# *Chapter-8* Rotodynamic Pumps

## 8.0 INTRODUCTION

Liquids have to be moved from one location to another and one level to another in domestic, agricultural and industrial spheres. The liquid is more often water in the domestic and agriculture spheres. In industries chemicals, petroleum products and in some cases slurries have to be moved, by pumping. **Three types of pumps** are in use.

(1) **Rotodynamic pumps** which move the fluid by dynamic action of imparting momentum to the fluid using mechanical energy. (2) **Reciprocating pumps** which first trap the liquid in a cylinder by suction and then push the liquid against pressure. (3) **Rotary positive displacement pumps** which also trap the liquid in a volume and push the same out against pressure.

**Reciprocating pumps** are limited by the **low speed of operation** required and **small volumes it can handle.**

**Rotary positive displacement pumps** are limited **by lower pressures** of operation and **small volumes** these can handle. Gear, vane and lobe pumps are of these type. **Rotodynamic pumps** *i.e.* centrifugal and axial flow pumps can be operated at **high speeds** often directly coupled to electric motors. These can handle from **small volumes to very large volumes.** These pumps **can handle corrosive and viscous**, **fluids and even slurries**. The **overall efficiency is high** in the case of these pumps. Hence these are found to be the most popular pumps in use. Rotodynamic pumps can be of **radial flow, mixed flow and axial flow** types according to the flow direction. **Radial flow** or purely centrifugal pumps generally handle **lower volumes at higher pressures. Mixed flow** pumps handle comparatively **larger volumes** at **medium range of pressures. Axial flow pumps** can handle **very large volumes,** but the **pressure** against which these pumps operate is **limited**. The overall efficiency of the three types are nearly the same.

## 8.1 CENTRIFUGAL PUMPS

These are so called because energy is imparted to the fluid by centrifugal action of moving blades from the inner radius to the outer radius. The **main components** of centrifugal pumps are (1) **the impeller,** (2) **the casing** and (3) **the drive shaft with gland and packing.**

Additionally suction pipe with one way valve (foot valve) and delivery pipe with delivery valve completes the system.

The liquid enters the eye of the impeller axially due to the suction created by the impeller motion. The impeller blades guide the fluid and impart momentum to the fluid, which increases the total head (or pressure) of the fluid, causing the fluid to flow out. The fluid comes out at a high velocity which is not directly usable. The casing can be of simple volute type or a diffuser can be used as desired. The volute is a spiral casing of gradually increasing cross section. A part of the kinetic energy in the fluid is  converted to pressure in the casing.

Figure 15.1.1 shows a sectional view of the centrifugal pump.

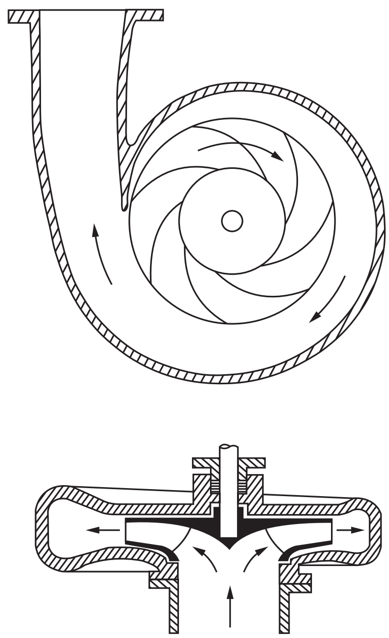


**Figure 8.1.1** Volute type centrifugal pump.

Gland and packing or so called stuffing box is used to reduce leakage along the drive shaft. By the use of the volute only a small fraction of the kinetic head can be recovered as useful static head. A diffuser can diffuse the flow more  efficiently and recover kinetic head as useful static head. A view of such arrangement is shown in figure 15.1.2. Diffuser pump are also called as turbine pumps as these resembles Francis turbine with flow direction reversed.

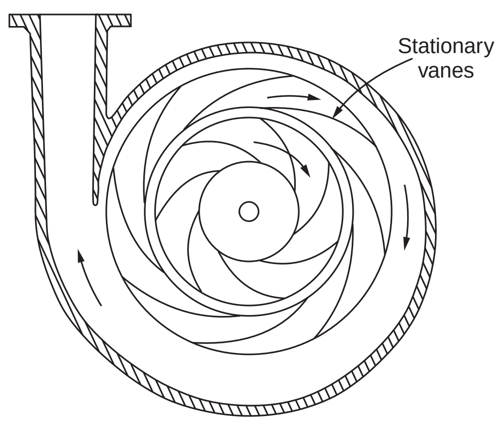



**Figure 8.1.2** Diffuser pump.

## 8.1.1 Impeller

The impeller consists of a disc with blades mounted perpendicularly on its surface. The blades may of three different orientations. These are ***(i)* Radial, *(ii)* Backward curved, and *(iii)* Forward curved.** Backward and forward refers to the direction of motion of the disc periphery. Of these the most popular one is the backward curved type, due to its desirable characteristics, which reference to the static head developed and power variation with flow rate. This will be discussed in detail later in this chapter.

A simple disc with blades mounted perpendicularly on it is called **open impeller.** If another disc is used to cover the blades, this type is called **shrouded impeller.** This is more popular with water pumps. Open impellers are well adopted for use with dirty or water containing solids. The **third type** is just the **blades spreading out from the shaft.** These are used to pump slurries. Impellers may be of cast iron or bronzes or steel or special alloys as required by the application. In order to maintain constant radial velocity, the width of the impeller will be wider at entrance and narrower at the exit. This may be also noted from figure 15.1.1.

The blades are generally cast integral with the disc. Recently even plastic material is used for the impeller. **To start delivery of the fluid the casing and impeller should be filled with the fluid without any air pockets. This is called priming.** If air is present the there will be only compression and no delivery of fluid. In order to release any air entrained an air valve is generally provided **The one way foot value keeps the suction line and the pump casing filled with water.**

## 8.1.2 Classification

As already mentioned, centrifugal pumps may be classified in several ways. On the **basis of speed** as low speed, medium speed and high speed pumps. On the basis of **direction of flow** of fluid, the classification is **radial flow**, **mixed flow** and **radial flow.** On the basis of **head** pumps may be classified as **low head** (10 m and below), **medium head** (10-50 m) **and high head** pumps. Single entry type and double entry type is another classification. Double entry pumps have blades on both sides of the impeller disc. This leads to reduction in axial thrust and increase in flow for the same speed and diameter. Figure 15.1.3 illustrates the same. When the head required is high and which cannot be developed by a single impeller, **multi staging** is used. In deep well submersible pumps the diameter is limited by the diameter of the bore well casing. In this case multi stage pump becomes a must. In multi stage pumps several impellers are mounted on the same shaft and the outlet flow of one impeller is led to the inlet of the next impeller and so on. The total head developed equals the sum of heads developed by all the stages.

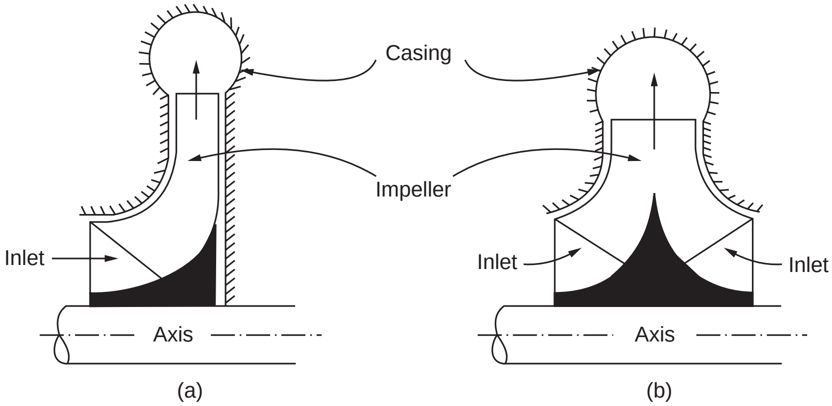



**Figure 8.1.3** Single and double entry pumps

Pumps may also be operated in parallel to obtain large volumes of flow. The characteristics under series and parallel operations is discussed later in the chapter. The classification may also be based on the **specific speed** of the pump. In chapter 9 the dimensionless parameters have been derived in the case of hydraulic machines. The same is also repeated in example 15.1. The expression for the dimensionless specific speed is given in equation 15.1.1.

$$N_s = \frac{N\sqrt{Q}}{(gH)^{3/4}} \tag{8.1.1}$$

More often dimensional specific speed is used in practise. In this case

$$N_s = \frac{N\sqrt{Q}}{H^{3/4}} \tag{8.1.1a}$$

The units used are : $N$ in rpm, $Q$ in $m^3/s$, and $H$ in meter.

Typical values are given in table 15.1

**Table 8.1 Specific speed classification of pumps.**

| **Flow direction** | **speed** | **Dimensional specific speed** | **Non Dimensional specific speed** |
|---|---|---|---|
| Radial | Low | 10 – 30 | 1.8 – 5.4 |
| | Medium | 30 – 50 | 5.4 – 9.0 |
| | High | 50 – 80 | 9.0 – 14.0 |
| Mixed flow | | 80 – 160 | 14 – 29 |
| Axial flow | | 100 – 450 | 18 – 81 |

The best efficiency is obtained for the various types of pumps in this range of specific speeds indicated.

## 8.2. PRESSURE DEVELOPED BY THE IMPELLER

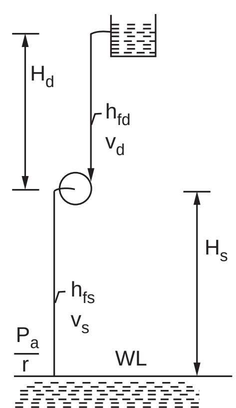
The general arrangement of a centrigugal pump system is shown in Figure 15.2.1.

$H_s$—Suction level above water level.

$H_d$—Delivery level above the impeller outlet.

$h_{fd}$, $h_{fs}$—frictionless $\dot{m}$, $m$.

$V_s$, $V_d$—pipe velocities.




Applying Bernoullis equation between the water level and pump suction,

$$\frac{P_a}{\gamma} + H_s + h_{fs} + \frac{V_s^{\,2}}{2g} = \frac{P_s}{\gamma} \tag{15.2.1}$$

$$\therefore \quad \frac{P_s}{\gamma} + \frac{P_a}{\gamma} H_s + h_{fs} + \frac{V_s^{\,2}}{2g} \tag{15.2.2}$$

Similarly applying Bernoulli's theorem between the pump delivery and the delivery at the tank,

$$\frac{P_d}{\gamma} + \frac{V_d^2}{2g} = \frac{P_a}{\gamma} + H_d + h_{fd} + \frac{V_d^2}{2g}$$

or $$\frac{P_d}{\gamma} = \frac{P_a}{\gamma} + H_d + h_{fd} \tag{8.2.3a}$$

where $P_d$ is the pressure at the pump delivery. From 15.2.2 and 15.2.3*a*

$$\frac{P_d}{\gamma} - \frac{P_s}{\gamma} = \frac{P_a}{\pi} + H_d + h_{fd} - \frac{P_a}{\gamma} + \frac{V_s^2}{2g} + H_s + h_{fs}$$

$$= H_d + H_s + h_f + \frac{V_s^2}{2g} = H_e + \frac{V_s^2}{2g} \tag{8.2.4}$$

where $H_e$ is the effective head.

### 8.2.1 Manometric Head

**The official code defines the head on the pump as the difference in total energy heads at the suction and delivery flanges. This head is defined as manometric head.**

The total energy at suction inlet (expressed as head of fluid)

$$\frac{P_s}{\gamma} + \frac{V_s^2}{2g} + Z_s$$

where $Z_s$ is the height of suction gauge from datum.

The total energy at the delivery of the pump

$$= \frac{P_d}{\gamma} + \frac{V_d^2}{2g} + Z_d$$

$Z_2$ is the height of delivery gauge from datum.

$\therefore$ The difference in total energy is defined as $H_m$

$$= \left(\frac{P_d}{\gamma} - \frac{P_s}{\gamma}\right) + \frac{V_d^2 - V_s^2}{2g} + (Z_d - Z_s)$$

From equation 15.2.4,

$$\frac{P_d}{\gamma} - \frac{P_s}{\gamma} = H_e + \frac{V_s^2}{2g}$$

Substituting $$H_m = H_e + \frac{V_d^2}{2g} + (Z_d - Z_s) \tag{8.2.5}$$

As $(Z_d - Z_s)$ is small and $\frac{V_d^2}{2g}$ is also small as the gauges are fixed as close as possible.

