-1}{n}\right]+1$ pigeon.

Since we have n holes, totally there are $n\left[\left[\frac{m-1}{n}\right]+1\right]$ pigeon.

$\Rightarrow m-1+n$ pigeons

$\Rightarrow m+n-1$ pigeons

Which is a contradiction to the assumption, that there are m pigeons.

Therefore, Atleast one hole has more than $\left[\frac{m-1}{n}\right]+1$ pigeon.

**Problems under Pigeonhole and Generalized pigeonhole principle**

**1. Show that, among 100 people, atleast 9 of them were born in the same month.**

**Solution:**

Here, Number of Pigeon = Number of people = 100

Number of holes = Number of month = 12

Then by generalized pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1=\left[\frac{100-1}{12}\right]+1=9$$

Were born in the same month.

**2. Show that, if seven colors are used to paint 50 bicycles, atleast 8 bicycles will be the same.**

**Solution:**

Here, Number of Pigeon = Number of bicycle = 50

Number of holes = Number of colors = 7

Then by generalized pigeon hole principle,

$$\left[\frac{m-1}{n}\right]+1=\left[\frac{50-1}{7}\right]+1=8$$

Therefore atleast 8 bicycles will have the same color.

**3. Show that, if 25 dictionaries in a library contain a total of 40,325 pages, then one of the dictionaries must have atleast 1614 pages.**

**Solution:**

Here, Number of Pigeon = Number of bicycle = 40325

Number of holes = Number of colors = 25

Then by generalized pigeon hole principle,

$$\left\lceil\frac{m-1}{n}\right\rceil + 1 = \left\lceil\frac{40325-1}{25}\right\rceil + 1 = 1614$$

Here, Number of Pigeon = Number of grades $= \ n \ = \ 5$

Let k be number of students (pigeon) in discrete mathematics class.

$$\Rightarrow k + 1 = 6$$

$$\Rightarrow k = 5$$

The total number of students $= kn + 1$

$$= 5 \times 5 + 1 = 26$$

Minimum number of students = 26.