HEAT TRANSFER EFFECTIVENESS

The method suggested by Nusselt and developed by Kays and London is now being extensively used. The effectiveness of a heat exchanger is defined as the ratio of the actual heat transferred to the maximum possible heat transfer.

Let $\dot{m}_h$ and $\dot{m}_c$ be the mass flow rates of the hot and cold fluids, c_h and c_c be the respective specific heat capacities and the terminal temperatures be Th and T h for the hot fluid at inlet and outlet, T_{h_i} and T_{h_0} for the cold fluid at inlet and outlet. By making an energy balance and assuming that there is no loss of energy to the surroundings, we write

$$\dot{Q} = \dot{m}_h\ c_h\left(T_{h_i} - T_{h_0}\right) = \dot{C}_h\left(T_{h_i} - T_{c_0}\right), \text{ and}$$

$$= \dot{m}_c c_c\left(T_{c_0} - T_{c_i}\right) = \dot{C}_c\left(T_{c_0} - T_{c_i}\right) \qquad (3.13)$$

From Eq. (10.13), it can be seen that the fluid with smaller thermal capacity, C, has the greater temperature change. Further, the maximum temperature change of any fluid would be $\left(T_{h_i} - T_{c_i}\right)$ and this Ideal temperature change can be obtained with the fluid which has the minimum heat capacity rate. Thus,

$$\text{Effectiveness, } \in = \dot{Q}/C_{min}\left(T_{h_i} - T_{c_i}\right) \qquad (3.14)$$

Or, the effectiveness compares the actual heat transfer rate to the maximum heat transfer rate whose only limit is the second law of thermodynamics. An useful parameter which also measures the efficiency of the heat exchanger is the 'Number of Transfer Units', NTU, defined as

NTU = Temperature change of one fluid/LMTD.

Thus, for the hot fluid: NTU = $\left(T_{h_i} - T_{h_0}\right)/\text{LMTD}$, and

for the cold fluid: $\text{NTU} = \left(T_{c_0} - T_{c_i}\right)/\text{LMTD}$

Since $\dot{Q} = UA(\text{LMTD}) = C_h\left(T_{h_i} - T_{h_0}\right) = \dot{C}_c\left(T_{c_0} - T_{c_i}\right)$

we have $\text{NTU}_h = UA/C_h$ and $\text{NTU}_c = UA/C_c$

The heat exchanger would be more effective when the NTU is greater, and therefore,

$$NTU = AU/C_{min} \tag{3.15}$$

An another useful parameter in the design of heat exchangers is the ratio the minimum to the maximum thermal capacity, i.e., $R = C_{min}/C_{max}$,

where R may vary between I (when both fluids have the same thermal capacity) and 0 (one of the fluids has infinite thermal capacity, e.g., a condensing vapour or a boiling liquid).

3.9. Effectiveness - NTU Relations

For any heat exchanger, we can write: $\in = f\left(NTU,\ C_{min}/C_{max}\right)$. In order to determine a specific form of the effectiveness-NTU relation, let us consider a parallel flow heat exchanger for which $C_{min} = C_h$. From the definition of effectiveness (equation 10.14), we get

$$\varepsilon = \left(T_{h_i} - T_{h_0}\right)/\left(T_{h_i} - T_{c_i}\right)$$

and, $C_{min}/C_{max} = C_h/C_c = \left(T_{c_0} - T_{c_i}\right)/\left(T_{h_i} - T_{h_0}\right)$ for a parallel flow heat exchanger, from Equation 10.4,

$$\ln\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = -UA\left(1/C_h + 1/C_c\right) = \frac{-UA}{C_{min}}\left(1 + C_{min}/C_{max}\right)$$

or, $\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = \exp\left[-NTU\left(1 + C_{min}/C_{max}\right)\right]$

But, $\left(T_{h_0} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right) = \left(T_{h_0} - T_{h_i} + T_{h_i} - T_{c_0}\right)/\left(T_{h_i} - T_{c_i}\right)$

$$= \left[\left(T_{h_0} - T_{h_i}\right) + \left(T_{h_i} - T_{c_i}\right) - \left\{R\left(T_{h_i} - T_{h_0}\right)\right\}\right]/\left(T_{h_i} - T_{c_i}\right)$$

$$= \in + 1 - R \in = 1 - \in\left(1 + R\right)$$

Therefore, $\in = \left[1 - \exp\left\{-NTU(1+R\right\}\right]/\left(1 + R\right)$

$$NTU = -\ln\left[1 - \in\left(1 + R\right)\right]/\left(1 + R\right)$$

Similarly, for a counter flow exchanger, $\varepsilon = \dfrac{\left[1 - \exp\left\{-NTU(1-R)\right\}\right]}{\left[1 - R\exp\left\{-NTU(1-R)\right\}\right]}$;

and, $NTU = \left[1/(R-1)\right] \ln\left[(\in -1)/(\in R-1)\right]$

Heat Exchanger Effectiveness Relation

Flow arrangement	relationship
Concentric tube	
Parallel flow	$\in = \dfrac{1-\exp\left[-N(1+R)\right]}{(1+R)}$; $R = C_{min} / C_{max}$
Counter flow	$\in = \dfrac{1-\exp\left[-N(1-R)\right]}{1-R\exp\left[-N(1-R)\right]}$; $R < 1$
	$\in = N/(1+N)$ for $R = 1$
Cross flow (single pass)	
Both fluids unmixed	$\in = 1-\exp\left[(1/R)(N)^{0.22}\left\{\exp\left(-R(N)^{0.78}\right)-1\right\}\right]$
C_{max} mixed , C_{min} unmixed	$\in = (1/R)\left[1-\exp\left\{-R\left(1-\exp(-N)\right)\right\}\right]$
C_{min} mixed, C_{max} unmixed	$\in = 1-\exp\left[-R^{-1}\left\{1-\exp(-RN)\right\}\right]$
All exchangers (R = 0)	$\in = 1-\exp(-N)$

Kays and London have presented graphs of effectiveness against NTU for Various values of R applicable to different heat exchanger arrangements, Fig. 3.11 to Fig. (3.15).