$\therefore$ **$H_m$ = Static head + all losses.**

## 8.3 ENERGY TRANSFER BY IMPELLER

The energy transfer is given by Euler Turbine equation applied to work absorbing machines,

$$W = -(u_1 V_{u1} - u_2 V_{u2}) = (u_2 V_{u2} - u_1 V_{u1})$$

This can be expressed as ideal head imparted as

$$H_{\text{ideal}} = \frac{u_2 V_{u2} - u_1 V_{u1}}{g} \tag{8.3.1}$$

The velocity diagrams at inlet and outlet of a backward curved vaned impeller is shown in figure 8.3.1. **The inlet whirl is generally zero.** There is no guide vanes at inlet to impart whirl. So the **inlet triangle is right angled.**

$$V_1 = V_{f1} \text{ and are radial}$$

$$\tan \beta_1 = \frac{V_1}{u_1} \quad \text{or} \quad \frac{V_f}{u_1}$$

$$V_{u1} = 0$$

$$\therefore \quad H_{\text{ideal}} = \frac{u_2\, V_{u2}}{g} \tag{15.3.2}$$

From the outlet triangle,

$$u_2 = \pi\, D_2\, N/60,$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan \beta_2}$$

$$\therefore \quad H_{\text{ideal}} = \frac{u_2}{g}\left[u_2 - \frac{V_{f2}}{\tan \beta_2}\right] \tag{15.3.3}$$

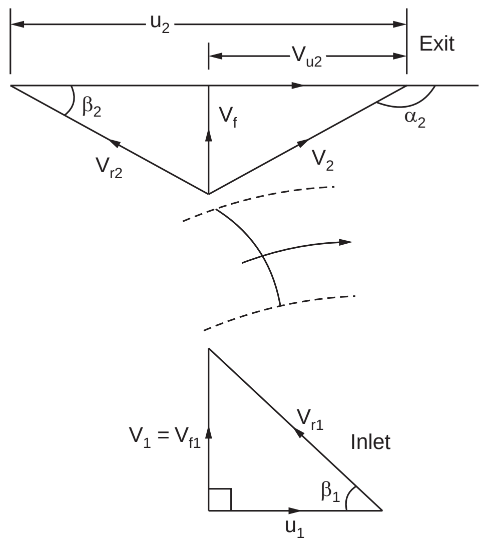



**Figure 8.3.1** Velocity triangles for backward curved bladed pump.

Manometric efficiency is defined as the ratio of manometric head and ideal head.

$$\eta_m = \frac{H_m \times g}{u_2(u_2 - V_{f2}/\tan \beta_2)}$$

$H_m$ = Static head + all losses (for practical purposes).

$$\text{Mechanical efficiency} \quad = \eta_{\text{mech}} = \frac{\text{Energy transferred to the fluid}}{\text{Work input}}$$

$$= \frac{(u_2\, V_{u2})\, Q\rho}{\text{power input}}$$

$$\text{Overall efficiency} \quad = \eta_o = \frac{\text{Static head } \times Q \times \rho \times g}{\text{Power input}} \tag{8.3.5}$$

There are always some leakage of fluid after being imparted energy by the impeller.

$$\text{Volumetric efficiency} \quad = \frac{\text{Volume delivered}}{\text{Volume passing through impeller}}$$

Thus $\quad \boldsymbol{\eta_o = \eta_m\, m_{mech} \cdot \eta_{vol}}$

$\dfrac{V_d^{\,2}}{2g}$ is not really useful as output of the pump. Hence the useful amount of energy transfer (as head, is taken as ($H_a$)

$$H_a = \frac{u_2\, V_{u2}}{g} - \frac{V_d^{\,2}}{2g}$$

By algebraic manipulation, this can be obtained as

$$H_a = (u_2^2 - V_f^2 \operatorname{cosec}^2 \beta_2)/2g \tag{8.3.7}$$

## 8.3.1 Slip and Slip Factor

In the analysis it is assumed that all the fluid between two blade passages have the same velocity (both magnitude of direction). Actually at the leading edge the pressure is higher and velocity is lower. On the trailing edge the pressure is lower and the velocity is higher. This leads to a circulation over the blades. Causing a non uniform velocity distribution. **The average angle at which the fluid leaves the blade is less than the blades angle.** The result is a reduction in the exit whirl velocity $V_{u2}$. This is illustrated in the following figure. The solid lines represent the velocity diagram without slip. The angle $\beta_2$ is the blade angle. The dotted lines represent the velocity diagram after slip. The angle $\beta_2' < \beta_2$. It may be seen that $V_{u2}' < V_{u2}$. The ratio $V_{u2}'/V_{u2}$ is known as slip factor. The result of the slip is that the energy transfer to the fluid is less than the theoretical value.

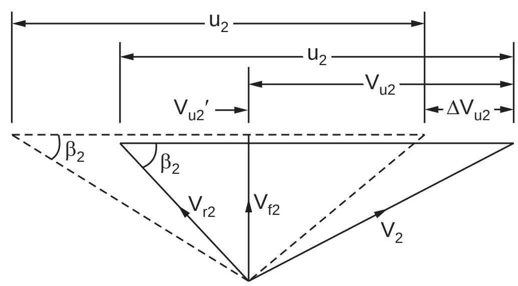


**Figure 8.3.2** Velocity triangle with slip

$$H_{th} = \sigma_s \cdot \frac{u_2 V_{u2}}{g}$$

where $\sigma_s$ is the slip coefficient or slip factor.

## 8.3.3 Losses in Centrifugal Pumps

Mainly there are three specific losses which can be separately calculated. These are

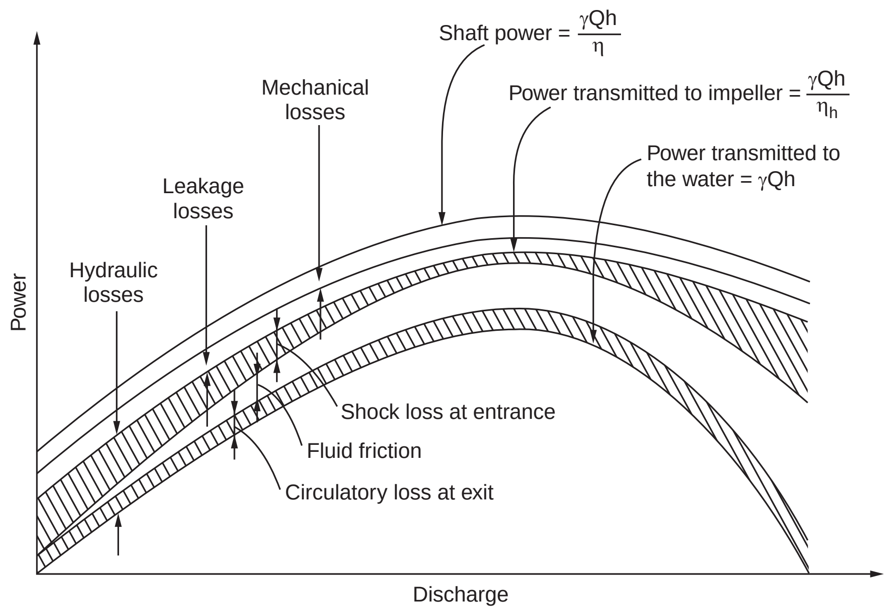


**Figure 8.3.3** Losses in pump

(*i*) **Mechanical friction** losses between the fixed and rotating parts in the bearings and gland and packing.

(*ii*) **Disc friction** loss between the impeller surfaces and the fluid.

(*iii*) **Leakage and recirculation** losses. The recirculation is along the clearance between the impeller and the casing due to the pressure difference between the hub and tip of the impeller. The various losses are indicated in figure 8.3.3.

## 8.3.4 Effect of Outlet Blade Angle

There are three possible orientation of the blade at the outlet. These are : forward curved, radial and Backward curved arrangements. The velocity triangles for the three arrangements are shown in Figure 8.3.4. In the case of forward curved blading $V_{u2} > u_2$ and $V_2$ is larger comparatively. In the case of radial blades $V_{u2} = u_2$. In the case of backward curved blading, $V_{u2} < u_2$.

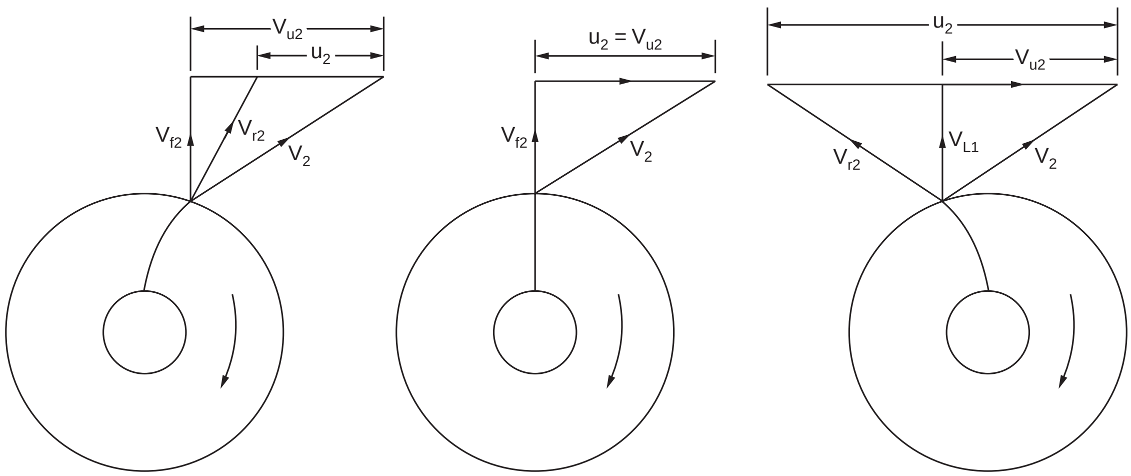



**Figure 8.3.4** Different blade arrangements

The head-flow rate curves are shown in Figure 15.3.5. The theoretical head variation can be expressed as

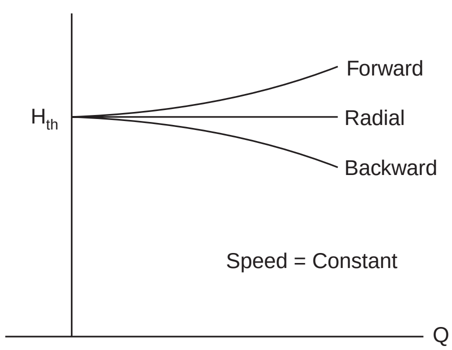



**Figure 8.3.5** Head variation

$$\mathbf{H_{th} = k_1 - k_2 \,.\, cot\, \beta_2\, Q}$$

where $k_1$ and $k_2$ are constants and $\beta_2$ is the outlet blade angle. cot $\beta_2$ becomes negative for forward curved blading. So head increases with flow rate. For radial blading cost $\beta_2 = 0$, and hence the head is constant with flow rate. In the case of backward curved blading, the head decreases with flow rate.

**The rising characteristics of the forward curved blading leads to increase of power input with increase of Q. The power curve is not self limiting and damage to motor is possible. The forward curved blading is rarely used.**

The backward curved blading leads to self limiting power characteristics and reduced losses in the exit kinetic energy.

So the backward curved blading is almost universally used. The radial blading also leads to rising power characteristics and it is used only in small sizes.

## 8.4 PUMP CHARACTERISTICS

We have seen that the theoretical head

$$H_{th} = \frac{u_2\, V_{u2}}{g} \quad \text{and} \quad V_{u2} = V_{f2} \cot \beta_2$$

$$V_{f2} = \frac{Q}{A}, \text{ where } A \text{ is the circumferential area.}$$

$$u_2 = \pi\, DN.$$

Substituting these relations in the general equation. We can write

$$H_{th} = \pi^2 D^2 N^2 - \left(\frac{\pi DN}{A} \,.\, \cot \beta_2\right) Q.$$

For a given pump, $D$, $A$, $\beta_2$ and $N$ are fixed . So at constant speed we can write

$$\mathbf{H_{th} = k_1 - k_2\, Q} \qquad (8.4.1)$$

where $k_1$ and $k_2$ are constants and

$$k_1 = \pi^2 D^2 N^2 \text{ and } k_2 = \left(\frac{\pi DN}{A} \,.\, \cot \beta_2\right)$$

Hence at constant speed this leads to a drooping linear characteristics for backward curved blading. This is shown by curve 1 in Figure 8.4.1.

The slip causes drop in the head, which can be written as $\sigma\, V_{u2}\, u_2/g$. As flow increases this loss also increases. Curve 2 shown the head after slip. The flow will enter without shock only at the design flow rate. At other flow rates, the water will enter with shock causing losses. This lose can be expressed as

$$\mathbf{h_{shock} = k_3\, (Q_{th} - Q)^2}$$

The reduced head after shock losses is shown in curve 5. The shock losses with flow rate is shown by curve 3. The mechanical losses can be represented by $\mathbf{h_f = k_4\, Q^2}$. The variation is shown by curve 4. With variation of speed the head characteristic is shifted near paralley with the curve 5 shown in Figure 8.4.1.

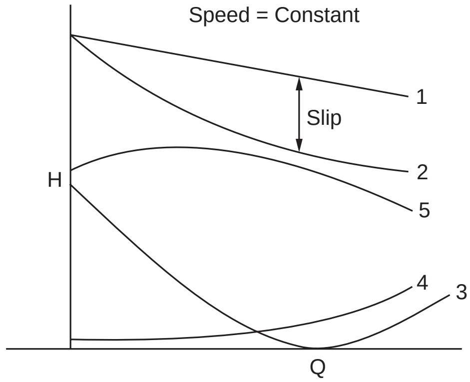



**Figure 8.4.1** Characteristics of a centrifugal pump

The characteristic of a centrigugal pump at constant speed is shown in Figure 8.4.2. It may be noted that the power increases and decreases after the rated capacity. In this way the pump is self limiting in power and the choice of the motor is made easy. The distance between the brake power and water power curves gives the losses.

