3.5 NUMBER SYSTEMS

Manipulation of numbers is one of the early skills that the present day child is trained to acquire. The present day technology and the way of life require the usage of several number systems. Usage of decimal numbers starts very early in one's life. Therefore, when one is confronted with number systems other than decimal, some time during the high-school years, it calls for a fundamental change in one's framework of thinking. There have been two types of numbering systems in use through out the world.

One type is symbolic in nature. Most important example of this symbolic numbering system is the one based on Roman numerals

I=1,V=5,X=10, L=50,C=100,D=500andM=1000IIMVII-2007

Roman number system is still used in some places like watches and release dates of movies.

The weighted-positional system based on the use of radix 10 is the most commonly used numbering system in most of the transactions and activities of today's world. However, the advent of computers and the convenience of using devices that have two well defined states brought the binary system, using the radix 2, into extensive use. The use of binary number system in the field of computersandelectronics alsoleadtotheuseofoctal(basedonradix 8)and hex-decimal system (based on radix 16). The usage of binary numbers at various levels has become so essential that it is also necessary to have a good understanding of all the binary arithmetic operations. Here we explore the weighted-position number systems and conversion from one system to the other.

Weighted-PositionNumberSystem

In a weighted-position numbering system using Indian numerals the value associated with a digit is dependent on its position. The value of a number is weighted sum of its digits. Consider the decimal number 2357.It can beexpressed as

2357=2x103+3x102+ 5x101 +7x100

Each weight is a power of 10 corresponding to the digit's position. A decimal point allows negative as well as positive powers of 10 to be used;

526.47=5x102+2x101+6x100+4x10-1+7x10-2

In a general positional number system, the radix may be any integer r > 2, and a digit position i has weight r i . The general form of a number in such asystem is

dp-1dp-2,....d1,d0.d-1d-2d-n

where there are p digits to the left of the point (called radix point) and n digits to the right of the point. The value of the number is the sum of each digit multiplied by the corresponding power of the radix.

Except for possible leading and trailing zeros, the representation of a number in positional system is unique (00256.230 is the same as 256.23). Obviously the values di's can take are limited by the radix value. For example a number like (356)5, where the suffix 5 represents the radix will be incorrect, as there can not be a digit like 5 or 6 in a weighted position number system withradix 5.

If the radix point is not shown in the number, then it is assumed to be located near the last right digit to its immediate right. The symbol used for the radix point is a point (.). However, a comma is used in some countries. Forexample 7,6 is used, instead of 7.6, to represent a number having seven as its integer component and six as its fractional. As much of the present day electronic hardware is dependent on devices that work reliably in two well defined states, a numbering system using 2 as its radix has become necessary and popular. Withthe radix value of 2, the binary number system will have only two numerals, namely 0 and 1. Consider the number

$(N)_2=(11100110)_2$.

It is an eight digit binary number. The binary digits are also known as bits. Consequently the above number would be referred to as an 8-bit number. Its decimal value is given by

$(N)_2$= 1x 27 +1x26+1x25+ 0x24+0x23+1x22+1x21 +0x20

=128+64+32+0+0+ 4+ 2+0

$=(230)_{10}$

Consider a binary fractional number

$(N)_2$ = 101.101.

Itsdecimalvalueisgiven by

$(N)_2$= 1x 22+0x 21+1x20+ 1x2-1+0x2-2 +1x2-3

=4+0+1+12+ 0+18

=5+0.5+ 0.125

$=(5.625)_{10}$

From here on we consider any number without its radix specifically mentioned, as a decimal number. With the radix value of 2, the binary number system requires very long strings of 1s and 0s to represent a given number. Some of the problems associated with handling large strings of binary digits may be eased by grouping them into three digits or four digits.

We can use the following groupings. Octal (radix 8 to group three binary digits)Hexadecimal(radix16togroupfourbinarydigits)Intheoctalnumber

systemthedigitswillhaveoneofthefollowingeightvalues0,1,2,3,4,5,6and
7. In the hexadecimal system we have one of the sixteen values 0 through 15. However, the decimal values from 10 to 15 will be represented by alphabet A (=10), B (=11), C (=12), D (=13), E (=14) and F (=15).

Conversion of a binary number to an octal number or a hexadecimal number is very simple, as it requires simple grouping of the binary digits into groups of three or four. Consider the binary number 11011011.

It may be converted into octal or hexadecimal numbers as

$(11011001)_2$ = (011) (011) (001)

$=(331)_8$

=(1101)(1001)

$=(D9)_{16}$

Note that adding a leading zero does not alter the value of the number.

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Similarly for grouping the digits in the fractional part of a binary number, trailing zeros may be added without changing the value of the number. Number System Conversions In general, conversion between numbers with different radicescannot be done by simple substitutions. Such conversions would involvearithmetic operations. Let us work out procedures for converting a number in any radix to radix 10, and viceversa.

Decimal value of the number is determined by converting each digit of the number to its radix-10 equivalent and expanding the formula using radix-10 arithmetic.Someexamplesare:

$(331)_8$

=3x82+ 3x81+1x80

=192+24 +1

$=(217)_{10}(D9)_{16}$

=13x 161+ 9x160

=208+9

$=(217)_{10}$

$(33.56)_8$

=3 x 81+ 3x80+5x8-1+ 6x 8-2

$=(27.69875)_{10}(E5.A)_{16}$

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=14x161+5x160+10 x16-1

$=(304.625)_{10}$

Consider the following examples.

	Quotient	Remainder
156÷2	78	0
78÷ 2	39	0
39÷ 2	19	1
19÷ 2	9	1
9÷ 2	4	1
4÷ 2	2	0
2÷ 2	1	0
1÷ 2	0	1

$(156)_{10}=(10011100)_2$

	Quotient	Remainder
678÷8	84	6
84÷ 8	10	4
10÷ 8	1	2

1÷ 8 0 1

$(678)_{10}=(1246)_8$

	Quotient	Remainder
678 ÷ 16	42	6
42÷ 16	2	A
2÷ 16	0	2

$(678)_{10}=(2A6)_{16}$

Representation of Negative Numbers In our traditional arithmetic we usethe "+" sign before a number to indicate it as a positive number and a "-" sign to indicate it as a negative number. We usually omit the sign before the number if itis positive. This method of representation of numbers is called "sign-magnitude" representation.But using"+" and"-"signson acomputeris notconvenient, and it becomes necessary to have some other convention to represent the signed numbers. We replace "+" sign with "0" and "-" with "1". These two symbolsalready exist in the binary system. Consider the following examples:

(+1100101)2(01100101)2

(+101.001)2(0101.001)2

(-10010)2(110010)2

(-110.101)2(1110.101)2

In the sign-magnitude representation of binary numbers the first digit is always treated separately. Therefore, in working with the signed binary numbersin sign-magnitude form the leading zeros should not be ignored. However, the leading zeros can be ignored after the sign bit is separated.

For example,

1000101.11=-101.11

While the sign-magnitude representation of signed numbers appears to be natural extension of the traditional arithmetic, the arithmetic operations with signed numbers in this form are not that very convenient, either for implementation on the computer or for hardware implementation. There are two other methods of representing signed numbers.