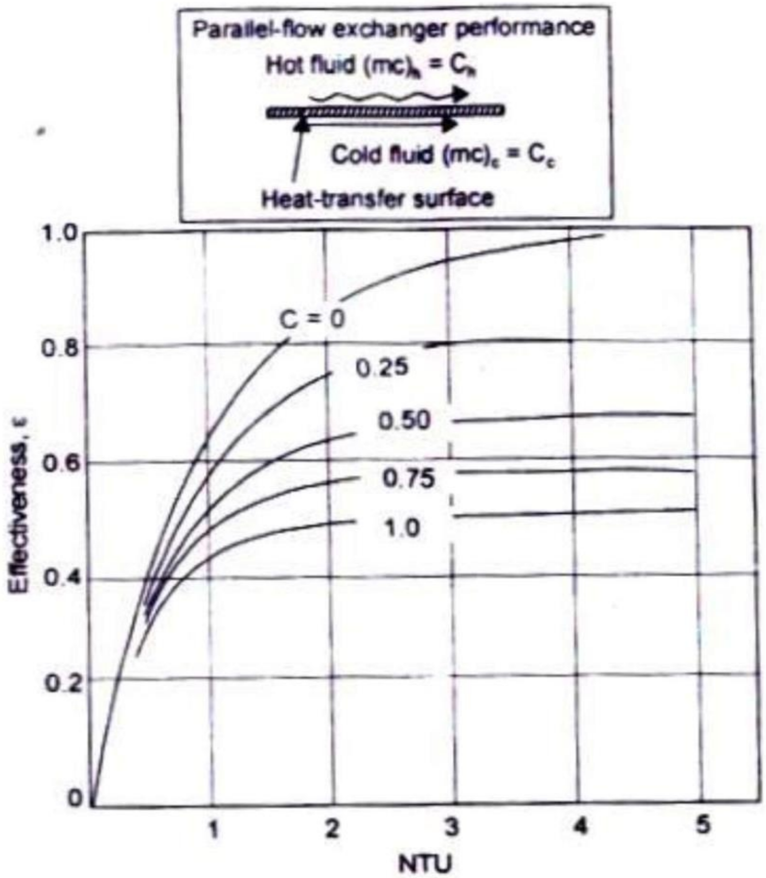


Fig 3.11 Heat exchanger effectiveness for parallel flow

Example 3.13 A single pass shell and tube counter flow heat exchanger uses exhaust gases on the shell side to heat a liquid flowing through the tubes (inside diameter 10 mm, outside diameter 12.5 mm, length of the tube 4 m). Specific heat capacity of gas 1.05 kJ/kgK, specific heat capacity of liquid 1.5 kJ/kgK, density of liquid 600 kg/m^3, heat transfer coefficient on the shell side and on the tube sides are: 260 and 590 W/m^{2}K respectively. The gases enter the exchanger at 675 K at a mass flow rate of 40 kg/s and the liquid enters at 375 K at a mass flow rate of 3 kg/s. If the velocity of liquid is not to exceed 1 m/s, calculate (i) the required number of tubes, (ii) the effectiveness of the heat exchanger, and (iii) the exit temperature of the liquid. Neglect the thermal resistance of the tube wall.

Solution: Volume flow rate of the liquid = 3/600 = 0.005 m/s. For a velocity of 1 mls through the tube, the cross-sectional area of the tubes will be 0.005 m^2. Therefore, the number of tubes would be

$n(0.005 \times 4)/(3.142 \times 0.01)^2 = 63.65 = 64$ tubes

The overall heat transfer coefficient based on the outside surface area of the tubes, after neglecting the thermal resistance of the tube wall, is

$U = 1/\left(1/h_0 + r_o/r_i h_i\right) = 1/\left[1/260 + 12.5/(10 \times 590)\right] = 167.65\ W/m^2K$

$C_{max} = 40 \times 1.05 = 42$; $C_{min} = 3 \times 1.5 = 4.5$; R = 4.5/42 = 0.107

$NTU = AU/C_{min} = 3.142 \times 0.0125 \times 4 \times 64 \times 167.65/(4.5 \times 1000) = 0.374$

From Fig. 10.12, for R = 0.107, and NTU = 0.3 74, E = 0.35 approximately Therefore, $0.35 = \left(T_{c_0} - 375\right)/(675 - 375)$ or $T_{c_0} = 207^o C$

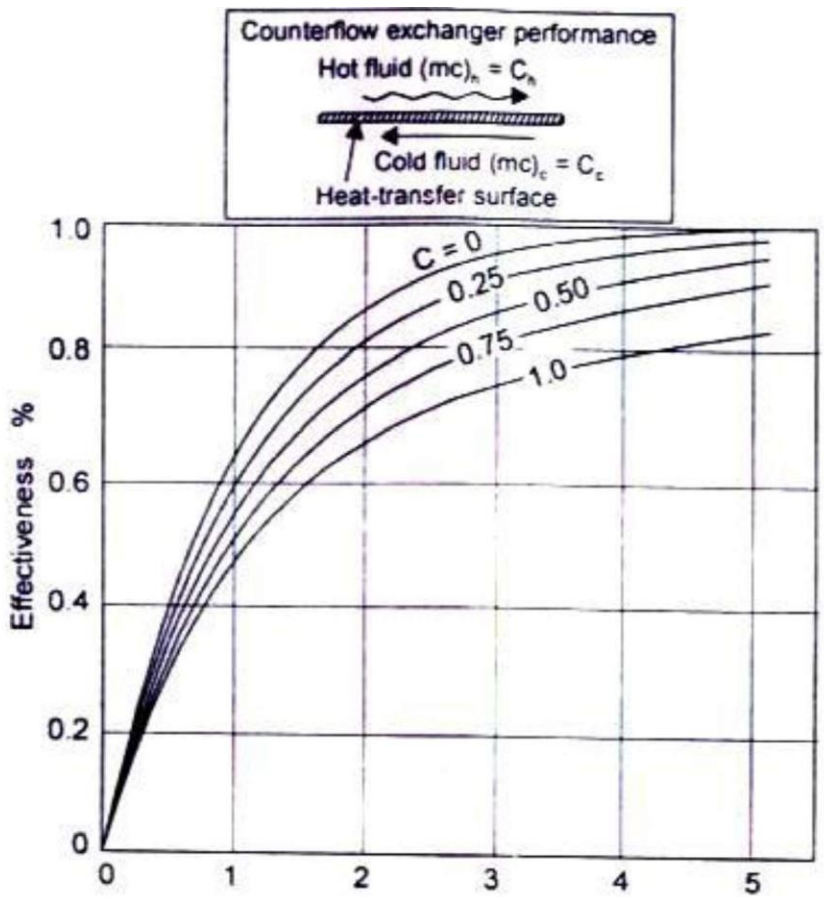


Fig 3.12 Heat exchanger effectiveness for counter flow

Example 3.14 Air at 25°C, mass flow rate 20 kg/min, flows over a cross-flow heat exchanger and cools water from 85°C to 50°C. The water flow rate is 5 kg/mm. If the overall heat transfer coefficient IS 80 W/m^2K and air is the mixed fluid, calculate the exchanger effectiveness and the surface area.

Solution: Let the specific heat capacity of air and water be 1.005 and 4.182 kJ/ kgK. By making an energy balance:

$$\dot{m}_c \times c_c \times \left(T_{c_0} - T_{c_i}\right) = \dot{m}_h \times c_h \times \left(T_{h_i} - T_{h_0}\right)$$

or, $5 \times 4182 \times (85 - 50) = 20 \times 1005 \times \left(T_{c_0} - 25\right)$

i.e., the air will come out at 61.4 °C.