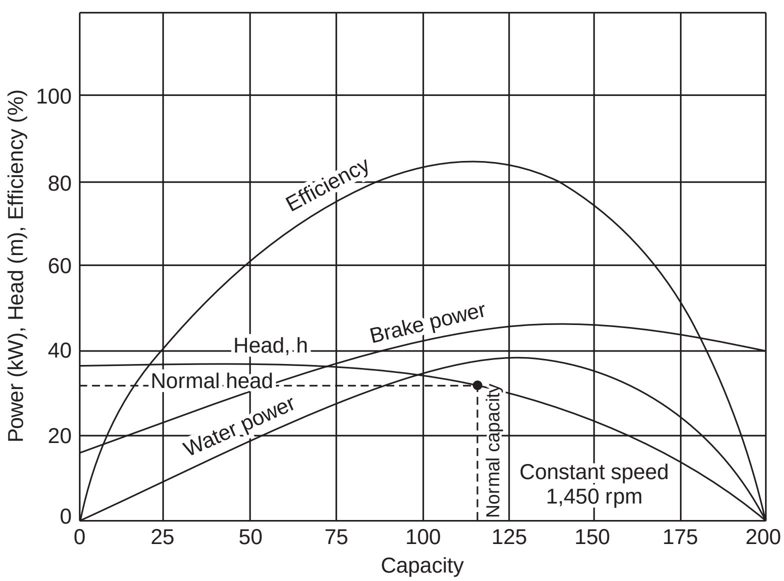



**Figure 8.4.2** Centrifugal pump characteristics at constant speed

The pump characteristics at **various speeds** including **efficiency contours** in shown in Figure 8.4.3. Such a plot helps in the development of a pump, particularly in specifying the head and flow rates.

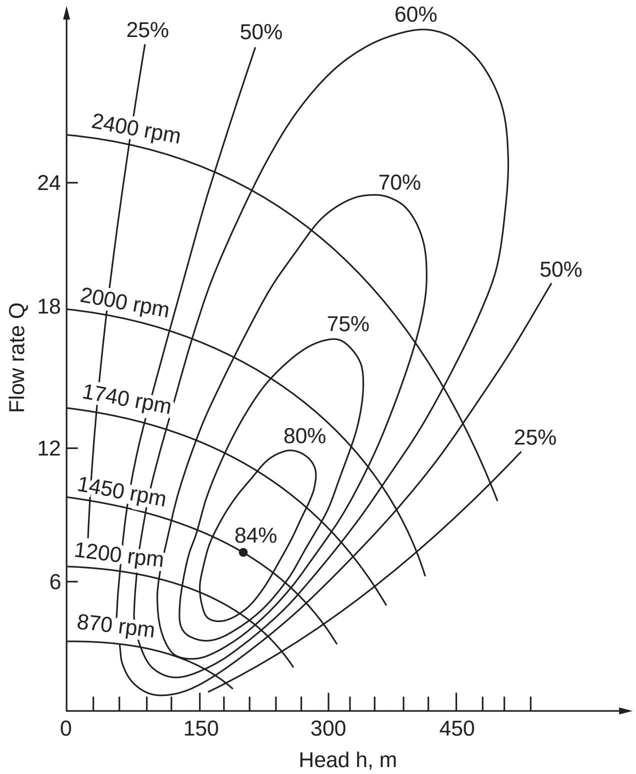



**Figure 8.4.3** Pump charateristics at various speeds

## 8.5 OPERATION OF PUMPS IN SERIES AND PARALLEL

Pumps are chosen for particular requirement. The requirements are not constant as per example the pressure required for flow through a piping system. As flow increases, the pressure required increases. In the case of the pump as flow increases, the head decreases. The operating condition will be the meeting point of the two curves representing the variation of head required by the system and the variation of head of the pump. This is shown in Figure 8.5.1. The operating condition decides about the capacity of the pump or selection of the pump.

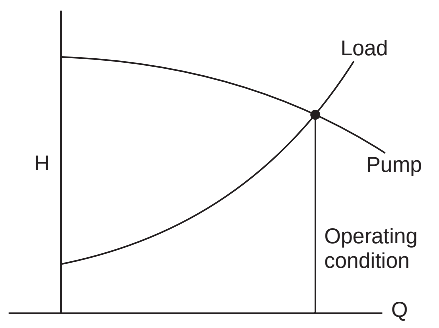



**Figure 8.5.1** Pump-load characteristics

If in a certain setup, there is a need for increased load, either a completely new pump may be chosen. This may be costlier as well as complete revamping of the setup. An additional pump can be the alternate choice. If the head requirement increases the old pump and the new pump can operate in series. In case more flow is required the old pump and the new pump will operate in parallel. There are also additional advantages in two pump operation. When the load is low one of the pump can operate with a higher efficiency when the load increases then the second pump can be switched on thus improving part load efficiency. The characteristics of parallel operation is depicted in Figure 8.5.2.

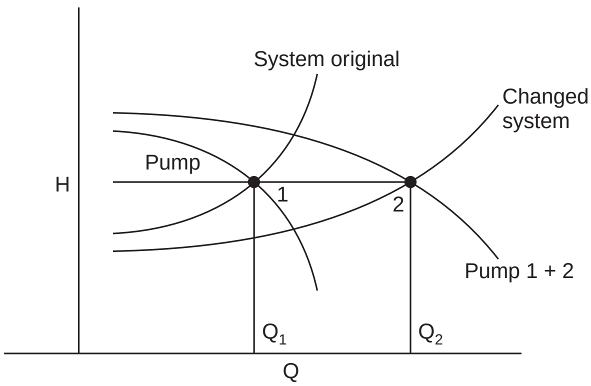



**Figure 8.5.2** Pumps in parallel

The original requirement was $Q_1$ at $H_1$. Pump 1 could satisfy the same and operating point is at 1. When the flow requirement and the system characteristic is changed such that $Q_2$ is required at head $H_1$, then two pumps of similar characteristics can satisfy the requirement. Providing a flow volume of $Q_2$ as head $H_1$. It is not necessary that similar pumps should be used. Suitable control system for switching on the second pump should be used in such a case.

When the head requirement is changed with flow volume being the same, then the pumps should work in series. The characteristics are shown in Figure 8.5.3.

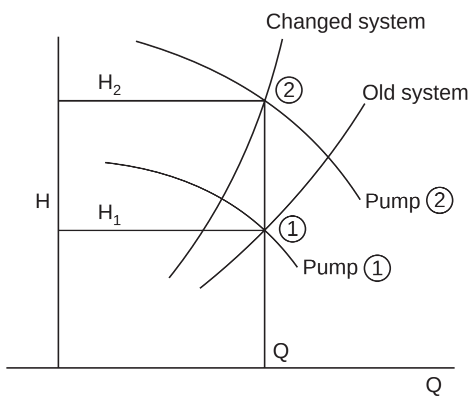



**Figure 8.5.3** Pumps in series

The flow requirement is $Q$. Originally head requirement was $H_1$ met by the first pump alone. The new requirement is flow rate $Q$ and head $H_2$. This can be met by adding in series the pump 2, which meets this requirement. It is also possible to meet changes in both head and flow requirements by the use of two pumps. Suitable control system should be installed for such purposes.

## 8.6 SPECIFIC SPEED AND SIGNIFICANCE

Some of the dimensionless parameters pertaining to pumps have been derived in the chapter on Dimensional analysis. These are derived from basics below :

1. Flow coefficient :

$$Q \propto V_f A \propto u\, Db \propto u\, DD \propto DN\, DD \propto ND^3$$

$$\therefore \qquad \mathbf{\frac{Q}{ND^3} = constant} \qquad (8.6.1)$$

For similar machine and also the **same machine.** In the case of same machine $D$ is constant.

$$\therefore \qquad \frac{Q}{N} = \text{constant or } \frac{Q_1}{N_1} = \frac{Q_2}{N_2}, \text{ unit quantity}$$

2. Head parameter :

$$H \propto u^2/g \propto D^2N^2/g$$

$$\therefore \qquad \frac{\mathbf{gH}}{\mathbf{N^2D^2}} = \text{constant} \qquad (8.6.2)$$

The head parameter is constant for similar machines. For the same machine

$$\frac{H_1}{N_1^{\,2}} = \frac{H_2}{N_2^{\,2}}, \text{ unit head}$$

3. Power parameter :

Multiplying the two parameters,

$$\frac{gH}{N^2D^2} \cdot \frac{Q}{ND^3} = \frac{\rho\, Q\, g\, H}{\rho\, N^3 D^5} = \frac{P}{\rho\, N^3 D^5}$$

4. Specific speed :

$$\text{Specific speed} = \frac{\sqrt{\text{Flow parameter}}}{3/4\sqrt{\text{Head parameter}}}$$

$$= \frac{\sqrt{Q}}{N^{1/2}D^{1.5}} \cdot \frac{N^{1.5}D^{1.5}}{(gH)^{3/4}}$$

$$N_s = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

This quantity is known as the specific speed of pumps. This is dimensionless. In practise $N_s = \frac{N\sqrt{Q}}{H^{3/4}}$ is in usage. **One definitions for the specific speed is the speed at which the pump will operate delivering unit flow under unit head.**

Actually the significance of the specific speed is its indication of the flow direction, width etc. of the impeller. This is illustrated in Figure 8.6.1. It is seen that different types of pumps have best efficiency at different specific speeds.

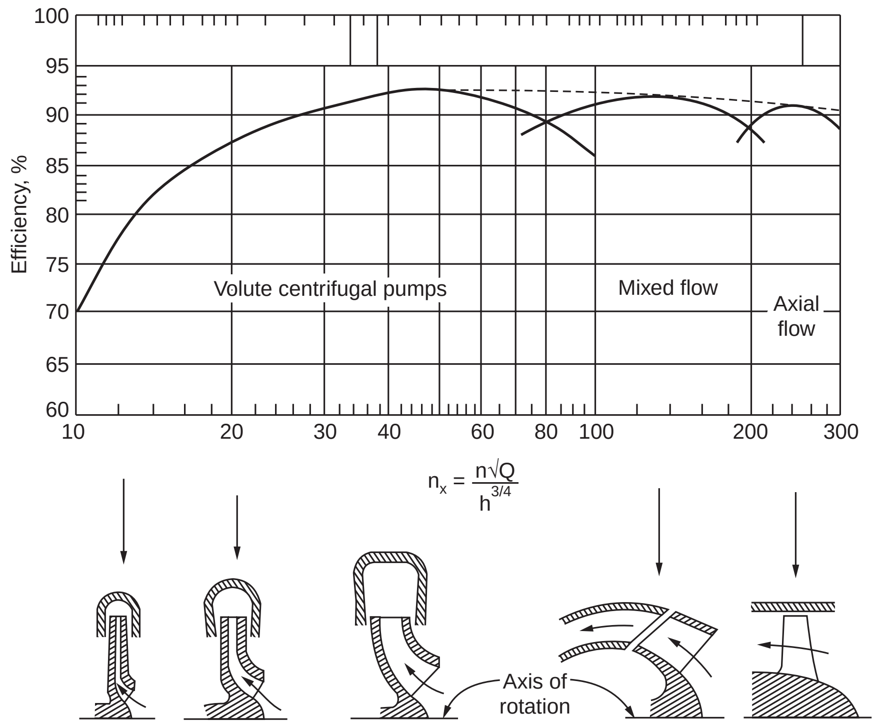



**Figure 8.6.1** Efficiency-specific speed and impeller shape relations

## 8.7 CAVITATION

What is cavitation and where and why it occurs has been discussed in the chapter on turbines. **In the case of pumps, the pressure is lowest at the inlet and cavitation damage occurs at the inlet.** For cavitation to occur the pressure at the location should be near the vapour pressure at the location.

Applying the energy equation between sump surface and the pump suction,

$$\frac{P_s}{\gamma} + \frac{V_s^2}{2g} + Z = \frac{P_a}{\gamma} - h_{fs}$$

where $Z$ is the height from sump surface and pump suction. The other terms have their usual significance. The term $h_{fs}$ should include all losses in the suction line.

**Net Positive Suction Head (NPSH) is** defined as the available total suction head at the pump inlet above the head corresponding to the vapour pressure at that temperature.

$$NPSH = \frac{P_s}{\gamma} + \frac{V_s^2}{2g} - \frac{P_v}{\gamma}$$

where $P_v$ is the vapour pressure.

From 8.7.1,

$$NPSH = \frac{P_a}{\gamma} - \frac{P_v}{\gamma} - Z - h_{fs}$$

Thoma cavitation parameter is defined by

$$\sigma = \frac{(NPSH)}{H} = \frac{(P_A/\gamma) - (P_v/\gamma) - Z - h_{fs}}{H}$$

At cavitation conditions,

$$\sigma = \sigma_c \quad \text{and} \quad \frac{P_s}{\gamma} = \frac{P_v}{\gamma}$$

$$\therefore \qquad \sigma_c = \frac{(P_a/\gamma) - (P_v/\gamma) - Z - h_{fs}}{H} \tag{8.7.4}$$

The height of suction, the frictional losses in the suction line play an important role for avoiding cavitation at a location. When pumps designed for one location is used at another location, atmospheric pressure plays a role in the onset of cavitation. Some authors use the term "suction specific speed, '$n_s$". Where $H$ in the general equation is substituted by $NPSH$. One correlation for critical cavitation parameter for pumps is given as

$$\sigma_c = \left(\frac{n_s}{175}\right)^{4/3}$$

These equations depend upon the units used and should be applied with caution.

## 8.8 AXIAL FLOW PUMP

A sectional view of axial flow pump is shown in Figure 8.8.1.

