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DEPARTMENT OF MECHANICAL ENGINEERING

ME3491 THEORY OF MACHINES

COURSE MATERIAL

UNIT II GEARS AND TRAINS

2.1Introduction

A **gear** is a rotating machine part having cut *teeth*, or *cogs*, which *mesh* with another toothed part in order to transmit torque.
The gears in a transmission are analogous to the wheels in a pulley. An advantage of gears is that the teeth of a gear prevent slipping.
When two gears of unequal number of teeth are combined a mechanical advantage is produced, with both the rotational speeds and the torques of the two gears differing in a simple relationship.
In transmissions which offer multiple gear ratios, such as bicycles and cars, the term **gear**, as in *first gear,* refers to a gear ratio rather than an actual physical gear.

2.1.1Fundamental Law of Gear-Tooth

Pitch point divides the line between the line of centres and its position decides the velocity ratio of the two teeth. The above expression is the **fundamental law of gear-tooth action**.

Formation of teeth:

Involute teeth
Cycloidal teeth

Involute curve:

The curve most commonly use d for gear-tooth profiles is the involute of a circle. This **involute curve** is the path traced by a point on a line as the line Rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string, which is originally wrapped on a circle when the string is un wrapped from the circle. The circle from which the involute is derived is called the **base circle**

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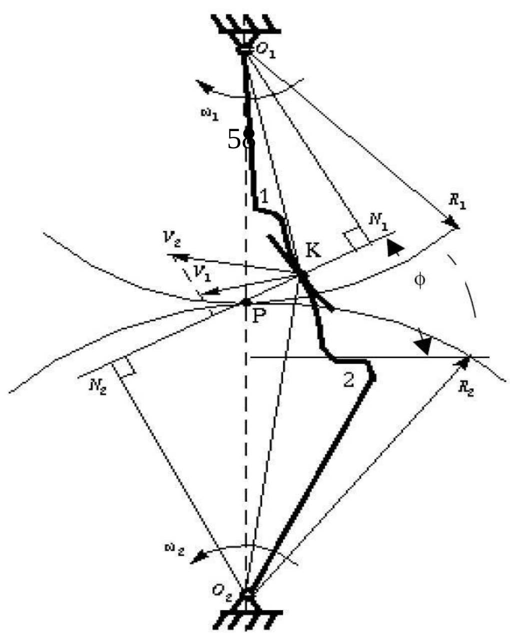
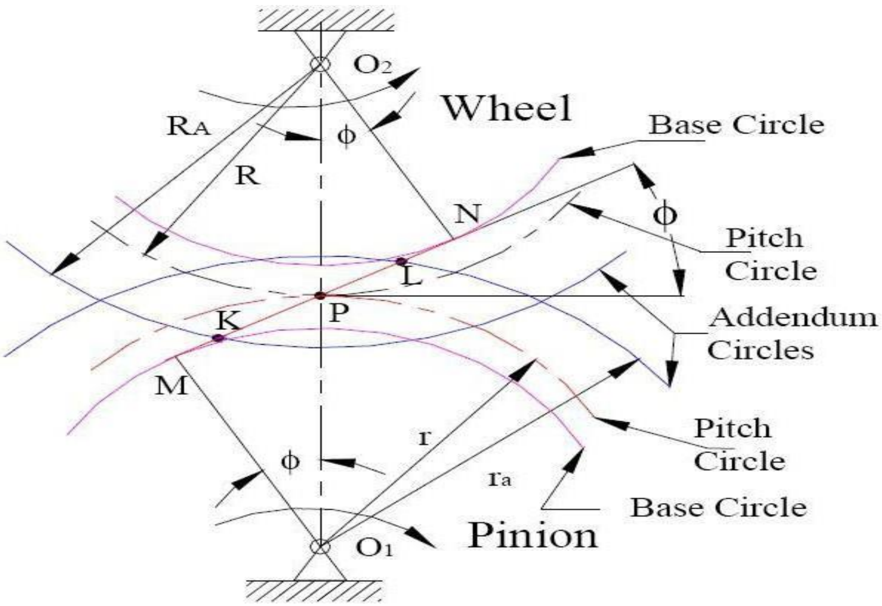