Heat capacity rates for water and air are:

$$C_w = 4182 \times 5/60 = 348.5; \quad C_a = 1005 \times 20/60 = 335$$

$$R = C_{min}/C_{max} = 335/348.5 = 0.96$$

The effectiveness on the basis of minium heat capacity rate is

$$\in = (61.4 - 25)/(85 - 25) = 0.6$$

From Fig. 10.13, for R = 0.96 and $\in$ = 06, NTU = 2.5

Since NTU = AU/C_{min}; A = 2.5 × 335/80 = 10.47 m^2

Since all the four terminal temperatures are easily obtained, we can also use the LMTD approach. Assuming a simple counter flow heat exchanger,

LMTD = (25 – 23.6)/ ln (25/23.6) = 24.3

The correction factor for using a cross-flow heat exchanger with one fluid mixed and the other unmixed, from Fig. 10.10(d), F = 0.55

$$\dot{Q} = U\ A\ F\ (LMTD)$$

Therefore, A = 348.5 × 35/(80 × 0.55 × 24.3) = 11.4 m^2

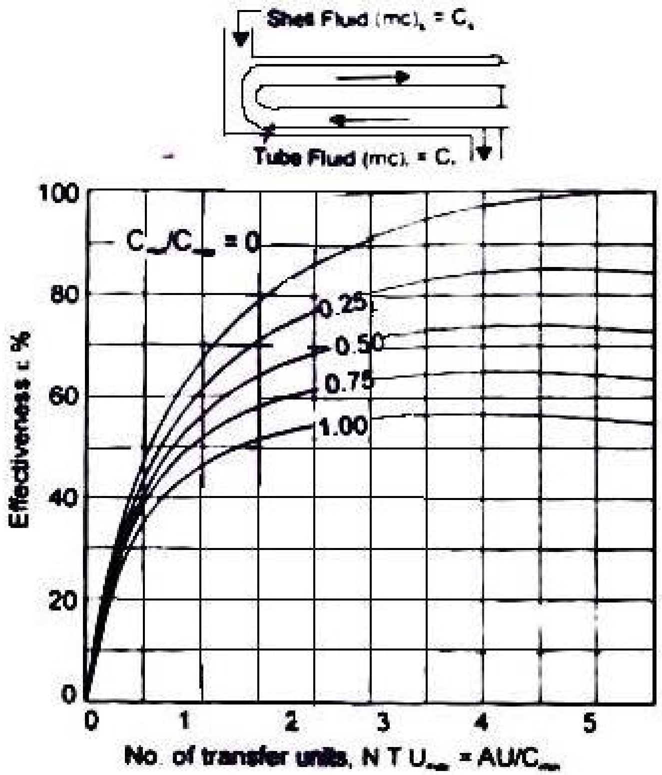
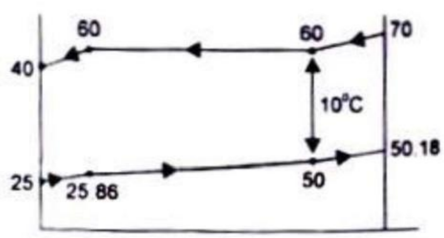


Fig. 3.13 Heat exchanger effectiveness for shell and tube heat exchanger with one shell pass and two, or a multiple of two, tube passes

Example 3.15 Steam at 20 kPa and 70°C enters a counter flow shell and tube exchanger and comes out as subcooled liquid at 40°C. Cooling water enters the condenser at 25°C and the temperature difference at the pinch point is 10°C. Calculate the (i) amount of water to be circulated per kg of steam condensed, and (ii) required surface area if the overall heat transfer coefficient is 5000 W/m^{2}K and is constant.

Solution: The temperature profile of the condensing steam and water is shown in the 40 accompanying sketch.



The saturation temperature corresponding to 20 kPa is 60°C and as such the temperature of the cooling water at the pinch pint is 50°C. The condensing unit may be considered as a combination of three sections:

(i) desuperheater - the superheated steam is condensed to saturated steam from 70°C to 60°C.

(ii) the condenser - saturated steam is condensed into saturated liquid.

(iii) subcooler - saturated liquid at 60°C is cooled to 40°C.

Assuming that the specific heat capacity of superheated steam is 1.8 kJ/kgK, heat given out in the desuperheater section is 1.8 × (70 - 60) = 18000 Jlkg. Heat given out in the condenser section = 2358600 J/kg (= hfg)

Heat given out in the subcooler = 4182 × (60 - 40) = 83640 J/kg

By making an energy balance, for subcooler and condenser section, we have

$\dot{m}_w \times 4182 \times (50-25) = (83640 + 2358600)$;

∴. Mass of water circulated, $\dot{m}_w$ = 23.36 kg/kg steam condensed.

The temperature of water at exit

= 25 + (83640 + 2358600 + 18000)/ (23.36 × 4182) = 50.18 °C

LMTD for desuperheater section

= [(70 - 50.18) - (60 - 50))/ln(20.18/1 0) = 14.5

LMTD for condenser section = [(60 - 50) - (60 - 25.86))/ln(1 0/34.14)

= 19.66

LMTD for subcooler section = [(34.14 - 15)/1n (34.14/15) = 23.27

Since U is constant through out,

Surface area for subcooler section = 83640/(5000 × 23.27) = 0.7188 m^2

Surface area for condenser section = 2358600/(5000 × 19.66) = 23.9939 m^2

Surface area for desuperheater section = 18000/(5000 × 14.5) = 0.2483 m^2

$\therefore$. Total surface area = 24.96 m^2 and average temperature difference = 19.71°C.

Example 3.16 In an economiser (a cross flow heat exchanger, both fluids unmixed) water, mass flow rate 10 kg/s, enters at 175°C. The flue gas mass flow rate 8 kg/s, specific heat 1.1 kJ/kgK, enters at 350°C. Estimate the temperature of the flue gas and water at exit, if U = 500 W/m^2K, and the surface area 20 m^2 What would be the exit temperature if the mass flow rate of flue gas is (i) doubled, and (ii) halved.

Solution: The heat capacity rate of water = 4182 × 10 = 41820 W /K

The heat capacity rate of flue gas = 1100 × 8 = 8800 W/K

C_{min}/C_{max} = 8800/41820 = 0.21

$NTU = AU/C_{min} = 500 \times 20/8800 = 1.136$

From Fig. 10.14. for NTU = 1.136 and $C_{min}/C_{max} = 0.21$, $\in = 0.62$

Therefore, 0.62 = (350 - T)/(350 - 175) and T = 241.5°C

The temperature of water at exit, T_w = 175 + 8800 × (350 - 241.5)/41820

= 197.83°C

When the mass flow rate of the flue gas is doubled. C_{gas} = 17600 W/K

$C_{min}/C_{max} = 0.42$, $NTU = AU/C_{min} = 0.568$

$\in$ = 0.39 = (350 - T)/(350 - 175);

T = 281. 75°C, an increase of 40°C

and T_w = 175 + 28.72 = 203.72°C, an increase of about 6°C.