The flow in these machines is purely axial and axial velocity is constant at all radii. The blade velocity varies with radius and so the velocity diagrams and blade angles will be different at different radii. Twisted blade or airfoil sections are used for the blading. Guide vanes are situated behind the impeller to direct the flow axially without whirl. In large pumps inlet guide vanes may also be used. Such pumps are also called as propeller pumps. The head developed per stage is small, but due to increased flow area, large volumes can be handled.

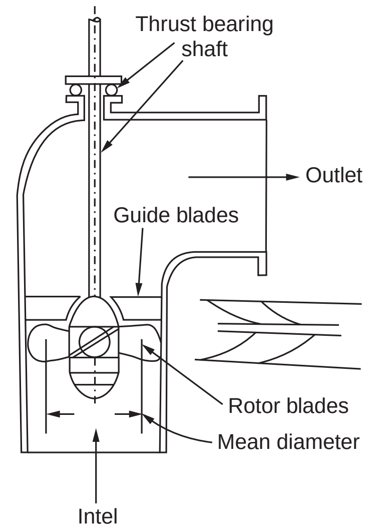



**Figure 8.8.1** Axial flow pums

A comparison of values of parameters is given in table 8.8.1.

**Table 8.8.1** Comparative values of parameter for different types of pumps

| | | | **Centrifugal** | |
|---|---|---|---|---|
| **Type of pump** | **Axial** | **Mixed flow** | **Diffuser pumps** | **Volute pumps** |
| Flow ratio $(V_f/\sqrt{2gH})$ | 0.25 – 0.6 | 0.3 | 0.15 | 0.2 |
| Speed ratio $\frac{u_2}{\sqrt{2gH}}$ | 2 – 2.7 | 1.35 | 0.9 – 1.05 | 1 – 1.2 |
| Specific speed $N\sqrt{Q}\,/H^{3/4}$ | 150 – 800 | 85 – 175 | 15 – 20 | 20 – 90 |

The whirl at inlet is zero. **The velocity triangles are given in Figure 8.8.2.**

$V_a$ is constant at all sections both at inlet and outlet.

$u$ varies with radius. Hence $\beta_1$ and $\beta_2$ will vary with radius.

$H_{th} = \dfrac{u_2\, V_{u2}}{g}$ as in the case of centrifugal pumps.

All other efficiencies are similar to the contrifugal pump.

The angle turned by the fluid during the flow over the blades is about 10 – 15°. Hence whirl imparted per

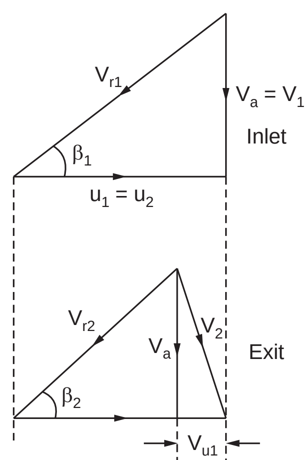



**Figure 8.8.2** Velocity triangles-axial pump

stage is small. The number of blades is limited as in the case of Kaplan turbine ranging between 2 and 8. The hub to tip ratio is in the range 0.3 to 0.6. Generally the blades are fixed. In rare designs the blades are rotated as in the case of Kaplan turbine by suitable governing mechanism.

## 8.9 POWER TRANSMITTING SYSTEMS

Ordinarily power is transmitted by mechanical means like gear drive or belt drive. In the case of gear drive there is a rigid connection between the driving and driven shafts. The shocks and vibrations are passed on from one side to the other which is not desirable. Also gear drives can not provide a stepless variation of speeds. In certain cases where the driven machine has a large inertia, the driving prime mover like electric motor will not be able to provide a large starting torque. Instead of the mechanical connection if fluids can be used for such drives, high inertia can be met. Also shock loads and vibration will not be passed on. Smooth speed variation is also possible. The power transmitting systems offer these advantages.

There are two types power transmitting devices. These are **(*i*) Fluid coupling and (*ii*) Torque converter or torque multiplier.**

### 8.9.1 Fluid Coupling

A sectional view of a fluid coupling is shown in figure 8.9.1.

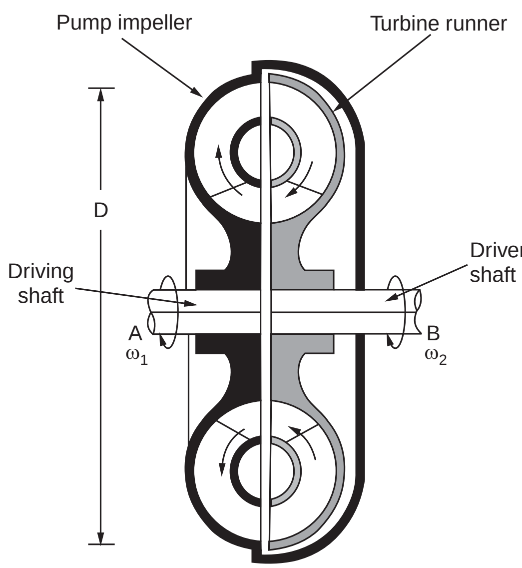



**Figure 8.9.1** Fluid coupling

In this device the driving and driven shafts are not rigidly connected. The drive shaft carries a pump with radial vanes and the driven shaft carries a turbine runner. Both of these are enclosed in a casing filled with oil of suitable viscosity. The pump accelerates the oil by imparting energy to it. The oil is directed suitably to hit the turbine vanes where the energy is absorbed and the oil is decelerated. The decelerated oil now enters the pump and the cycle is repeated. There is no flow of fluid to or from the outside. The oil transfers the energy from the drive shaft to the driven shaft. As there is no mechanical connection between the shafts, stock loads or vibration will not be passed on from one to the other. The turbine will start rotating only after a certain level of energy picked up by the oil from the pump.

Thus the prime mover can pick up speed with lower starting torque before the power is transmitted. In this way heavy devices like power plat blowers can be started with motors with lower starting torque. The pump and turbine can not rotate at the same speeds. In case these do run at the same speed, there can be no circulation of oil between them as the centrifugal heads of the pump and turbine are equal, and no energy will be transferred from one to the other. The ratio of difference in speeds to the driver speed is known as slips, $S$.

$$S = \frac{\omega_p - \omega_T}{\omega_p} \tag{8.9.1}$$

where $\omega_p$ is the pump speed and $\omega_T$ is the turbine speed. The variation of slip with pump speed is shown in figure 8.9.2.

As shown up to the pump speed $\omega_{ps}$ the turbine will not run and slip is 100%. As the driver speed increases slip rapidly decreases and at the operating conditions reaches values of about 2 to 5%.

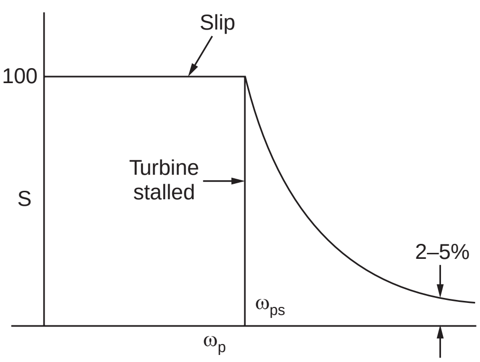



**Figure 8.9.2** Slip variation with pump speed

The efficiency of transmission

$$= \frac{\tau_t \,.\, N_t}{\tau_p \; N_p}$$

In the absence of mechanical friction $\tau_t = \tau_p$

So, $$\eta = \frac{N_t}{N_p}$$

As slip, $$S = \frac{N_p - N_t}{N_p} = 1 - \frac{N_t}{N_p}$$

$$\eta = (1 - S) \tag{8.9.2}$$

## 8.9.2 Torque Converter

In the case of fluid coupling the torque on the driver and driven members are equal. The application is for direct drives of machines. But there are cases where the torque required at the driven member should be more than the torque on the driver. Of course the speeds in this case will be in the reverse ratio. Such an application is in automobiles where this is achieved in steps by varying the gear ratios. The desirable characteristics is a stepless variation of torque. This is shown in figure 8.9.3. The torque converter is thus superior to the gear train with few gear ratios. A sectional view of torque converter is shown in figure 8.9.4. Torque

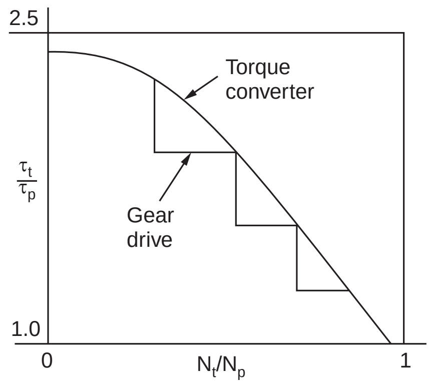



**Figure 8.9.3** Torque variation in torque converter and gear train

converter consists of **three elements namely pump impeller, a turbine runner and a fixed guide wheel** as shown in figure 8.9.4. The pump is connected to the drive shaft. The guide vanes are fixed. The turbine runner is connected to the driven shaft. All the three are enclosed in a casing filled with oil. The oil passing through the pump impeller receives energy. Then it passes to the turbine runner where energy is extracted from the oil to turn the shaft. Then the oil passes to the stationary guide vanes where the direction is changed. This introduces a reactive torque on the pump which increases the torque to be transmitted. The shape and size and direction of the guide vanes controls the increase in torque. More than three elements have also been used in advanced type of torque converters. It may be noted that the speed ratio will be the inverse of torque ratio. The efficiency is found to be highest at speed ratio of about 0.6.

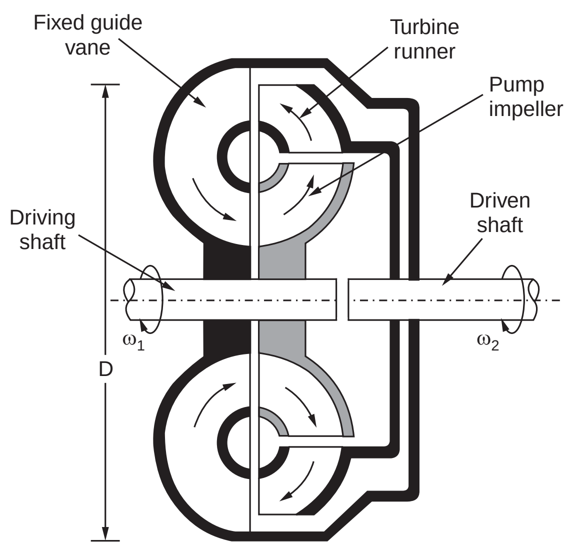



**Figure 8.9.4** Torque converter

## SOLVED EXAMPLES

**Problem 8.1.** *The following details refer to a centrifugal pump. Outer diameter : 30 cm. Eye diameter : 15 cm. Blade angle at inlet : 30°. Blade angle at outlet : 25°. Speed 1450 rpm. The flow velocity remains constant. The whirl at inlet is zero.* ***Determine the work done per kg****. If the manometric efficiency is 82%,* ***determine the working head****. If width at outlet is 2 cm,* ***determine the power*** $\eta_o$ *= 76%.*

$$u_1 = \frac{\pi \times 0.3 \times 1450}{60} = 22.78 \text{ m/s}$$

$$u_2 = 11.39 \text{ m/s.}$$

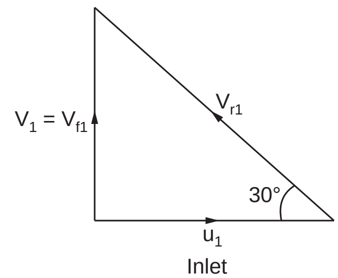



**Figure P. 8.1(a)**

From inlet velocity diagram.

$$V_{f1} = u_1 \tan \beta_1$$
$$= 11.39 \times \tan 30 = 6.58 \text{ m/s}$$

From the outlet velocity diagram,

$$V_{u2} = u_1 - \frac{V_{f2}}{\tan \beta_2} = 22.78 - \frac{6.58}{\tan 25} = 8.69 \text{ m/s}$$

**Work done per kg** $= u_2\, V_{u2} = 22.78 \times 8.69$

$= \mathbf{197.7\ Nm/kg/s}$

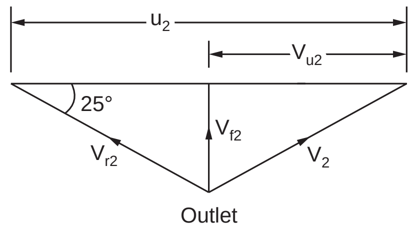



**Figure P. 8.1(b)**

$$\eta_m = 0.82 = \frac{g\,H}{197.7}$$

$\therefore$ **H = 16.52 m**

Flow rate = $\pi \times 0.3 \times 0.02 \times 6.58 = 0.124$ m$^3$/s

**Power** $= \dfrac{0.124 \times 10^3 \times 9.81 \times 16.52}{0.76 \times 10^3} = \mathbf{26.45\ kW.}$

**Problem 8.2** *A homologus model of a centrifugal pump runs at 600 rpm against a head of 8 m, the power required being 5 kW. If the prototype 5 times the model size is to develop a head of 40 m* ***determine its speed, discharge and power****. The overall efficiency of the model is 0.8 while that of the prototype is 0.85.*

(1) $Q \propto D^2 H^{1/2}$ (as $Q = AV_f$, $A \propto Db$, $b \propto D$, $V_f \propto u \propto \sqrt{H}$)

(2) $$u \propto DN \propto \sqrt{H} \quad \therefore \quad \frac{ND}{\sqrt{H}} = \text{const.}$$

$$Q_m = \frac{P_m \times \eta_m}{\rho\, g\, H_m} = \frac{5 \times 10^3 \times 0.8}{10^3 \times 9.81 \times 8} = 0.05097 \text{ m}^3\text{/s}$$

From (1) $$\mathbf{Q_p} = Q_m \cdot \frac{D_p^{\,2}}{D_m^{\,2}} \cdot \frac{H_p^{\,1/2}}{H_m^{\,1/2}}$$

$$= 0.05097 \times 5^2 \cdot \left(\frac{40}{8}\right)^{1/2} = \mathbf{2.8492\ m^3/s}$$

From (2) $$\mathbf{N_p} = N_m \cdot \left(\frac{H_p}{H_m}\right)^{1/2} \cdot \frac{D_m}{D_p} = 600 \cdot 5^{1/2} \cdot \frac{1}{5} = \mathbf{268.32\ rpm}$$

$$\textbf{Power} = \frac{2.8492 \times 9.81 \times 40 \times 10^3}{0.85 \times 10^3} = \mathbf{1315.3\ kW.}$$

**Problem 8.3** *The diameter and width of a contrifugal pump impeller are 50 cm and 2.5 cm. The pump runs at 1200 rpm. The suction head is 6 m and the delivery head is 40 m. The frictional drop in suction is 2 m and in the delivery 8 m. The blade angle at out let is 30°. The manometric efficiency is 80% and the overall efficiency is 75%.* ***Determine the power required to drive*** *the pump. Also* ***calculate the pressures at the suction and delivery side of the pump****.*

Inlet swirl is assumed as zero.