Path of contact:



- Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begin sat *K* (on the near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).
- *MN* is the common normal at the point of contacts and the common tangent to the base circles. The point *K* is the intersection of the addendum circle of wheel and the common tangent. The point *L* is the intersection of the addendum circle of pinion and common tangent.
- The length of path of contact is the length of common normal cut-off by the addendum circles of the wheel and the pinion.
- Thus, the length of part of contact is *KL* which is the sum of the parts of path of Contacts *KP* and *PL*. Contact length *KP* is called as **path of approach** and contact length *PL* is called as **path of recess**.

Path of approach: *KP*

$$\boxed{\begin{aligned} KP &= KN - PN \\ &= \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \end{aligned}}$$

Path of recess: *PL*

$$\begin{aligned} PL &= ML - MP \\ &= \sqrt{(r_a)^2 - r^2\cos^2\phi} - r\sin\phi \end{aligned}$$

Length of path of contact :

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$$\boxed{\begin{aligned} KL &= KP + PL \\ &= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R+r)\sin\phi \end{aligned}}$$

Arc of contact: Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is *EPF* or *GPH*.

The arc *GP* is known as *arc of approach and* the arc *PH* is called *arc of recess*. The angles subtended by the SE arcs at O1arecalled *angle of approach* and *angle of recess* respectively.

$$\text{Length of arc of approach} = \text{arc } GP \boxed{= \frac{Lenght\ of\ path\ of\ approach}{\cos\phi} = \frac{KP}{\cos\phi}}$$

$$\text{Length of arc of recess} = \text{arc } PH \boxed{= \frac{Lenght\ of\ path\ of\ recess}{\cos\phi} = \frac{PL}{\cos\phi}}$$

$$\text{Length of arc contact} = \text{arc } GPH \boxed{= arc\ GP + arc\ PH}$$

$$\boxed{= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi} = \frac{Length\ of\ path\ of\ contact}{\cos\phi}}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

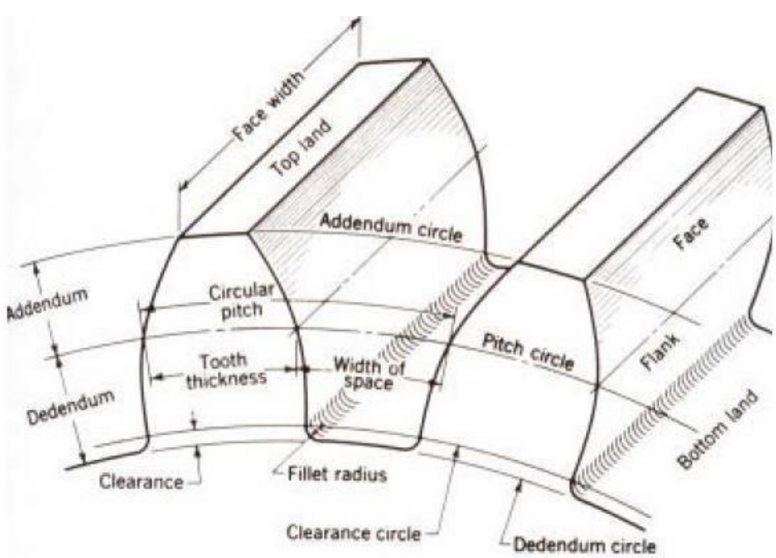
The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

$$\boxed{Contat\ ratio = \frac{Length\ of\ the\ arc\ of\ contact}{P_C}}$$

$\boxed{P_C = Circular\ \ pitch = \pi \times m}$ and m = Module.

2.2Spur Gear Terminology

1. Pitch circle. It is an imaginary circle which by pure rolling action, would give the same motion as an actual gear



2. *Pitch circle diameter*. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as ***pitch diameter***.

3. *Pitch point*. It is a common point of contact between two pitch circles.