When the mass flow rate of the flue gas is halved, C_{min} = 4400 W/K

$C_{min}/C_{max} = 0.105$, NTU = 2.272, and from the figure, $\in$ = 0.83, an increase and T_g = 204.75 and T_w = 190.3°C

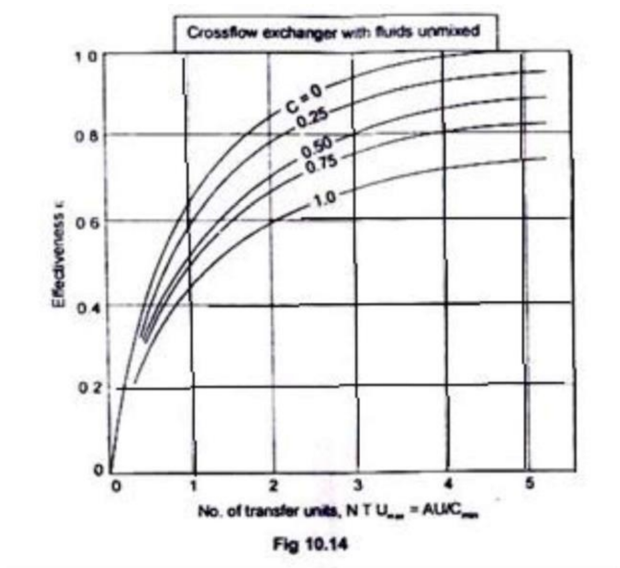


Fig 3.14

Example 3.17 In a tubular condenser, steam at 30 kPa and 0.95 dry condenses on the external surfaces of tubes. Cooling water flowing through the tubes has mass flow rate 5 kg/s, inlet temperature 25°C, exit temperature 40°C. Assuming no subcooling of the condensate, estimate the rate of condensation of steam, the effectiveness of the condenser and the NTU.

Solution: Since there is no subcooling of the condensate, the steam will lose its latent heat of condensation = 0.95 × h_{fg} = 0.95 × 2336100 = 2.22 × 10^6 J/kg. At pressure, 30kPa, saturation temperature is 69.124°C

Steam condensation rate × 2.22 × 10^6 = Heat gained by water

= 5 × 4182 × (40 – 25) = 313650 J

Therefore, m, = 313650/2.22 × 106 = 847.7 kg/hour.

When the temperature of the evaporating or condensing fluid remains constant, the value of LMTD is the same whether the system is having a parallel flow or counter flow arrangement, therefore,

LMTD = [(69.124 - 25) - (69.124-40)]/ln(44.124/29.124) = 36.1

Q = UA(LMTD)

Therefore, UA = 5 × 4182 × (40 - 25)/36.1 = 8688.36 W/K

$NTU = UA/C_{min} = 8688.36/(5 \times 4182) = 0.4155$

Effectiveness= Actual temp. difference; Maximum possible temp. difference

= (40 - 25)/(69.124 - 25) = 34%.

Example 3.18 A single shell 2 tube pass steam condenser IS used to cool steam entering at 50°C and releasing 2000 MW of heat energy. The cooling water, mass flow rate 3 × 104 kg/s, enters the condenser at 25°C. The condenser has 30,000 thin walled tube of 30 mm diameter. If the overall heat transfer coefficient is 4000 Wim^2K, estimate the (I) rise m temperature of the cooling water, and (II) length of the tube per pass.

Solution: By making an energy balance:

Heat released by steam = heat taken in by cooling water,

or, $2000 \times 10^6 = 3 \times 10^4 \times 4182 \times (\square T)$; $\square\square\square\square T = 15.94°C$.

Since in a condenser, heat capacity rate of condensing steam is usually very large m comparison with the heat capacity rate of cooling water, the effectiveness

$$\in = \left(T_{c_0} - T_{c_i}\right) / \left(T_{h_i} - T_{c_i}\right) = 15.94 / (50 - 25) = 0.6376$$

And, for $C_{min} / C_{max} = 0$, $\in = 1 - \exp(-NTU)$

$$\therefore \exp(-NTU) = 1.0 - 0.6376 = 0.3624$$

And, $NTU = 1.015 = AU / C_{min} = (2 \times 3.142 \times 0.03 \times L \times 30000) \times 4000 / (1.25 \times 108)$

$L = 5.546$ m

3.10. Heat Exchanger Design-Important Factors

A comprehensive design of a heat exchanger involves the consideration of the thermal, mechanical and manufacturing aspect. The choice of a particular design for a given duty depends on either the selection of an existing design or the development of a new design. Before selecting an existing design, the analysis of his performance must be made to see whether the required performance would be obtained within acceptable limits.

In the development of a new design, the following factors are important:

(a) Fluid Temperature - the temperature of the two fluid streams are either specified for a given inlet temperature, or the designer has to fix the outlet temperature based on flow rates and heat transfer considerations. Once the terminal temperatures are defined, the effectiveness of the heat exchanger would give an indication of the type of flow path-parallel or counter or cross-flow.

(b) Flow Rates - The maximum velocity (without causing excessive pressure drops, erosion, noise and vibration, etc.) in the case of liquids is restricted to 8 m1s and m case of gases below 30 m/s. With this restriction, the flow rates of the two fluid streams lead to the selection of flow passage cross-sectional area required for each of the two fluid steams.

(c) Tube Sizes and Layout - Tube sizes, thickness, lengths and pitches have strong influence on heat transfer calculations and therefore, these are chosen with great care. The sizes of tubes vary from 1/4" O.D. to 2" O.D.; the more commonly used sizes are: 5/8", 3/4" and 1" O.D. The sizes have to be decided after making a compromise between higher heat transfer from smaller tube sizes and the easy clean ability of larger tubes. The tube thickness will depend pressure, corrosion and cost. Tube pitches are to be decided on the basis of heat transfer calculations and difficulty in cleaning. Fig. 3.16 shows several arrangements for tubes in bundles. The two standard types of pitches are the square and the triangle. the usual number 0 f tube passes in a given shell ranges from one to eight. In multipass designs, even numbers of passes are generally used because they are Simpler to design.

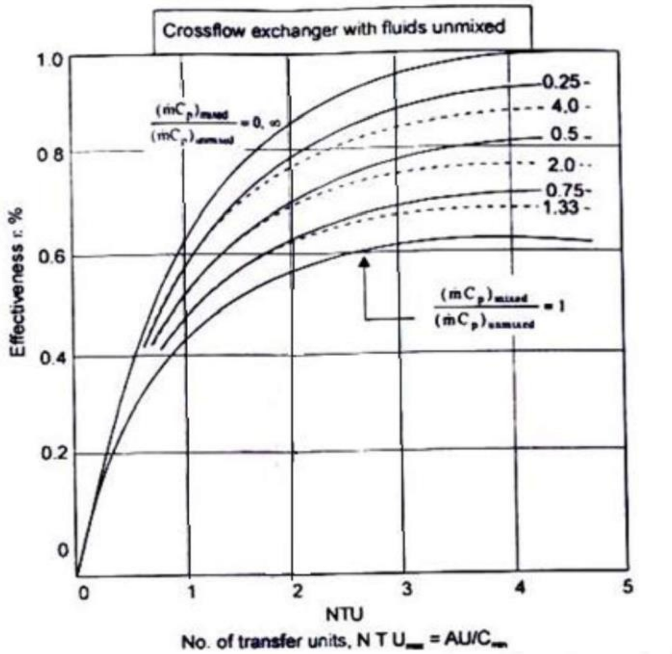


Fig 3.15: Heat exchanger effectiveness for crossflowwith one fluid mixed and the other unmixed