Total head against the pump is

40 + 6 + 2 + 8 = 56 m.

$$u_2 = \pi \times 0.5 \times 1200/60 = 31.42 \text{ m/s}$$

$$\eta_m = \frac{g\,H}{u_2\,V_{u2}} = 0.8$$

$$\therefore \quad \frac{9.81 \times 56}{31.42 \times V_{u2}} = 0.8, \text{ solving } V_{u2} = 21.86 \text{ m/s}$$

To calculate $V_f$, the velocity triangle is used.

$$\tan \beta_2 = \frac{V_{f2}}{u_2 - V_{u2}}$$

$$\therefore \quad V_{f2} = \tan 30\,(31.42 - 21.86) = 5.52 \text{ m/s}$$

Flow rate $$= \pi\, D_2\, b_2\, V_{f2} = \pi \times 0.5 \times 0.15 \times 5.52$$
$$= 0.13006 \text{ m}^3/\text{s}$$

$$\therefore \quad \textbf{Power} = \frac{0.13006 \times 10^3 \times 9.81 \times 56}{0.75 \times 10^3} = \mathbf{95.3\ kW}$$

Considering the water level and the suction level as 1 and 2

$$\frac{P_1}{\gamma} + 0 + 0 = \frac{P_2}{\gamma} + Z + \frac{V_2^{\,2}}{2g} + \text{losses}$$

$$10 = \frac{P_2}{\gamma} + 6 + \frac{5.52^2}{2 \times 9.81} + 2, \text{ solving,}$$

$$\frac{\mathbf{P_2}}{\gamma} = \mathbf{0.447\ m\ absolute\ (vacuum)}$$

Consider suction side and delivery side, as 2 and 3

$$\frac{P_2}{\gamma} + \frac{V_2^{\,2}}{2g} + \frac{u_2\,V_{u2}}{g} = \frac{P_3}{\gamma} + \frac{V_3^{\,2}}{2g}$$

$$V_3 = \sqrt{21.86^2 + 5.52^2} = 22.55 \text{ m/s}$$

$$\frac{P_3}{\gamma} = 0.447 + \frac{5.52^2}{2 \times 9.81} + \frac{31.42 \times 21.86}{9.81} - \frac{22.55^2}{2 \times 9.81} = \mathbf{40.1\ m\ absolute}$$

**Problem 8.4** *It is proposed to design a homologous model for a centrifugal pump. The prototype pump is to run at 600 rpm and develop 30 m head the flow rate being 1 m³/s. The model of 1/4 scale is to run at 1450 rpm.* ***Determine the head developed discharge and power required for the model.*** *Overall efficiency = 80%.*

In this case the speeds and diameter ratios are specified.

$$Q = AV_f,\ A = \pi\, Db,\ b \propto D, \quad \therefore \quad A \propto D^2$$

$$V_f \propto u \propto DN \propto \sqrt{H}$$

$\therefore$ $\quad \mathbf{Q \propto D^3\, N}$ ...(1)

Also $\quad u \propto \sqrt{H} \propto DN$

$\therefore$ $\quad \mathbf{Q \propto D^2\, H^{1/2}}$ ...(2)

$\quad \mathbf{P \propto Q\, H}$ ...(3)

As $\quad u \propto DN \propto \sqrt{H}$

$\therefore$ $\quad \mathbf{H \propto N^2\, D^2}$ ...(4)

$$\mathbf{P_p} = \frac{30 \times 1 \times 10^3 \times 9.81}{0.8 \times 1000} = \mathbf{367.9\ kW}$$

Using (1) $\quad \mathbf{Q_m} = Q_p \left(\frac{D_m}{D_p}\right)^3 \cdot \frac{N_m}{N_p} = 1 \times \left(\frac{1}{4}\right)^3 \times \frac{1450}{600} = \mathbf{0.03776\ m^3/s}$

Using (4) $\quad \mathbf{H_m} = H_p \left(\frac{D_m}{D_p}\right)^2 \cdot \left(\frac{N_m}{N_p}\right)^2 = 30 \times \left(\frac{1}{4}\right)^2 \cdot \left(\frac{1450}{600}\right)^2 = \mathbf{10.95\ m}$

Using (3) $\quad \mathbf{P_m} = P_p \frac{Q_m}{Q_p} \cdot \frac{H_m}{H_p} = 367.9 \times \frac{0.03776}{1} \cdot \frac{10.95}{30} = \mathbf{5.07\ kW}$

**Check :** $\quad P_m = \frac{0.03776 \times 10^3 \times 9.81 \times 10.95}{0.8} = \mathbf{5.07\ kW}$

**Problem 8.5** *A centrifugal pump has been designed to run at 950 rpm delivering 0.4 $m^3/s$ against a head of 16 m. If the pump is to be coupled to a motor of rated speed 1450 rpm.* ***Calculate the discharge, head and power input.*** *Assume that the overall efficiency is 0.82 remains constant.*

For a given pump, diameter, blade angles and physical parameters remain the same.

Hence, we can derive the following relations. (Similar to unit quantities).

$$Q = AV_f, A \text{ is constant} \qquad \therefore \quad Q \propto V_f$$

$$V_f \propto u \text{ and } u \propto N \qquad \therefore \quad Q \propto N \quad \text{or } Q/N = \text{constant}$$

$$\therefore \qquad \frac{Q_2}{Q_1} = \frac{N_2}{N_1} \tag{1}$$

For centrifugal pump, $H \propto u^2 \propto N^2$

$$\therefore \qquad \frac{H}{N^2} = \text{constant}$$

$$\therefore \qquad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \tag{2}$$

Power $\propto QH \propto N\,N^2 \propto N^3$

$$\therefore \qquad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 \tag{3}$$

Using the equation (1), (2), (3)

$$P_1 = \frac{1000 \times 0.4 \times 9.81 \times 16}{1000 \times 0.82} = 76.57 \text{ kW}$$

$$\mathbf{Q_2} = Q_1 \cdot \frac{N_2}{N_1} = 0.4 \times \frac{1450}{950} = \mathbf{0.61\ m^3/s}$$

$$\mathbf{H_2} = H_1 \cdot \left(\frac{N_2}{N_1}\right)^2 = 16 \times \left(\frac{1450}{950}\right)^2 = \mathbf{37.27\ m}$$

$$\mathbf{P_2} = 76.57 \times \left(\frac{1450}{950}\right)^3 = \mathbf{272\ kW}$$

**Check :** $$\mathbf{P_2} = \frac{1000 \times 0.61 \times 37.27 \times 9.81}{1000 \times 0.82} = \mathbf{272\ kW.}$$

**Problem 8.6** *A centrifugal pump running at 1450 rpm has an impeller diameter of 0.4 m. The backward curved blade outlet angle is 30° to the tangent. The flow velocity at outlet is 10 m/s.* ***Determine the static head*** *through which water will be lifted. In case a diffuser reduces the outlet velocity to 40% of the velocity at the impeller outlet, what will be the* ***increase in the static head.***

The whirl at inlet is assumed as zero.

The velocity diagram at outlet is shown.

$$u_2 = \frac{\pi DN}{60} = \frac{\pi \times 0.4 \times 1450}{60} = 30.37 \text{ m/s}$$

From velocity triangle

$$V_{u2} = u_2 - V_{f2} / \tan \beta_2 = 30.27 - \frac{10}{\tan 30} = \mathbf{13.05 \text{ m/s}}$$

Total head developed by the impeller

$$= \frac{u_2 V_{u2}}{g} = \frac{30.37 \times 13.05}{9.81} = 40.4 \text{ m}$$

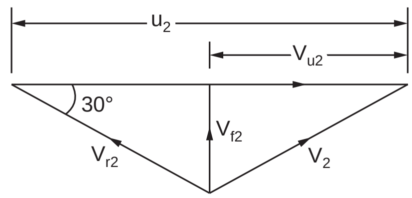



Figure P.8. 15.6

Absolute velocity at exit $= (V_f^{\,2} + V_{u2}^{\,2})^{0.5}$

$= (10^2 + 13.05^2)^{0.5}$

$= 16.44$ m/s

Kinetic head $= \dfrac{V_2^{\,2}}{2g} = \dfrac{16.44^2}{2 \times 9.81} = 13.77$

**If diffuser is not used**, static lift

$= 40.4 - 13.77 = \mathbf{26.63 \text{ m}}$

The diffuser outlet velocity $= 0.4 \times 16.44 = 6.576$ m/s

Kinetic head at outlet $= 6.576^2/2 \times 9.81 = 2.2$ m

**With diffuser use, the static lift** $= 40.4 - 2.2 = \mathbf{38.2 \text{ m}}$

**Increase in static head** $= 38.2 - 26.63$ m $= \mathbf{11.57 \text{ m}}$

**Problem 8.7** *A form stage centrifugal pump running at 600 rpm is to deliver 1 m* $^3$*/s of water against a manometric head of 80 m. The vanes are curved back at 40° to the tangent at outer periphery. The velocity of flow is 25% of the peripheral velocity at outlet. The hydraulic losses are 30% of the velocity head at the outlet of the impeller.* ***Determine the diameter of the impeller and the manometric efficiency.***

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_2 \times 600}{60} = 31.42 D_2 \text{ m/s}$$

Velocity of flow at outlet $= \dfrac{1}{4} \times 31.42 = 7.85 D_2$ m/s

$$V_{u2} = 31.42 D_2 - \frac{7.85 D_2}{\tan 40} = 2.06 D_2 \text{ m/s}$$

$$V_2 = [V_{u2}^{\,2} + V_{f2}^{\,2}]^{0.5} = [22.06^2 D_2^{\,2} + 7.85^2 D_2^{\,2}]^{0.5}$$

$$= 23.41 D_2 \text{ m/s}$$

$$\frac{0.3 V_2^{\,2}}{2g} = 8.383 D_2^{\,2}$$

Energy imparted to impeller

$$= \frac{u_2 V_{u2}}{g} = \frac{31.42 \times 22.06}{9.81} D_2^2 = 70.655\ D_2^2$$

$70.655\ D_2^2$ = manometric head per stage + losses.