4. *Pitch surface*. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity*. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and $20°$.

6. *Addendum*. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum*. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle*. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle*. It is the circle drawn through the bottom of the teeth. It is also called root circle.

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

10. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively, then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

11. *Diametral pitch*. It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d. Mathematically,

Diametral pitch, $$p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because\ p_c = \frac{\pi D}{T}\right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. *Module*. It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m. Mathematically,

Module, $m = D/T$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

13. *Clearance*. It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle***.

14. *Total depth*. It is the radial distance between the addendum and the dedendum circles of a gear. It is equal to the sum of the addendum and dedendum.

15. *Working depth*. It is the radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. *Tooth thickness*. It is the width of the tooth measured along the pitch circle.

17. *Tooth space* . It is the width of space between the two adjacent teeth measured along the pitch circle.

18. *Backlash*. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

19. *Face of tooth*. It is the surface of the gear tooth above the pitch surface.

20. *Flank of tooth*. It is the surface of the gear tooth below the pitch surface.

21. *Top land*. It is the surface of the top of the tooth.

22. *Face width*. It is the width of the gear tooth measured parallel to its axis.

23. *Profile*. It is the curve formed by the face and flank of the tooth.

24. *Fillet radius*. It is the radius that connects the root circle to the profile of the tooth.

25. *Path of contact*. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement.

26. **Length of the path of contact*. It is the length of the common normal cut-off by the addendum circles of the wheel and pinion.

27. *** Arc of contact*. It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. The arc of contact consists of two parts, *i.e.*

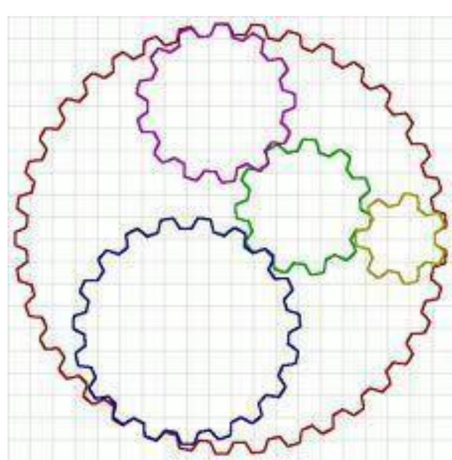
(*a*) *Arc of approach*. It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(*b*) *Arc of recess*. It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Note : The ratio of the length of arc of contact to the circular pitch is known as ***contact ratio*** *i.e.* number of pairs of teeth in contact.

Epicyclic gear trains:

- If the axis of the shafts over which the gears are mounted are moving relative to a fixed axis , the gear train is called the epicyclic gear train.
- Problems in epicyclic gear trains.



Differentials:

- Used in the rear axle of an automobile.
- To enable the rear wheels to revolve at different speeds when negotiating a curve.
- To enable the rear wheels to revolve at the same speeds when going straight.

12.20. Minimum Number of Teeth on the Pinion in Order to Avoid Interference

We have already discussed in the previous article that in order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency. The limiting condition reaches, when the addendum circles of pinion and wheel pass through points N and M (see Fig. 12.13) respectively.

Let t = Number of teeth on the pinion,,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion $= m.t/2$

G = Gear ratio $= T/t = R/r$

ϕ = Pressure angle or angle of obliquity.

From triangle O_1NP,

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2r.R \sin\phi \cos(90° + \phi)$$

$$\quad ...(\because PN = O_2P \sin\phi = R\sin\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r.R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$\therefore$ Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi} = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t} + 2\right]\sin^2\phi}$$

Let $A_p m$ = Addendum of the pinion, where A_p is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

We know that the addendum of the pinion

$$= O_1N - O_1P$$

$$\therefore \quad A_p.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{mt}{2} \qquad ...(\because O_1P = r = mt/2)$$

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

or

$$A_p = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore \quad t = \frac{2A_p}{\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1} = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

12.21. Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

and $A_W.m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should be multiplied.

