Bayes Theorem

State and Prove Bayes Theorem

(OR)

State and Prove Theorem of Probability of Causes.

Statement

If $B_1, B_2, \ldots$ B_n be a set of exhaustive and mutually exclusive events associated with random experiment and A is another event associated with (or caused)by Bi. Then

$$P(A \mid B_i) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) P(A/B_i)}$$

Proof:

Given $B_1, B_2, \ldots, B_n$ are mutually exclusive events

$A \cap B_1, A \cap B_2, \ldots, A \cap B_n$ are mutually exclusive events

Let $A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n)$

By addition theorem,

$\Rightarrow P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

$\Rightarrow P(A) = \sum_{i=1}^{n} P(A \cap B_i)$

$$\Rightarrow P(A \cap B_i) = P(B) \cdot P\left({}^{A}\!/_{B}\right)$$

$$\Rightarrow P(B_i \mid A) = \frac{P(B_i) \cdot P(A/B_i)}{P(A)} \dots \dots \dots (1)$$

Substitute $P[A]$ in eqn (1)

$$(1) \Rightarrow P[B_i/A] = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) P(A/B_i)}$$

Hence the proof.

1.(a) Four boxes A, B, C, D contain fuses. The boxes contain 5000, 3000, 2000 and 1000 fuses respectively. The percentages of fuses in boxes which are defective are 3%, 2%, 1% and 0.5% respectively.one fuse in selected at random arbitrarily from one of the boxes. It is found to be defective fuse. Find the probability that it has come from box D.

(OR)

(b) Four boxes A, B, C, D contain fuses. Box A contain 5000 fuses, box B contain 3000 fuses, box C contain 2000 fuses and box D contain 1000 fuses. The percentage of fuses in boxes which are defective are 3%, 2%, 1% and 0.5% respectively. One fuse is select at random from one of the boxes. It is found to be defective fuse. What is the probability that it has come from box *D*.

Solution:

Since selection ratio is not given

Assume selection ratio is $1 : 1 : 1 : 1$

Total = $1+1+1+1 \;=\; 4$

$\Rightarrow P(A) = \frac{1}{4}$

$\Rightarrow P(B) = \frac{1}{4}$

$\Rightarrow P(C) = \frac{1}{4}$

$\Rightarrow P(D) = 1/4$

Let E be the event selecting a defective fuse from any one of the machine

$\Rightarrow P(E/A) = 3\% = 0.03$

$\Rightarrow P(E/B) = 2\% = 0.02$

$\Rightarrow P(E/C) = 1\% = 0.01$

$\Rightarrow P(E/D) = 5\% = 0.05$

$P(E) \;= P(A)P(E/A) + P(B)P(E/B) + P(C)P(F/\mathrm{C}) + P(D)(F/\mathrm{D})$

$= \frac{1}{4} \times 0.03 + 1/4 \times 0.02 + 1/4 \times 0.01 + 1/4 \times 0.05$

$= 0.0275$

$$P(D/E) = \frac{P(D)P(E/D)}{P(E)}$$

$$= \frac{\frac{1}{4}\times 0.05}{0.0275} = 0.4545$$

$$= 0.4545$$

2. (a) In a bolt Factory, Machines A, B and C manufacture respectively 25%, 35% and 40% of total output. also out of these output of A, B, C are 5, 4, 2 percent respectively are defective. A bolt is drawn at random from the total output and it is found to be defective. What is the probability that it was manufactured by the machine B?

(OR)

(b) In a company machine A, B and C manufactured bolts, 25%, 35% and 40% of total output. also out of these output of A, B, C are 5,4,2 percent respectively are defective. A bolt is taken random from the total output and it is found to be defective. Find the probability that it was manufactured by the machine B?

Solution:

Given ,P(E_1) =P(A) =25% = 0.25

$\Rightarrow \ P(E_2) = P(B) = 35\% = 0.35$

$\Rightarrow P(E_3) = P(C) = 40\% = 0.40$

Let D be the event of drawing defective bolt

$\Rightarrow P(D/E_1) = 5\% = \frac{5}{100} = 0.05$

$\Rightarrow P(D/E_2) = 4\% = 0.04$

$\Rightarrow P(D/E_3) = 2\%) = 0 \cdot 02$

To find $P(E_2/D)$

By Bayes theorem

$$P(E_2/D) = \frac{P(E_2)P(D/E_2)}{P(E_1)\, P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)}$$

$$= \frac{(0.35)(0.04)}{(0.25)(0.05) + (0.35)(0.04) + (0.4)(0.02)}$$

$$= \frac{0.014}{0.0345}$$

$$= 0.406$$

3. (a) A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the Probability that it was drawn from bag B

(OR)

(b) A box A contains 2 white and 3 red balls and a box B contains 4 white and 5 red balls at random one ball is taking and is found to be red. What is the probability that it was drawn from bag B?

Solution:

Let B_1 be the event that the ball is drawn from the bag A.

Let B_2 be the event that the ball is drawn from the bag B.

Let A be the event that the drawn ball is red

$\Rightarrow P(B_1) = P(B_2) = \frac{1}{2}$

$\Rightarrow P(A/B_1) = \frac{3C_1}{5C_1} = \frac{3}{5}$

$\Rightarrow P(A/B_2) = \frac{5C_1}{9C_1} = \frac{5}{9}$

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{(\frac{1}{2})(\frac{5}{9})}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+(\frac{1}{2})(\frac{5}{9})}$$

$$= \frac{\frac{5}{18}}{\frac{52}{90}}$$

$\Rightarrow P(B_2/A) = \frac{25}{52}$