$= (80/4) + 8.838\ D_2^2$

$\therefore \qquad 62.272\ D_2^2 = 20$

$\therefore$ **Solving** $\qquad \mathbf{D_2 = 0.57\ m}$

$$\eta_m = \frac{20}{70.655 \times 0.57^2} = \mathbf{0.8712 \text{ or } 87.12\%}$$

**Problem 8.8** *A centrifugal pump works at 900 rpm and is required to work against a head of 30m. The blades are curved back at 25° to the tangent at outlet. The flow velocity is 2.5 m/s.* ***Determine the diameter (i) If all the kinetic energy is lost (ii) It the velocity is reduced to 50%*** *converting kinetic energy to pressure energy.*

Energy imparted to the impeller $\qquad = u\left(u - \dfrac{2.5}{\tan 25}\right)/g$

Kinetic head at exit $\qquad = \dfrac{V_2^2}{2g} = \left\{\left(u - \dfrac{2.5}{\tan 25}\right)^2 + 2.5^2\right\}/2g$

The difference is the head against which the pump works (simplyfying)

$$2(u^2 - 5.361\ u) - (u^2 + 5.361^2 - 2 \times 5.361\ u + 6.25) = 30 \times 2 \times 9.81$$

This reduces to

$$u^2 = 623.59 \quad \therefore \quad u = 24.97\ m/s$$

$$u = \frac{\pi\ DN}{60},\ 24.97 = \frac{\pi \times D \times 900}{60},$$

**Solving** $\qquad \mathbf{D = 0.53\ m}$

In case the velocity at exit is reduced to half its value,

$$\frac{u(u - 5.361\, u)}{g} - \frac{(u - 5.361)^2 + 2.5^2}{8g} = 30$$

This reduces to

$$7\ u^2 - 32.166\ u - 2392.35 = 0$$

Solving $\qquad u = 20.93$ m/s

**This leads to D = 0.44 m.**

**Problem 8.9** *The dimensionless specific speed of a centrifugal pump is 0.06. Static head is 30 m. Flow rate is 50 l/s. The suction and delivery pipes are each of 15 cm diameter. The friction factor is 0.02. Total length is 55 m other losses equal 4 times the velocity head in the pipe. The vanes are forward curved at 120°. The width is one tenth of the diameter. There is a 6% reduction in flow area due to the blade thickness. The manometric efficiency is 80%.* ***Determine the impeller diameter.*** *Inlet is radial.*

Frictional head is calculated first. Velocity in the pipe

$$= \frac{0.05 \times 4}{\pi \times 0.15^2} = 2.83 \text{ m/s}$$

Total loss of head $= \dfrac{fl\,V^2}{2g\,D} + \dfrac{4\,V^2}{2g}$

$$= \frac{0.02 \times 55 \times 2.83^2}{2 \times 9.81 \times 0.15} + \frac{4 \times 2.83^2}{2 \times 9.81} = 4.63 \text{ m}$$

Total head against which pump operates = 34.63 m

Speed is calculated from specific speed $N_s = N\sqrt{Q}\,/(gH)^{3/4}$

$$N = \frac{0.06 \times (9.81 \times 34.63)^{3/4}}{0.05^{1/2}} = 21.23 \text{ rps}$$

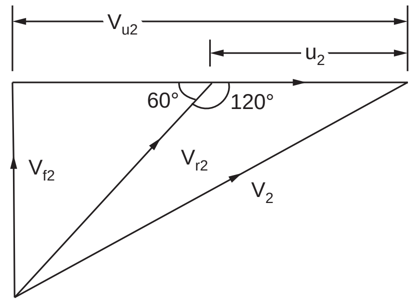


**Figure P. 8.9** Outlet velocity diagram (forwerd curved)

Flow velocity is determined :

Flow area $= \pi \times D \times \dfrac{D}{10} \times 0.94 = 0.2953\,D^2$

$$V_{f2} = \frac{0.05}{0.2953\,D^2} = 0.1693/D^2 \quad \text{...(1)}$$

$$u_2 = \pi\,DN = 21.23 \times \pi \times D = 66.7\,D \quad \text{...(2)}$$

$$\eta_m = 0.8 = \frac{9.81 \times 34.63}{66.7\,D \times V_{u2}}$$

$$\therefore \quad V_{u2} = \frac{6.367}{D} \quad \text{...(3)}$$

From velocity diagram,

$$\tan 60 = \frac{V_{f2}}{V_{u2} - u_2} = \frac{0.1693}{D^2} \cdot \frac{1}{\left(\dfrac{6.367}{D} - 66.7\,D\right)}$$

Rearranging, $115.52\,D^3 - 11.028\,D + 0.1693 = 0$

Solving, **D = 0.3 m.**

**Problem 8.10** *The* ***head developed*** *by a centrifugal pump running at 900 rpm is 27 m. The flow velocity is 3 m/s. The blade angle at exit is 45°.* ***Determine the impeller diameter.***

The head developed $= \dfrac{u_2\,V_{u2}}{g} = 27\,m$

$$V_{u2} = u_2 - \frac{V_f}{\tan 45} = u_2 - 3$$

$$\therefore \qquad u_2 \times (u_2 - 3) = 27 \times 9.81$$

$$u_2^2 - 3u_2 - 264.87 = 0$$

$$u_2 = 17.84 \text{ (or } - 14.84\text{, trivial)}$$

$$\frac{\pi DN}{60} = 17.84, N = 900$$

$$\therefore \qquad \mathbf{D} = \frac{17.84 \times 60}{\pi \times 900} = \mathbf{0.397 \text{ m or } 379 \text{ mm.}}$$

**Problem 8.11** *A radial vaned centrifugal compressor delivers 0.3 m* $^3$*/s against a head of 20 m. The flow velocity is constant at 3 m/s. The manometric efficiency is 80%. If the width is 1/10th of the diameter* ***Calculate the diameter, width and speed.*** *The eye diameter is 0.5 of outer diameter. Calculate the dimensions of inlet.*

Assume zero whirl at inlet. The velocity diagram at outlet is shown

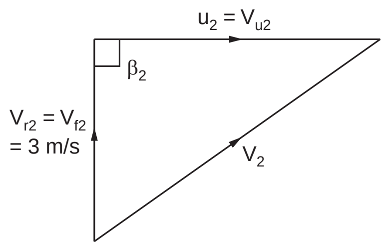


**Figure P. 8.11**

$$\eta_m = \frac{g H}{u_2 V_{u2}} \text{ here } u_2 = V_{u2}$$

$$\therefore \qquad u_2^2 = \frac{g H}{\eta_m} = \frac{9.81 \times 20}{0.8} = 245.25$$

$$\therefore \qquad u = 15.66 \text{ m/s}$$

$$Q = \pi D_2 b_2 V_{f2}$$

$$\therefore \qquad 0.3 = \pi \times D_2 \times 0.1 D_2 \times 3$$

$$D_2^2 = \frac{0.3}{\pi \times 0.1 \times 3},$$

Solving, $\mathbf{D_2 = 0.5642 \text{ m}}$

$\therefore$ $\mathbf{b_2 = 0.05642 \text{ m}}$

$\mathbf{D_1 = 0.2821 \text{ m}}$

$\mathbf{b_1 = 0.1128 \text{ m}}$

$$u_2 = \frac{\pi DN}{60}, 15.66 = \frac{\pi \times 0.5642}{60} \times N$$

Solving **N = 530 rpm.**

**Problem 8.12** *A centrifugal pump running at 900 rpm and delivering 0.3 m* $^3$*/s of water against a head of 25 m, the flow velocity being 3 m/s. If the manometric efficiency is 82%* ***determine the diameter and width of the impeller.*** *The blade angle at outlet is 25°.*

The velocity diagram at outlet is as shown. The inlet whirl is generally assumed as zero unless mentioned.

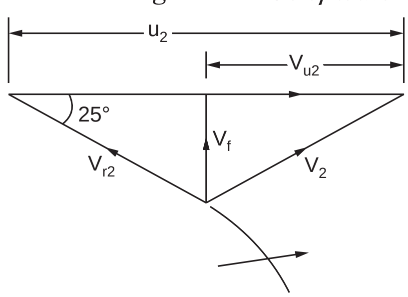


**Figure P. 8.12**

$$\eta_m = \frac{g H}{u_2 V_{u2}},$$

$\therefore \quad u_2 V_{u2} = \dfrac{9.81 \times 25}{0.82} = 299.09$ (A)

$\tan 25 = \dfrac{V_f}{u_2 - V_{u2}} \quad \therefore \quad u_2 - V_{u2} = \dfrac{3}{\tan 25} = 6.43352$ (B)

$V_{u2} = u_2 - 6.43352$

$\therefore \quad u_2 \times (u_2 - 6.43352) = 299.09$

or $\quad u_2^2 - 6.43352\, u_2 - 299.09 = 0$

Solving $\quad u_2 = 20.808$ m/s (the other solution being negative).

$u_2 = \dfrac{\pi DN}{60}$,

$\therefore \quad \mathbf{D} = \dfrac{u_2 \times 60}{\pi N} = \dfrac{20.808 \times 60}{\pi \times 900} =$ **0.4416 m or 44.16 cm**

$Q = \pi D_2 b_2 V_{f2}$

$\therefore \quad \mathbf{b_2} = \dfrac{Q}{\pi D_2 V_{f2}} = \dfrac{0.3}{\pi \times 0.4416 \times 3} =$ **0.0721 m or 7.21 cm.**

**Problem 8.13.** *An axial flow pump running at 600 rpm deliver 1.4 m³/s against a head of 5 m. The speed ratio is 2.5. The flow ratio is 0.5. The overall efficiency is 0.83.* ***Determine the power required*** *and the* ***blade angles at the root and tip and the diffuser blade inlet angle.*** *Inlet whirl is zero.*

$\beta_1$ – Blade angle at inlet

$\beta_2$ – Blade angle at outlet

$\alpha_2$ – Diffuser blade inlet angle

**Power** $= \dfrac{1.4 \times 10^3 \times 5 \times 9.81}{0.83 \times 10^3} =$ **82.73 kW**

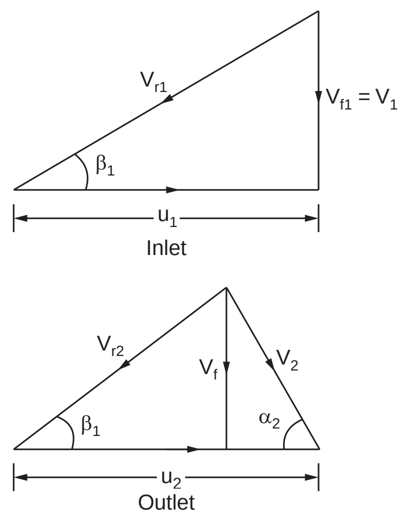



**Figure P. 8.13**

$u_t = 2.5\sqrt{2g\,H}$

$= 2.5\sqrt{2 \times 9.81 \times 5}$

$= 24.76$ m/s,

$V_f = 0.5\sqrt{2g\,H} = 4.95$ m/s

$D_0 = \dfrac{24.76 \times 60}{\pi \times 600} = 0.788$ m

$Q = \dfrac{\pi (D_0^2 - D_1^2) V_f}{4}$

$\therefore \quad 1.4 = \pi(0.788^2 - D_1^2)\, 4.95$

Solving, $D_1 = 0.51$ m

**At Tip :** From inlet triangle,

$$\beta_{1t} = \tan^{-1}\left(\frac{4.95}{24.76}\right) = \mathbf{11.3°}$$

From outlet triangle, $\beta_{2t} = V_f/(u - V_{u2})$

$$u_2 V_{u2} = gH,\ 24.76 \times V_{u2} = 9.81 \times 5,\ V_{u2} = 1.981 \text{ m/s}$$

$$\therefore \quad \beta_{2t} = \tan^{-1}\left(\frac{4.95}{24.76 - 1.981}\right) = \mathbf{12.26°}$$

$$\alpha_{2t} = \tan^{-1}\left(\frac{4.95}{1.981}\right) = 68.2°, \text{ can also be given as } (180 - 68.2°)$$

**At Root :** $u_2 V_{u2} = gH,\ u_2 = \pi \times 0.51 \times 600/60 = 16.02$ m/s

$\therefore \quad V_{u2} = 5 \times 9.81/16.02 = 3.06$ m/s, $V_f$ = constant

$$\beta_{1R} = \tan^{-1}\left(\frac{4.95}{16.02}\right) = \mathbf{17.17°}$$

$$\beta_{2R} = \tan^{-1}(4.95/(16.02 - 3.06) = \mathbf{20.9°}$$

$$\alpha_{2R} = \tan^{-1}\left(\frac{4.95}{3.06}\right) = \mathbf{58.3°}$$

**$\alpha$ values can also be given as (180 – 58.3)°.**

**Problem 8.14** *A centrifugal pump of impeller diameter 0.4 m runs at 1450 rpm. The blades are curved back at 30° to the tangent at the outlet. The velocity of flow is 3 m per second.* ***Determine the theoretical maximum lift*** *if the outlet velocity is reduced by the diffuser by 50%.*

Inlet whirl is assumed to be zero

$$u_2 = \frac{\pi \times 0.4 \times 1450}{60} = 30.37 \text{ m/s}$$

$$V_{u2} = 30.37 - \frac{3}{\tan 30} = 25.17 \text{ m/s}$$

$$V_2 = (25.17^2 + 3^2)^{0.5} = 25.35 \text{ m}$$

**Head imparted** $= \dfrac{30.37 \times 25.17}{9.81} = \mathbf{77.92\ m}$

Static head $= 77.92 - \dfrac{25.35^2}{2 \times 9.81} = 45.17$ m

Without diffuser the pump can pump to a head of 45.17 m theoretically.

If velocity is reduced to 50% of the value

New velocity = 12.675 m/s

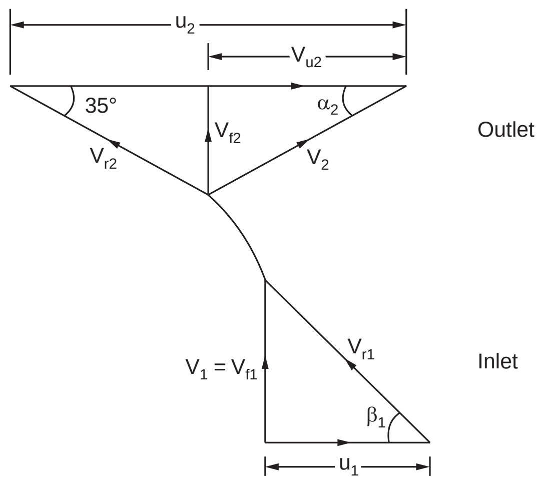
$\therefore$ Head recovered $= \dfrac{25.35^2 - 12.675^2}{2 \times 9.81} = 24.57$ m

$\therefore$ **Theoretical maximum lift**

$$= 45.17 + 24.57 = \mathbf{69.74\ m}$$

**Problem 8.15** *A centrifugal pump running at 900 rpm has an impeller diameter of 500 mm and eye diameter of 200 mm. The blade angle at outlet is 35° with the tangent. Determine assuming zero whirl at inlet,* ***the inlet blade angle. Also calculate the absolute velocity at outlet and its angle with the tangent.*** *The flow velocity is constant at 3 m/s. Also calculate the manometric head.*

The velocity diagrams are as shown.



