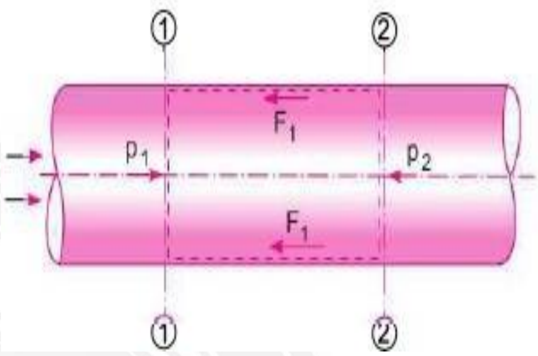
FRICTIONAL LOSS IN PIPE FLOW – DARCY WEISBACK EQUATION

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent tothe pipe wall is zero. The velocity of liquidgoes on increasing from the wall and thus velocity gradient and hence shear stressesare produced in the whole liquid due to viscosity. This viscous action causes loss of energy, which is known as frictional loss.



Consider a uniform horizontal pipe having steady flow. Let 1-1, 2-2 are two sections of pipe.

Let P_1 = Pressure intensity at section

1-1V_1 = Velocity of flow at section 1-1

L = Length of pipe between section 1-1 and

2-2d = Diameter of pipe

f' = Fractional resistance for unit wetted area per a unit velocity

hf = Loss of head due to friction

And P_2, V_2 = are values of pressure intensity and velocity at section

2-2Applying Bernoulli's equation between sections 1-1 and 2-2

Total head at 1-1 = total head at 2-2 + loss of head due to friction between 1-1 and 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\therefore \quad \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \text{ or } h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \quad \ldots(i)$$

But h_f is head is lost due to friction and hence the intensity of pressure will be reducedin the direction flow by frictional resistance.

Now, Frictional Resistance = Frictional resistance per unit wetted area per unit velocity unit velocity × Wetted Area × (velocity)^2

$$F_1 = f' \times \pi d L \times V^2 \quad [\because \text{ wetted area} = \pi d \times L, \text{ velocity} = V = V_1 = V_2]$$
$$= f' \times P \times L \times V^2 \quad [\because \pi d = \text{Perimeter} = P] \ldots(ii)$$

The forces acting on the fluid between section 1-1 and 2-2 are
Pressure force at section 1-1 = P1 ×A where A = area of pipePressure force at section 2-2 = P2 ×A
Frictional force = F1

Resolving all forces in the horizontal direction, we have

$$p_1A - p_2A - F_1 = 0 \qquad \ldots(1)$$

or $$(p_1 - p_2)A = F_1 = f' \times P \times L \times V^2 \qquad [\because \text{ From } (ii), F_1 = f'PLV^2]$$

or $$p_1 - p_2 = \frac{f' \times P \times L \times V^2}{A}$$

But from equation (i), $p_1 - p_2 = \rho g h_f$

Equating the value of $P_1 - P_2$, we get

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \ldots(iii)$$

In equation (iii), $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$$\therefore \quad h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \qquad \ldots(iv)$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (iv), becomes as $h_f = \frac{4 . f}{2g} . \frac{LV^2}{d} = \frac{4f . L . V^2}{d \times 2g}$

$$h_f = \frac{f . L . V^2}{d \times 2g}$$

This Equation is known as Darcy – Weisbach equation, commonly used for finding loss of head due to friction in pipes
Then f is known as a friction factor or co-efficient of friction which is a dimensionless

quantity. f is not a constant but, its value depends upon the roughness condition of pipe surface and the Reynolds number of the flow.

MOODY DIAGRAM

Moody’s diagram is plotted between various values of friction factor(f),Reynolds number(Re) and relative roughness(R/K) as shown in figure 2.6.For any turbulent flowproblem,the values of friction factor(f) can therefore be determined from Moody’s diagram,if the numerical values of R/K for the pipe and Re of flow are known.

The Moody’s diagram has plotted from the equation

$1/\sqrt{f}$-2.0 log10(R/K)=1.74-2.0 log10[1+18.7/(R/K/Re/ f)]

Where,R/K=relative roughness

f=friction factor and Re=Reynolds number.