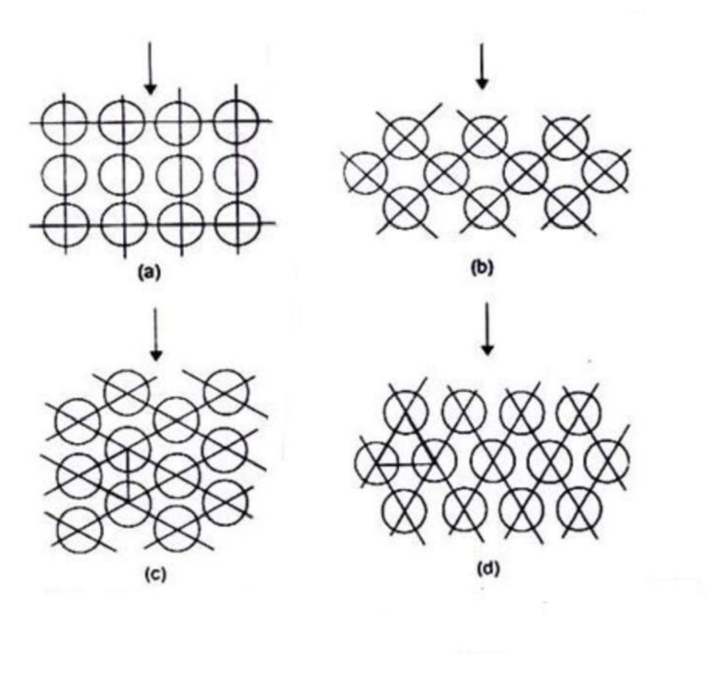


Fig 3.16 Several arrangements of tubes in bundles : (a) I line arrangement with square pitch, (b) staggered arrangement with triangular pitches (c) and(d) stagged arrangement with triangular pitches

Fig 3.17 shows three types of transverse baffles used to increase velocity on the shell side. The choice of baffle spacing and baffle cut is a variable and the optimum ratio of baffle cuts and spacing cannot be specified because of many uncertainties and insufficient data.

(d)Dirt F actor and Fouling - the accumulation of dirt or deposits affects significantly the r ate of heat transfer and the pressure drop. P roper allowance for the fouling factor and dirt factor should receive the greatest attention design because they cannot be avoided. A heat exchanger requires frequent cleaning. Mechanical cleaning will require removal of the tube bundle for cleaning. Chemical cleaning will require the use of non-corrosive materials for the tubes.

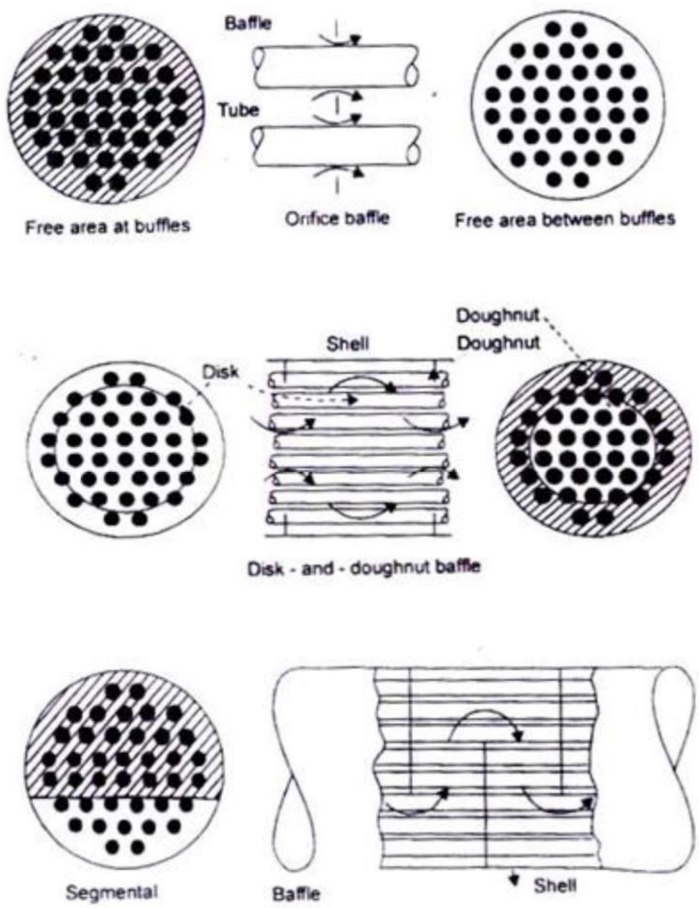


Fig. 3.17 Three types of transverse baffles

(e) Size and Installation - In designing a heat exchanger, It is necessary that the constraints on length, height, width, volume and weight is known at the outset. Safety regulations should also be kept in mind when handling fluids under pressure or toxic and explosive fluids.

(f) Mechanical Design Consideration - While designing, operating temperatures, pressures, the differential thermal expansion and the accompanying thermal stresses require attention.

And, above all, the cost of materials, manufacture and maintenance cannot be Ignored.

Example 3.19 In a counter flow concentric tube heat exchanger cooling water, mass flow rate 0.2 kg/s, enters at 30°C through a tube inner diameter 25rnnl. The oil flowing through the annulus, mass flow rate 0.1 kg/s, diameter 45 rum, has temperature at inlet 100°C. Calculate the length of the tube if the oil comes out at 60°C. The properties of oil and water are:

Oil: C_p – 2131 J/kgK, □ = 3.25×10^{-2} Pa-s, k = 0.138 W/mK,

Water; C_p = 4178 J/kg K, □ = 725×10^{-6} Pa-s,

k = 0.625 W/mK, Pr = 4.85

Solution: By making an energy balance: Heat given out by oil = heat taken in by water.

$$0.1 \times 2131 \times (100 - 60) = 0.2 \times 4187 \times \left(T_{c_0} - 30\right)$$

$$T_{c_0} = 40.2^\circ \text{ C}$$

$$\text{LMTD} = \left[\left(T_{h_i} - T_{c_0}\right) - \left(T_{h_0} - T_{c_i}\right)\right] / \ln\left[\left(T_{h_i} - T_{c_0}\right) / \left(T_{h_0} - T_{c_0}\right)\right]$$

$$= \left[(100 - 40.2) - (60 - 30)\right] / \ln(59.8/30) = 43.2^\circ \text{C}$$

Since water is flowing through the tube,

$$\text{Re} = 4\dot{m} / \pi D\mu = \frac{4 \times 0.2}{3.142 \times 0.025 \times 725 \times 10^{-6}} = 14050\text{, a turbulent flow.}$$

□□□□Nu = 0.023 $\text{Re}^{0.8}$ $\text{Pr}^{0.4}$, fluid being heated.

= 0.023 $(14050)^{0.8}$ $(4.85)^{0.4}$ = 90; $\therefore$ h_i = 90 × 0.625/0.025 = 2250 $\text{W/m}^2\text{K}$

The oil is flowing through the annulus for which the hydraulic diameter is:

(0.045 - 0.025) = 0.02 m

$$Re = 4\dot{m} / \pi(D_o + D_i)\mu = 4 \times 0.1 / \left(3.142 \times 0.07 \times 3.25 \times 10^{-2}\right) = 56.0$$

laminar flow.