**Figire P. 8.15**

**Consider inlet**

$$u_1 = \frac{\pi \times 0.2 \times 900}{60} = 9.42 \text{ m/s}$$

$$V_{f1} = 3 \text{ m/s}$$

Blade angle at inlet

$$\tan \beta_1 = \frac{V_{f1}}{u_1} = \frac{3}{9.42}$$

$$\therefore \quad \boldsymbol{\beta_1 = 17.66°}$$

**Considering outlet**

$$u_2 = \frac{\pi \times 0.5 \times 900}{60} = 23.56 \text{ m/s}$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan 35} = 23.56 - \frac{3}{\tan 35} = 19.28 \text{ m/s}$$

$$\tan \alpha_2 = \frac{3}{19.28}$$

$$\therefore \quad \boldsymbol{\alpha_2 = 8.85°}$$

$$\boldsymbol{V_2} = \sqrt{3^2 + 19.28^2} = \mathbf{19.51\ m/s}$$

The outlet velocity is 19.51 m/s at an angle of 8.85° to the tangent. (taken in the opposite direction of $u$).

**Manometric head** $= \dfrac{23.56 \times 19.28}{9.81} = \mathbf{46.3\ m}$

**Problem 15.16** *A centrifugal pump running at 900 rpm delivers 800 l/s against a head of 70 m. The outer diameter of the impeller is 0.7 m and the width at outlet is 7 cm. There is recirculation of 3% of volume delivered. There is a mechanical loss of 14 kW. If the manometric efficiency is 82%* ***determine the blade angle at outlet, the motor power and the overall efficiency.***

The velocity diagram at outlet is as shown. The inlet whirl is assumed as zero. Backward curved vane is assumed (This is the general case).

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.7 \times 900}{60} = 32.99 \text{ m/s}$$

From manometric efficiency, $V_{u2}$ is determined

$$\eta_m = \frac{g H}{u_2 V_{u2}},$$

$$0.82 = \frac{9.81 \times 70}{32.99 \times V_{u2}},$$

$$\therefore \quad V_{u2} = 25.385 \text{ m/s}$$

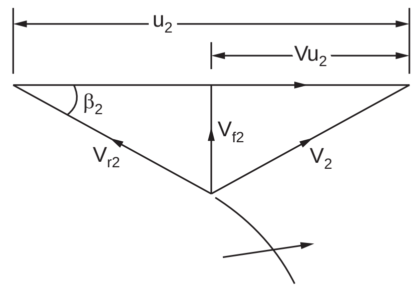



Figure P. 8.15.16

The flow through the impeller is increased by 3%

$$\therefore \quad V_{f2} = \frac{Q \times 1.03}{\pi D_2 b_2} = \frac{0.8 \times 1.03}{\pi \times 0.7 \times 0.07} = 5.353 \text{ m/s}$$

From velocity triangle,

$$\tan \beta_2 = \frac{V_{f2}}{u_2 - V_{u2}} = \frac{5.353}{32.99 - 25.385}$$

**Solving** $\boldsymbol{\beta_2 = 35.14°}$

**Motor Power** $= Q \times 1.03 \times u_2 V_{u2}$ + Mechanical losses

$= (0.8 \times 1.03 \times 32.99 \times 25.385) + 14$

$= 690.06 + 14 =$ **704.06 kW**

**Overall efficiency** $= 1.03 \times 0.8 \times 9.81 \times 70 \times \frac{1}{704.06}$

= **0.8036 or 80.36%.**

**Problem 15.17** *The pressure difference between the suction and delivery sides of a pump is 25 m. The impeller diameter is 0.3 m and the speed is 1450 rpm. The vane angle at outlet 30° with the tangent. The velocity of flow is 2.5 m.* ***Determine the manometric efficiency.*** *If frictional losses in the impeller is 2 m* ***calculate the fraction of total energy converted to pressure energy*** *in the impeller. Also calculate the* ***pressure rise in the pump casing.***

$$u_2 = \frac{\pi \times 0.3 \times 1450}{60} = 22.78 \text{ m/s}$$

$$V_{u2} = 22.78 - \frac{2.5}{\tan 30} = 18.45 \text{ m/s}$$

$$V_2 = (18.45^2 + 2.5^2)^{0.5} = 18.62 \text{ m/s}$$

Energy input $= \dfrac{22.78 \times 18.45}{9.81} = 42.84$ m

$$\eta_m = \frac{25}{42.84} = \mathbf{0.5835 \text{ or } 58.35\%}$$

Kinetic energy at exit of impeller

$= 18.62^2/2 \times 9.81 = 17.67$ m

∴ Energy conversion in the impeller

$= 42.84 - 17.67 = 25.17$ m

Frictional loss $= 2$ m

**Energy fraction for pressure**

$$= \frac{25.17 - 2}{42.84} = \mathbf{0.5409 \text{ or } 54.09\%}$$

Total pressure rise $= 25$ m

Pressure rise in the impeller $= (25.17 - 2) = 23.17$ m

∴ **Pressure rise in the pump casing = 1.83 m.**

**Problem 8.18** *A centrifugal pump running at 1450 rpm has impeller of 350 mm OD and 150 mm ID. The blade angles as* ***measured with the radial direction*** *are 60° and 65° at inlet and outlet. The hydraulic efficiency is 85%. The width of the impeller at inlet is 50 mm. Flow velocity is constant.*

***Determine the static and stagnation pressure risc across*** *the impeller and the power input to the impeller.*

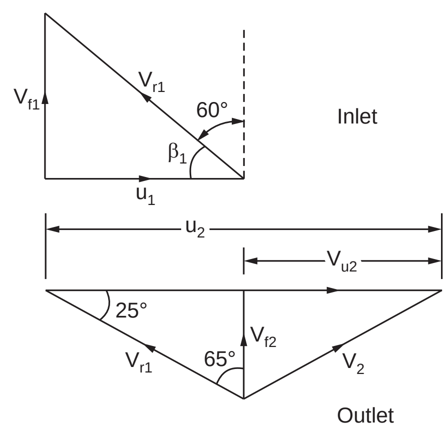


**Figure P. 8.18**

$$u_1 = \frac{\pi \times 0.15 \times 1450}{60} = 11.39 \text{ m/s}$$

$$V_{f1} = u_1 \,.\, \tan(90 - 60)$$
$$= 11.39 \times \tan 30 = 6.575 \text{ m/s}$$

$$u_2 = \frac{\pi \times 0.35 \times 1450}{60} = 26.572 \text{ m/s}$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan \beta_2} = 26.572 - \frac{6.575}{\tan 25^\circ} = 12.472 \text{ m/s}$$

Energy input to the pump $= \dfrac{u_2 V_{u2}}{g} = 33.78$ m

Energy to the fluid $= 0.85 \times 33.78 = 28.72$ m

**Stagnation pressure rise** through the impeller

$$= \left[\frac{P_2 - P_1}{\gamma} + \frac{V_2^{\,2} - V_1^{\,2}}{2g}\right] = \mathbf{28.72\ m}$$

**Static pressure rise** $= 28.72 - \left[\dfrac{V_2^{\,2} - V_1^{\,2}}{2g}\right]$

$$V_2 = (V_{f2}^{\,2} + V_{u2}^{\,2})^{1/2} = (6.575^2 + 12.472^2)^{1/2} = 16.1 \text{ m/s}$$

$\therefore$ **Static pressure rise**

$$= 28.72 - \frac{16.1^2 - 6.575^2}{2 \times 9.81} = \mathbf{17.71\ m}$$

(Heads can be converted to pressure using $H\gamma = P$)

Flow rate $= \pi \times 0.15 \times 0.05 \times 6.575 = 0.1549 \text{ m}^3\text{/s}$

$\therefore$ **Power input** $= \dfrac{33.78 \times 01549 \times 10^3 \times 9.81}{10^3} = \mathbf{51.3\ kW.}$

**Problem 8.19** *A centrifugal pump with OD = 0.6 m and ID = 0.3 m runs at 900 rpm and discharges 0.2 m³/s of water against a head of 55 m. The flow velocity remains constant along the flow. The peripheral area for flow is 0.0666 m². The vane angle at outlet is 25°. The entry is radial.* ***Determine the manometric efficiency and the inlet vane angle.***

Radial entry means zero whirl at inlet. The velocity triangles are as shown.

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.6 \times 900}{60} = 28.27 \text{ m/s}$$

**From outlet triangle,**

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan 25}$$

$$V_{f2} = 0.2/0.0666 = 3 \text{ m/s}$$

$$\therefore \quad V_{u2} = 28.27 - \frac{3}{\tan 25} = 21.84 \text{ m/s}$$

$$\therefore \quad \eta_m = \frac{g\,H}{u_2\,V_{u2}} = \frac{55 \times 9.81}{28.27 \times 21.84} = 0.8739$$

**or 87.39%**

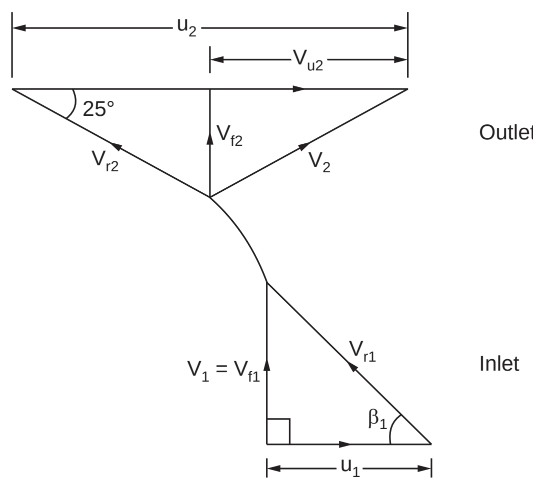



**Figure P. 8.19**

**From inlet triangle,**

$$\tan \beta_1 = \frac{V_{f1}}{u_1},\ u_1 = \frac{\pi \times 0.3 \times 900}{60} = 14.14 \text{ m/s}$$

$$\therefore \qquad \tan \beta_1 = \frac{3}{14.14} \quad \therefore \quad \boldsymbol{\beta_1 = 12°}$$

**Problem 8.20** *A five stage centrifugal pump with blades radial at outlet runs at 500 rpm delivering 0.25 m³/s against a total head of 100 m. The diameter of the impellers is 0.6 m and the flow velocity is 5 m/s.* ***Determine the manometric efficiency and the width of the impellers.***

Inlet whirl is assumed zero. The velocity diagram is shown.

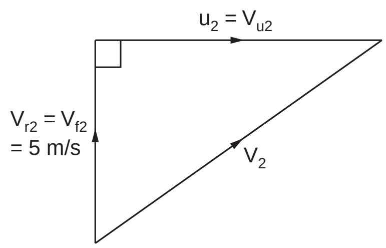



Figure P. 8.20

**Here $\mathbf{u_2 = V_{u2}}$** $\qquad u_2 = \frac{\pi D N}{60} = \frac{\pi \times 0.6 \times 500}{60}$

$= 15.708$ m/s

Manometric head $\qquad = \frac{u_2 V_{u2}}{g} = \frac{u_2^{\,2}}{g} = \frac{15.708^2}{9.81}$

$= 25.15$ m

Head delivered by each impeller is 100/5 = 20 m

$\therefore$ **Manometric efficiency** = 20/25.15 = 0.7952 = **79.52%**

Flow rate $= \pi D b V_f = 0.25 = \pi \times 0.6 \times 5 \,.\, b_2$

Solving $\mathbf{b_2}$ = 0.0265 m or **26.5 mm.**

**Problem 8.21** *A centrifugal pump with an impeller diameter of 0.4 m runs at 1450 rpm. The angle at outlet of the backward curved vane is 25° with tangent. The flow velocity remains constant at 3 m/s. If the manometric efficiency is 84%* ***determine the fraction of the kinetic energy at outlet recovered as static head.***

The whirl at inlet is zero. The velocity triangle is as shown.