Using the same notations as in Art. 12.20, we have from triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2 \phi - 2\, R.r \sin \phi \cos (90° + \phi) \qquad ...(\because PM = O_1P \sin \phi = r)$$

$$= R^2 + r^2 \sin^2 \phi + 2R.r \sin^2 \phi$$

$$= R^2 \left[1 + \frac{r^2 \sin^2 \phi}{R^2} + \frac{2r \sin^2 \phi}{R}\right] = R^2 \left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi\right]$$

$\therefore$ Limiting radius of wheel addendum circle,

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2 \phi} = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi}$$

We know that the addendum of the wheel

$$= O_2M - O_2P$$

$$\therefore \quad A_W m = \frac{m.T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - \frac{m.T}{2} \qquad ...(\because O_2P = R = m.T/2)$$

$$= \frac{m.T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

or

$$A_W = \frac{T}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1\right]$$

$$\therefore \quad T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2 \phi} - 1} = \frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1}$$

Notes : 1. From the above equation, we may also obtain the minimum number of teeth on pinion.

Multiplying both sides by $\frac{t}{T}$,

$$T \times \frac{t}{T} = \frac{2A_W \times \frac{t}{T}}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1}$$

$$t = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2 \phi} - 1\right]}$$

2. If wheel and pinion have equal teeth, then $G = 1$, and

$$T = \frac{2A_W}{\sqrt{1 + 3\sin^2 \phi} - 1}$$

Example 12.11. *A pair of involute spur gears with 16° pressure angle and pitch of module 6 mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. When the gear ratio is 1.75, find in order that the interference is just avoided ;* **1.** *the addenda on pinion and gear wheel ;* **2.** *the length of path of contact ; and* **3.** *the maximum velocity of sliding of teeth on either side of the pitch point.*

Solution. Given : $\phi = 16°$; $m = 6$ mm ; $t = 16$; $N_1 = 240$ r.p.m. or $\omega_1 = 2\pi \times 240/60 = 25.136$ rad/s ; $G = T/t = 1.75$ or $T = G.t = 1.75 \times 16 = 28$

1. ***Addenda on pinion and gear wheel***

We know that addendum on pinion

$$= \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1 + \frac{28}{16}\left(\frac{28}{16} + 2\right)\sin^2 16°} - 1\right]$$

$$= 48\ (1.224 - 1) = 10.76 \text{ mm } \textbf{Ans.}$$

and addendum on wheel

$$= \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 28}{2}\left[\sqrt{1 + \frac{16}{28}\left(\frac{16}{28} + 2\right)\sin^2 16°} - 1\right]$$

$$= 84\ (1.054 - 1) = 4.56 \text{ mm } \textbf{Ans.}$$

2. ***Length of path of contact***

We know that the pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 28/2 = 84 \text{ mm}$$

and pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 16/2 = 48 \text{ mm}$$

$\therefore$ Addendum circle radius of wheel,

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76 \text{ mm}$$

and addendum circle radius of pinion,

$$r_A = r + \text{Addendum of pinion} = 48 + 4.56 = 52.56 \text{ mm}$$

We know that the length of path of approach,

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(94.76)^2 - (84)^2\cos^2 16°} - 84\sin 16°$$

$$= 49.6 - 23.15 = 26.45 \text{ mm}$$

and the length of the path of recess,

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(52.56)^2 - (48)^2\cos^2 16°} - 48\sin 16°$$

$$= 25.17 - 13.23 = 11.94 \text{ mm}$$

$\therefore$ Length of the path of contact,

$$KL = KP + PL = 26.45 + 11.94 = 38.39 \text{ mm } \textbf{Ans.}$$

3. ***Maximum velocity of sliding of teeth on either side of pitch point***

Let ω_2 = Angular speed of gear wheel.