Assuming Uniform temperature along the Inner surface of the annulus and a perfectly insulated outer surface.

Nu = 5.6, by interpolation (chapter 6)

$h_o = 5.6 \times 0.138/0.02 = 38.6$ W/m^{2}K.

The overall heat transfer coefficient after neglecting the tube wall resistance,

U = 1/(1/2250 + 1/38.6) = 38 W/m^{2}K

$\dot{Q} = UA(LMTD)$, where where $A = \pi D_i \times L$

L = (0.1 × 2131 × 40)/(38 × 3.142 × 0.025 × 43.2) = 66.1 m requires more than one pass.

Example 3.20 A double pipe heat exchanger has an effectiveness of 0.5 for the counter flow arrangement and the thermal capacity of one fluid is twice that of the other fluid. Calculate the effectiveness of the heat exchanger if the direction of flow of one of the fluids is reversed with the same mass flow rates as before.

Solution: For a counter flow arrangement and R = 0.5, $\in = 0.5$

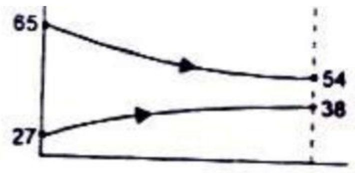
$$NTU = \left[1/(R-1)\right]\ln(\in R - 1) = -2.0 \ln(0.5/0.75) = 0.811$$

For parallel flow, $\in = \left[1 - \exp\{-NTU(1+R)\}\right]/(1+R)$

$$= \left[1 - \exp(-0.811 \times 1.5)\right]/1.5 = 0.469$$

Example 3.21 Oil is cooled in a cooler from 65°C to 54°C by circulating water through the cooler. The cooling load is 200 kW and water enters the cooler at 27°C. If the overall heat transfer coefficient, based on the outer surface area of the tube is 740 W/m^{2}K and the temperature rise of cooling water is 11°C, calculate the mass flow rate of water, the effectiveness and the heat transfer area required for a single pass In a parallel flow and in a counter flow arrangement.

Solution: Cooling load = 200 kW = mass of water × sp. heat × temp. rise Mass of water = 200/(4.2 × 11) = 4.329 kg/s



(i) Parallel flow:

From the temperature profile:

LMTD = (38 – 16)/ln(38/16) = 25.434 Q = U A (LMTD);

Area A = 200 × 10^3/(740 × 25.434) = 10.626 m^2

Effectiveness, $\in$ = (38 - 27)/(54 - 27) = 0.407.

(ii) Counter flow:

From the temperature profile:

LMTD = mean temperature difference = 27°C

Area A = 200 × 10^3/(740 × 27) = 10 m^2

Effectiveness, E = (38 - 27)/(65 - 27) = 0.289.

Example 3.22 Oil (mass flow rate 1.5 kg/s C_p = 2 kJ/kgK) is cooled in a single pass shell and tube heat exchanger from 65 to 42°C. Water (mass flow rate 1 kg/s, C_p = 4.2 kJ/kgK) has an inlet temperature of 28°C. If the overall heat transfer coefficient is 700 W/m^2K, calculate heat transfer area for a counter flow arrangement using $\in$NTU method.

Solution: Heat capacity rate of 011; 1.5 × 2.0 = 3 kW/K

Heat capacity rate of water = 1 × 4.2; 4.2 kW/K

$C_{min} = 3.0\ kW/K$ and $R = C_{min}/C_{max} = 3/4.2 = 0.714$

For a counter flow arrangement, $NTU = [1/(R-1)] \ln[(\in -1)/(\in R-1)]$

Effectiveness, $\in = (65-42)/(65-28) = 0.6216$

and NTU = 1.346 = AU/C_{min}; A = 1.346 × 3000 / 700 = 5.77 m^2

By making an energy balance, we can compute the water temperature at outlet.

or 3.0 × (65 - 42); 4.2 × (T - 28), T; 44.428

LMTD for a counter flow arrangement:

LMTD; (20.572 - 14)/ln (20.572/14) = 17.076

Area, A = $\dot{Q}/U \times (LMTD) = 3 \times 10^3 \times (65-42)/(700 \times 17.076) = 5.77\ m^2$

Example 3.23 A fluid (mass flow rate 1000 kg/min, sp. heat capacity 3.6 kJ/kgK) enters a heat exchanger at 700 C. Another fluid (mass flow rate 1200 kg/mm, sp. heal capacity 4.2 kJ/kgK) enters al 100 C. If the overall heat transfer coefficient is 420 W/m^2K and the surface area is 100m2, calculate the outlet temperatures of both fluids for both counter flow and parallel flow arrangements.

Solution: Heat capacity rate for the hot fluid

$$1000 \times 3.6 \times 10^3 \times 60 = 60 \times 10^3 W/K$$

Heat capacity rate for the cold fluid = 1200 × 4.2 × 10^3/60 = 84 × 10^3 W/K

R = C_{min}/C_{max} = 60/84; 0.714, NTU = U A/C_{min} = 420 × 100/60000 = 0.7

(i) For counter flow heat exchanger:

$$\in = [1-\exp\{-N(1-R)\}]/[1-\mathrm{Re}\,\mathrm{xp}\{-N(1-R)\}]$$

$$[1-\exp\{-0.7(1-0.714)\}]/1[1-0.714\ \exp\{-0.7(1-0.714)\}] = 0.4367$$

Since heat capacity rate of the hot fluid IS lower,

$$\in = \left(700 - T_{h_0}\right) / \left(700 - 100\right)$$

and $T_{h_0} = 700 - 0.4367 \times 600 = 438^o C$

By making an energy balance, $60 \times 10^3 \left(700 - 438\right) = 84 \times 10^3 \left(T_{c_0} - 100\right)$

or, $T_{c_0} = 60 \times 262 / 84 + 100 = 87.14^o$ C

(ii) For parallel flow heat exchanger

$$\in = \left[1 - \exp\left\{-N\left(1 + R\right)\right\}\right] / \left(1 + R\right) = \left[1 - \exp\left\{0.7\left(1 + 0.714\right)\right\}\right] / \left(1.714\right)$$

$\in = 0.4077$, a lower value

and $\left(T_{h_i} - T_{h_0}\right) / \left(T_{h_i} - T_{c_0}\right) = 0.4077 = \left(700 - T_{h_0}\right) / \left(700 - T_{c_0}\right)$

By making an energy balance: $60 \times 10^3 \times \left(700 - T_{h_0}\right) = 84 \times 10^3 \times \left(T_{c_0} - 100\right)$

or, $\left(700 - T_{c_0}\right) = \left(700 - T_{h_0}\right) / 0.4077$

and $84 \times \left(T_{c_0} - 100\right) / 60 = \left(1.4 T_{c_0} - 140\right)$

Therefore, $T_{c_0} = 237.5^o C$

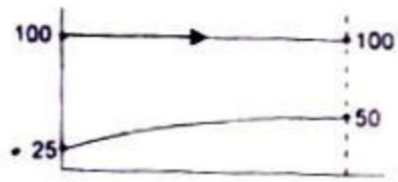
and $T_{h_0} = 511.4^o$ C

Example 3.24 Steam enters the surface condenser at 100°C and water enters at 25°C with a temperature rise of 25°C. Calculate the effectiveness and the NTU for the condenser. If the water temperature at inlet changes to 35 C, estimate the temperature rise for water.