We know that $\frac{\omega_1}{\omega_2} = \frac{T}{t} = 1.75$ or $\omega_2 = \frac{\omega_1}{1.75} = \frac{25.136}{1.75} = 14.28$ rad/s

$\therefore$ Maximum velocity of sliding of teeth on the left side of pitch point *i.e.* at point K

$$= (\omega_1 + \omega_2)\ KP = (25.136 + 14.28)\ 26.45 = 1043 \text{ mm/s } \textbf{Ans.}$$

and maximum velocity of sliding of teeth on the right side of pitch point *i.e.* at point L

$$= (\omega_1 + \omega_2)\ PL = (25.136 + 14.28)\ 11.94 = 471 \text{ mm/s } \textbf{Ans.}$$

Example 12.13. *Two gear wheels mesh externally and are to give a velocity ratio of 3 to 1. The teeth are of involute form ; module = 6 mm, addendum = one module, pressure angle = 20°. The pinion rotates at 90 r.p.m. Determine : 1. The number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel, 2. The length of path and arc of contact, 3. The number of pairs of teeth in contact, and 4. The maximum velocity of sliding.*

Solution. Given : $G = T/t = 3$; $m = 6$ mm ; $A_P = A_W = 1$ module = 6 mm ; $\phi = 20°$; $N_1 = 90$ r.p.m. or $\omega_1 = 2\pi \times 90/60 = 9.43$ rad/s

1. Number of teeth on the pinion to avoid interference on it and the corresponding number of teeth on the wheel

We know that number of teeth on the pinion to avoid interference,

$$t = \frac{2A_P}{\sqrt{1 + G(G+2)\sin^2\phi} - 1} = \frac{2 \times 6}{\sqrt{1 + 3(3+2)\sin^2 20°} - 1}$$

$= 18.2$ say 19 **Ans.**

and corresponding number of teeth on the wheel,

$$T = G.t = 3 \times 19 = 57 \text{ Ans.}$$

2. Length of path and arc of contact

We know that pitch circle radius of pinion,

$$r = m.t/2 = 6 \times 19/2 = 57 \text{ mm}$$

$\therefore$ Radius of addendum circle of pinion,

$$r_A = r + \text{Addendum on pinion } (A_P) = 57 + 6 = 63 \text{ mm}$$

and pitch circle radius of wheel,

$$R = m.T/2 = 6 \times 57/2 = 171 \text{ mm}$$

$\therefore$ Radius of addendum circle of wheel,

$$R_A = R + \text{Addendum on wheel } (A_W) = 171 + 6 = 177 \text{ mm}$$

We know that the path of approach (*i.e.* path of contact when engagement occurs),

$$KP = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi \qquad \text{...(Refer Fig. 12.11)}$$

$$= \sqrt{(177)^2 - (171)^2\cos^2 20°} - 171\sin 20° = 74.2 - 58.5 = 15.7 \text{ mm}$$

and the path of recess (*i.e.* path of contact when disengagement occurs),

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$= \sqrt{(63)^2 - (57)^2\cos^2 20°} - 57\sin 20° = 33.17 - 19.5 = 13.67 \text{ mm}$$

$\therefore$ Length of path of contact,

$$KL = KP + PL = 15.7 + 13.67 = 29.37 \text{ mm Ans.}$$

We know that length of arc of contact

$$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{29.37}{\cos 20°} = 31.25 \text{ mm Ans.}$$

3. Number of pairs of teeth in contact

We know that circular pitch,

$$p_c = \pi \times m = \pi \times 6 = 18.852 \text{ mm}$$

$\therefore$ Number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{p_c} = \frac{31.25}{18.852} = 1.66 \text{ say } 2 \text{ Ans.}$$

4. Maximum velocity of sliding

Let ω_2 = Angular speed of wheel in rad/s.

We know that $\dfrac{\omega_1}{\omega_2} = \dfrac{T}{t}$ or $\omega_2 = \omega_1 \times \dfrac{t}{T} = 9.43 \times \dfrac{19}{57} = 3.14$ rad/s

$\therefore$ Maximum velocity of sliding,

$$v_S = (\omega_1 + \omega_2)\, KP \qquad \text{...}(\because KP > PL)$$

$$= (9.43 + 3.14)\, 15.7 = 197.35 \text{ mm/s Ans.}$$