Solution: Effectiveness, $\in$ 25/(100 25) = 0.33

For R = 0, $\in$ 1 - exp(- N)

or, N = -ln(1 - $\in$) = 0.405



Since other parameters remain the same,

$$25/(100 - 25) = \Delta T/(100 - 35)$$

and $\Delta T = 21.66$; or, $T_{c_0} = 35 + 21.66 = 56.66°C$.

3.11. Increasing the Heat Transfer Coefficient

For a heat exchanger, the heat load is equal to Q = UA (LMTD). The effectiveness of the heat exchanger can be increased either by increasing the surface area for heat transfer or by increasing the heat transfer coefficient. Effectiveness versus NTU(AU/C_{min}) curves, Fig. 10.10 - 15, reveal that by increasing the surface area beyond a certain limit (the knee of the curves), there is no appreciable improvement in the performance of the exchangers. Therefore, different methods have been employed to increase the heat transfer coefficient by increasing turbulence, improved mixing, flow swirl or by the use of extended surfaces. The heat transfer enhancement techniques is gaining industrial importance because it is possible to reduce the heat transfer surface area required for a given application and that leads to a reduction in the size of the exchanger and its cost, to increase the heating load on the exchanger and to reduce temperature differences.

The 'different techniques used for increasing the overall conductance U are: (a) Extended Surfaces - these are probably the most common heat transfer enhancement methods. The analysis of extended surfaces has been discussed in Chapter 2. Compact heat exchangers use extended surfaces to give the required heat transfer surface area in a small volume. Extended surfaces are very effective when applied in gas side heat transfer. Extended surfaces find their application in single phase natural and forced convection pool boiling and condensation.

(b) Rough Surfaces - the inner surfaces of a smooth tube is artificially roughened to promote early transition to turbulent flow or to promote mixing between bulk flow and the various sub-layer in fully developed turbulent flow. This method is primarily used in single phase forced convection and condensation.

(c) Swirl Flow Devices - twisted strips are inserted into the flow channel to impart a rotational motion about an axis parallel to the direction of bulk flow. The heat transfer coefficient increases due to increased flow velocity, secondary flows generated by swirl, or increased flow path length in the flow channel. This technique is used in flow boiling and single phase forced flow.

(d) Treated Surfaces - these are used mainly in pool boiling and condensation.

Treated surfaces promote nucleate boiling by providing bubble nucleation sites. The rate of condensation increases by promoting the formation of droplets, instead of a liquid film on the condensing surface. This can be accomplished by coating the surface with a material that makes the surface non-wetting.

All of these techniques lead to an increase in pumping work (increased frictional losses) and any practical application requires the economic benefit of increased overall conductance. That is, a complete analysis should be made to determine the increased first cost because of these techniques, increased heat exchanger heat transfer performance, the effect on operating costs (especially a substantial increase in pumping power) and maintenance costs.

3.12. Fin Efficiency and Fin Effectiveness

Fins or extended surfaces increase the heat transfer area and consequently, the amount of heat transfer is increased. The temperature at the root or base of the fin is the highest and the temperature along the length of the fm goes on decreasing Thus, the fin would dissipate the maximum amount of heat energy if the temperature all along the length remains equal to the temperature at the root. Thus, the fin efficiency is defined as:

η_{fin} = (actual heat transferred) / (heat which would be transferred if the entire fin area were at the root temperature)

In some cases, the performance of the extended surfaces is evaluated by comparing the heat transferred with the fin to the heat transferred without the fin. This ratio is called 'fin effectiveness' E and it should be greater than 1, if the rate of heat transfer has to be increased with the use of fins.

For a very long fin, effectiveness $E = \dot{Q}_{with\ fin} / \dot{Q}_{without\ fin}$

$= (hpkA)^{1/2}\theta_0 / hA\ \theta_0 = (kp/hA)^{1/2}$

And $\eta_{fin} = (hpkA)^{1/2}\ \theta_0\ (hpL\ \theta_0) = (hpkA)^{1/2} / (hpL)$

$$\frac{E}{\eta_{fin}} = \frac{(kp/hA)^{1/2}}{(hpkA)^{1/2}} \times hpL = \frac{pL}{A} = \frac{\text{Surface area of fin}}{\text{Cross-sectional area of the fin}}$$

i.e., effectiveness increases by increasing the length of the fin but it will decrease the fin efficiency.

Expressions for Fin Efficiency for Fins of Uniform Cross-section

1. Very long fins: $(hpkA)^{1/2}(T_0 - T_\infty)/\left[hpL(T_0 - T_\infty)\right] = 1/mL$

2 For fins having insulated tips:

$$\frac{(hpkA)^{1/2}(T_0 - T_\infty)\tan h(mL)}{hpL(T_0 - T_\infty)} = \frac{\tan h\,(mL)}{mL}$$

Example 3.25 The total efficiency for a finned surface may be defined as the ratio of the total heat transfer of the combined area of the surface and fins to the heat which would be transferred if this total area were maintained at the root temperature T_0 . Show that this efficiency can be calculated from

$\eta_t = 1 - A_f / A(1 - \eta_t)$ where η = total effltiency, A_f = surface area of all fins, A = total heat transfer area, η = fin efficiency

Solution: Fin efficiency,

$$\eta_f = \frac{\text{Actual heat transfered}}{\text{Heat that would be transferred if the entire fin were at the root temperature}}$$

$$\text{or, } \eta_f = \frac{\text{Actual heat transfer}}{hA_f(T_0 - T_\infty)}$$

$\therefore$ Actual heat transfer from finned surface = $\eta_f hA_f(T_0 - T_\infty)$

Actual heat transfer from un finned surface which are at the root temperature: $h(A-A_f)$

$(T_0 - T_\infty)$

Actual total heat transfer = $h(A - A_f)(T_0 - T_\infty) + \eta_f h A_f (T_0 - T_\infty)$

By the definition of total efficiency,

$$\eta_t = \left[h\left(A - A_f\right)\left(T_0 - T_\infty\right) + \eta_f h A_f \left(T_0 - T_\infty\right)\right] / \left[hA\left(T_0 - T_\infty\right)\right]$$

$$= \frac{\left(A - A_f\right) + \eta_f h A_f}{A} = 1 - A_f / A + \eta_f A_f / A$$

$$= 1 - \left(A_f / A\right) + \left(1 - \eta_f\right).$$

3.13. Extended Surfaces do not always Increase the Heat Transfer Rate

The installation of fins on a heat transferring surface increases the heat transfer area but it is not necessary that the rate of heat transfer would increase. For long fins, the rate of heat loss from the fin is given by $(hpkA)^{1/2}\theta_0 = kA(hp/kA)^{1/2}\theta_0 = kAm\,\theta_0$. When h/ mk = 1, $Q = hA\,\theta_0$ which is equal to the heat loss from the primary surface with no extended surface. Thus, when h = mk, an extended surface will not increase the heat transfer rate from the primary surface whatever be the length of the extended surface.

For h/mk > 1, $Q < hA\,\theta_0$ and hence adding a secondary surface reduces the heat transfer, and the added surface will act as an insulation. For h/mk < 1, $Q > hA\,\theta_0$, and the extended surface will increase the heat transfer, Fig. 2.31. Further, $h/mk = (h^2.kA/k^2hp)^{1/2} = (hA/kP)^{1/2}$, i.e. when h/mk < 1, the heat transfer would be more effective when h/k is low for a given geometry.

3.14. An Expression for Temperature Distribution for an Annular Fin of Uniform Thickness

In order to increase the rate of heat transfer from cylinders of air-cooled engines and in certain type of heat exchangers, annular fins of uniform cross-section are employed. Fig. 2.32 shows such a fin with its nomenclature.

In the analysis of such fins, it is assumed that:

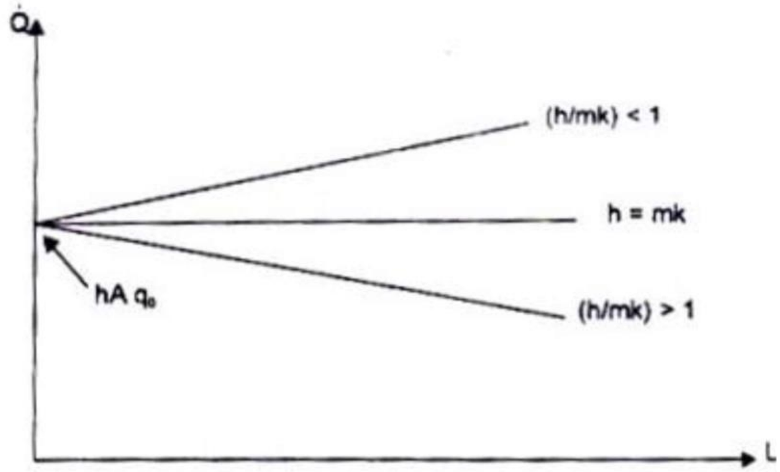
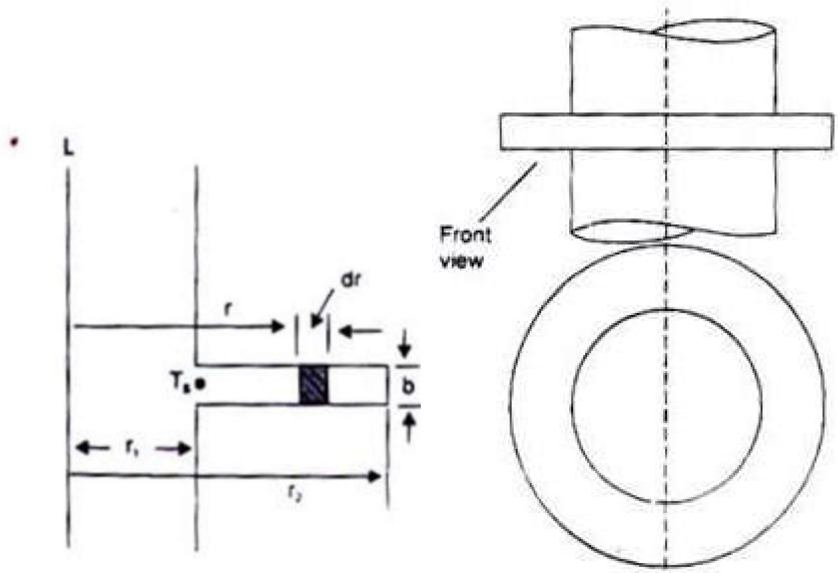


Fig 3.18

(For increasing the heat transfer rate by fins, we should have (i) higher value of thermal conductivity, (ii) a lower value of h, fins are therefore generally placed on the gas side, (iii) perimeter/cross-sectional area should be high and this requires thin fins.)

(i) the thickness b is much smaller than the radial length (r_2 - r_1) so that one-dimensional radial conduction of heat is valid;

(ii) steady state condition prevails.



Annular fin of uniform thickness Top view of annular fin

Fig 3.19

We choose an annular element of radius r and radial thickness dr. The cross-sectional area for radial heat conduction at radius r is $2\pi rb$ and at radius r + dr is $2\pi(r + dr)b$. The surface area for convective heat transfer for the annulus is $2(2\pi r.dr)$. Thus, by making an energy balance,

$$-k2\pi rb\frac{dT}{dr} = -k2\pi(r+dr)b\left(\frac{dT}{dr}+\frac{d^2T}{dr^2}dr\right) + h\times 4\pi r.dr(T-T_\infty)$$

or, $d^2T / dr^2 + (1/r)dT/dr - 2h/kb(T - T_\infty) = 0$

Let, $\theta = (T - T_\infty)$ the above equation reduces to

$d^2\theta / dr^2 + (l/r)\, d\theta/dr - (2h\ kb)\ \theta = 0$

The equation is recognised as Bessel's equation of zero order and the solution is $\theta = C_1 I_0(nr) + C_2 K_0(nr)$, where $n=(2h/kb)^{1/2}$, I_0 is the modified Bessel function, 1st kind and K_0 is the modified Bessel function, 2nd kind, zero order, The constants C_1 and C_2 are evaluated by applying the two boundary conditions:

at $r = r_l$, $T = T_s$ and $\theta = T_s - T_\infty$

at $r = r_2$, $dT/dr = 0$ because $b << (r_2 - r_l)$

By applying the boundary conditions, the temperature distribution is given by

$$\frac{\theta}{\theta_0} = \frac{I_0(nr)K_1(nr_2) + K_0(nr)I_1(nr_2)}{I_0(nr_1)K_1(nr_2) + K_0(nr_1)I_1(nr_2)} \qquad (3.16)$$

$I_1(nr)$ and $K_1(nr)$ are Bessel functions or order one.

And the rate of heat transfer is given by:

$$Q = 2\pi knb\ \theta_0 r_1 \frac{K_1(nr_1)I_1(nr_2) - I_1(nr_1)K_1(nr_2)}{K_0(nr_1)I_1(nr_2) + I_0(nr_1)K_1(nr_2)} \qquad (3.17)$$

Table 2.1 gives selected values of the Modified Bessel Functions of the First and Second kinds, order Zero and One. (The details of solution can be obtained from: C.R. Wylie, Jr: Advanced Engineering Mathematics, McGraw-Hill Book Company, New York.)

The efficiency of circumferential fins is also obtained from curves for efficiencies

$(\text{along Y-axis}) \,\square\, \left(r_2 + \frac{b}{2} - r_1\right)^{\frac{3}{2}} \left(\frac{2h}{Kb}(r_2 - r_1)\right)^{\frac{1}{2}}$ for different values of $\left(r_2 + \frac{b}{2}\right)/r_1$.