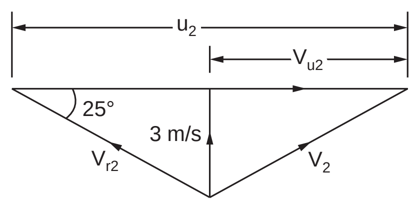



Figure 8. P.21

$$u_2 = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 1450}{60}$$

$= 30.37$ m/s

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan \beta_2}$$

$= 30.37 - (3/\tan 25) = 23.94$

m/s

$$V_2 = \sqrt{V_{u2}^{\,2} + V_{f2}^{\,2}} = \sqrt{23.94^2 + 3^2} = 24.12 \text{ m/s}$$

Total head developed $\qquad = \frac{30.37 \times 23.94}{9.81} = 74.11$ m

Kinetic head $= \dfrac{24.12^2}{2 \times 9.81} = 29.65$ m

Kinetic head at inlet $= \dfrac{3^2}{2 \times 9.81} = 0.46$ m

$\therefore$ Static head at impeller exit (using Bernoulli equation between inlet and outlet)

$$= 74.11 + 0.46 - 29.65 = 44.92 \text{ m}$$

Actual static head $= \eta_m \times \dfrac{u_2 V_{u2}}{9.81} = 0.84 \times 74.11 = 62.25$ m

Static head recovered $= 62.25 - 44.92 = 17.33$ m

Let the fraction be $\phi$

$$\frac{\phi \times 24.12^2}{2 \times 9.81} = 17.33,$$

**Solving** $\boldsymbol{\phi = 0.5823}$

**Problem 8.22** *A centrifugal pump running at 900 rpm delivers 1 m $^3$/s against a head of 12 m. The impeller diameters are 0.5 m and 0.3 m respectively. The blade angle at outlet is 20° to the tangent.* ***Determine the manometric efficiency and the power required.*** *Mechanical efficiency = 98%. Also* ***estimate the minimum*** *speed at which the pump will* ***start delivery. The impeller width at outlet is 10 cm.***

Whirl at inlet is zero is assumed

$$u_2 = \frac{\pi \times 0.5 \times 900}{60} = 23.56 \text{ m/s}$$

$$V_{f2} = 1/\pi \times 0.5 \times 0.1 = 6.366 \text{ m/s}$$

$$V_{u2} = u_2 - \frac{V_{f2}}{\tan 20} = 23.56 - \frac{6.366}{\tan 20} = 6.07 \text{ m/s}$$

$$\frac{u_2 V_{u2}}{g} = \frac{23.56 \times 6.07}{9.81} = 14.58 \text{ m}$$

$\therefore$ Manometric efficiency

$$= \frac{12}{14.58} = 0.8232 = 83.32\%$$

Power required $= \dfrac{1 \times 10^3 \times 14.58 \times 9.81}{0.98 \times 10^3} = 145.9$ kW

Starting speed is given by the expression

$$\frac{u_2^2 - u_1^2}{2g} \geq \frac{u_2 V_{u2}}{g}$$

$$\frac{\pi^2 N^2 (0.5^2 - 0.3^2)}{60^2 \times 2 \times 9.81} = 14.58$$

**Solving** $\qquad$ **N = 801.6 rpm**

**The pump will start delivering at 801.6 rpm.**

**Problem 8.23** *If the backward curved bladed impeller of 40° outlet angle, running an 1440 rpm is operated in the opposite direction,* ***find the ratio of power and exit velocities.*** *The diameter of the impeller is 0.3 m and the flow velocity is 0.20 of blade velocity.*

The respective velocity diagrams are shown.

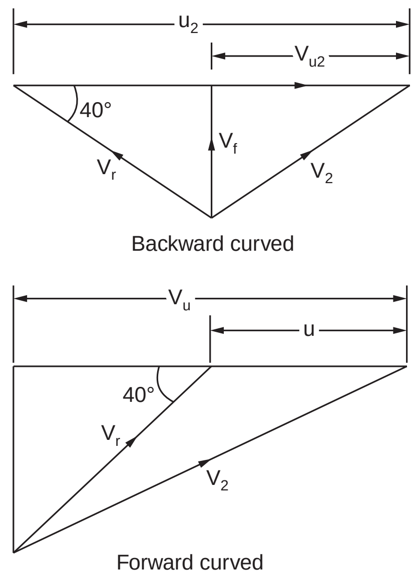



**Figure P. 8.23**

**Backward curved**

$$u_2 = \frac{\pi \times 0.3 \times 1440}{60} = 22.62 \text{ m/s}$$

$$V_{f2} = 22.62 \times 0.2 = 4.52 \text{ m/s}$$

$$V_{u2} = 22.62 - \frac{4.52}{\tan 40} = 17.23 \text{ m/s}$$

**Input head** $= \dfrac{u_2 V_{u2}}{g} = \dfrac{22.62 \times 17.23}{9.81}$

$= \mathbf{39.72\ m}$

$\mathbf{V_2} = [17.23^2 + 4.52^2]^{0.5} = \mathbf{17.81\ m/s}$

**Static pressure rise** in the impeller

$$= 39.72 - \frac{V_2^{\,2}}{2g} = 39.72 - \frac{17.81^2}{2 \times 9.81} = \mathbf{23.55\ m}$$

**Forward curved**

$$u_2 = 22.62 \text{ m/s}, V_{f2} = 4.52 \text{ m/s}$$

$$V_{u2} = u_2 + \frac{V_{f2}}{\tan 40} = 28 \text{ m/s}$$

**Input head** $\qquad = \dfrac{u_2 V_{u2}}{g} = \dfrac{22.62 \times 28}{9.81} = \mathbf{64.58\ m}$

$\mathbf{V_2} = [28^2 + 4.52^2]^{0.5} = \mathbf{28.36\ m/s}$

**Static pressure rise in** the impeller

$$= 64.58 - \frac{28.36^2}{2 \times 9.81} = 64.58 - 41 = \mathbf{25.58\ m}$$

**Ratio of power :** $\qquad FB/BB = \dfrac{64.58}{39.72} = \mathbf{1.63}$

**Ratio of velocities at outlet :** $\qquad FB/BB = \dfrac{28.36}{17.81} = \mathbf{1.59}$

**Static pressure rise is found to be nearly equal.**

**Problem 8.24** *A centrifugal pump when tested with Brine of density 1190 kg/m $^3$ discharged 60 l/s against a pressure of 300 kPa. It is desired to investigate the change in power when a similar pump is used to pump petrol of density 700 kg/m$^3$ against the same pressure. It is desired to keep the speed the same.* ***Check whether any change in the drive motor is required.***

Assume an overall efficiency of 70% in both cases.

**With Brine :**

$$\text{Head developed} = \frac{300 \times 10^3}{1190 \times 9.81} = 25.7 \text{ m}$$

$$\textbf{Power} = \frac{60 \times 1.19 \times 9.81 \times 25.7}{0.7 \times 10^3} = \textbf{25.71 kW}$$

**With petrol :**

$$\text{Head developed} = \frac{300 \times 10^3}{700 \times 9.81} = 43.69 \text{ m}$$

$$\textbf{Power} = \frac{60 \times 0.7 \times 9.81 \times 43.69}{0.7 \times 10^3} = \textbf{25.71 kW}$$

**There is no need to change the motor.**

As long as $\gamma H$ is the same, other conditions remaining constant, the power will be the same.

**Problem 8.25** *A centrifugal pump was tested for cavitation initiation. Total head was 40 m and flow rate was 0.06 m$^3$/s. Cavitation started when the total head at the suction side was 3 m. The atmospheric pressure was 760 mm Hg and the vapour pressure at this temperature was 2 kPa. It was proposed to install the pump where the atmospheric pressure is 700 mm Hg and the vapour pressure at the location temperature is 1 kPa. If the pump develops the same total head and flow, can the pump be fixed as the same height as the lab setup ?* ***What should be the new height.***

It is necessary to consider the suction point

Total head = Vapour pressure + velocity head.

∴ Velocity head = Total head – Vapour pressure in head of water

$$\therefore \quad \frac{V_s^2}{2g} = 3 - \frac{2 \times 10^3}{10^3 \times 9.81} = 2.769 \text{ m}$$

Cavitation parameter σ is defined by

$$\sigma = \frac{V_s^2}{2g\,H} = 2.796/40 = 0.0699$$

Considering the point at the sump level and the suction point

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{\mathrm{P_{atm}}}{\gamma} - h_f$$

$$\therefore \qquad (Z_1 + h_f) = \frac{\mathrm{P_{atm}}}{\gamma} - \sigma H - \frac{P_v}{\gamma}$$

$$= \left(\frac{760 \times 13.6}{1000}\right) - 2.796 - \frac{2 \times 10^3}{10^3 \times 9.81} = 7.336 \text{ m}$$

At the new location (head and flow being the same, friction loss will be the same)

$$(Z' + h_f') = \frac{700 \times 13.6}{1000} - 2.796 - \frac{1 \times 10^3}{10^3 \times 9.81} = 6.622 \text{ m}$$

As $\qquad h_f = h_f'$

$$Z - Z' = 0.716 \text{ m}$$

The pump cannot be set at the same height.

It should be lowered by **0.716 m**

**The new height is 6.622 m.**

## REVIERW QUESTIONS

1. What are the types of casings used in centrifugal pumps?
2. What are the advantages of centrifugal pumps over reciprocating pumps?
3. What are the advantages of double suction pumps?
4. Define manometric head and manometric efficiency of a centrifugal pump.
5. Explain why priming is necessary to start pumping by centrifugal pump.
6. Define shut off head.
7. List the types of impellers and indicate where each of them are used.
8. Explain the functions of a foot value. Indicate how it works.
9. Explain what is meant by slip. What are the effects of slip?
10. What is cavitation? Where does it occur in centrifugal pumps?
11. Define critical cavitation parameter and write down the expression for the same.
12. Explain why backward curved blades are more popularly used.

## OBJECTIVE QUESTIONS

**I. Choose the correct answer**

1. Manometric head of a centrifugal pump is given by
   (*a*) Static head + losses  (*b*) Static head
   (*c*) $u_2 V_{u2} / g$  (*d*) Static head + losses + exit kinetic head.
2. The dimensionless specific speed of a centrifugal pump
   (*a*) $\dfrac{N\sqrt{P}}{H^{3/4}}$  (*b*) $\dfrac{N\sqrt{Q}}{H^{5/4}}$  (*c*) $\dfrac{N\sqrt{Q}}{(gH)^{3/4}}$  (*d*) $\dfrac{N\sqrt{Q}}{H^{3/4}}$.

# *Chapter-9* Reciprocating Pumps

## INTRODUCTION

There are two main types of pumps namely the dynamic and positive displacement pumps. Dynamic pumps consist of centrifugal, axial and mixed flow pumps. In these cases pressure is developed by the dynamic action of the impeller on the fluid. Momentum is imparted to the fluid by dynamic action. This type was discussed in the previous chapter. Positive displacement pumps consist of reciprocating and rotary types. These types of pumps are discussed in this chapter. In these types a certain volume of fluid is taken in an enclosed volume and then it is forced out against pressure to the required application.

## COMPARISON

| Dynamic pumps | Positive displacement pumps |
|---|---|
| 1. Simple in construction. | More complex, consists of several moving parts. |
| 2. Can operate at high speed and hence compact. | Speed is limited by the higher inertia of the moving parts and the fluid. |
| 3. Suitable for large volumes of discharge at moderate pressures in a single stage. | Suitable for fairly low volumes of flow at high pressures. |
| 4. Lower maintenance requirements. | Higher maintenance cost. |
| 5. Delivery is smooth and continuous. | Fluctuating flow. |

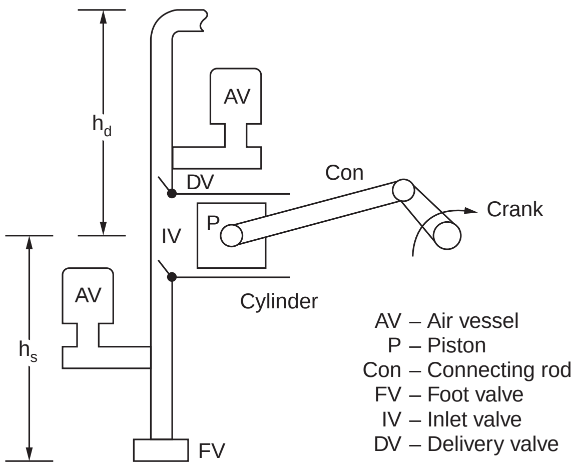
## DESCRIPTION AND WORKING

The main components are:

1. Cylinder with suitable valves at inlet and delivery.
2. Plunger or piston with piston rings.
3. Connecting rod and crank mechanism.

4. Suction pipe with one way valve.
5. Delivery pipe.
6. Supporting frame.
7. Air vessels to reduce flow fluctuation and reduction of acceleration head and friction head.

A diagramatic sketch is shown in Fig. 16.2.1.




The action is similar to that of reciprocating engines. As the crank moves outwards, the piston moves out creating suction in the cylinder. Due to the suction water/fluid is drawn into the cylinder through the inlet valve. The delivery valve will be closed during this outward stroke. During the return stroke as the fluid is incompressible pressure will developed immediately which opens the delivery valve and closes the inlet valve. During the return stroke fluid will be pushed out of the cylinder against the delivery side pressure. The functions of the air vessels will be discussed in a later section. The volume delivered per stroke will be the product of the piston area and the stroke length. In a single acting type of pump there will be only one delivery stroke per revolution. Suction takes place during half revolution and delivery takes place during the other half. As the piston speed is not uniform (crank speed is uniform) the discharge will vary with the position of the crank. The discharge variation is shown in figure

In a single acting pump the flow will be fluctuating because of this operation.

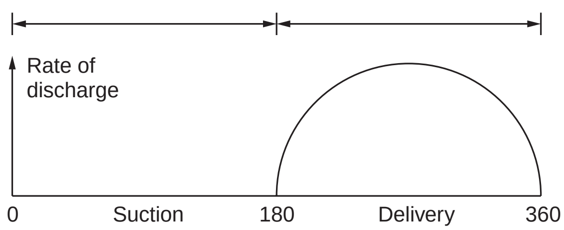



**Figure** Flow variation during crank movement of single acting pump

Fluctuation can be reduced to some extent by double acting pump or multicylinder pump.

The diagramatic sketch of a double acting pump is shown in figure

In this case the piston cannot be connected directly with the connecting rod. A gland and packing and piston rod and cross-head and guide are additional components. There will be nearly double the discharge per revolution as compared to single acting pump. When one side of the piston is under suction the other side will be delivering the fluid under pressure. As can be noted, the construction is more complex.

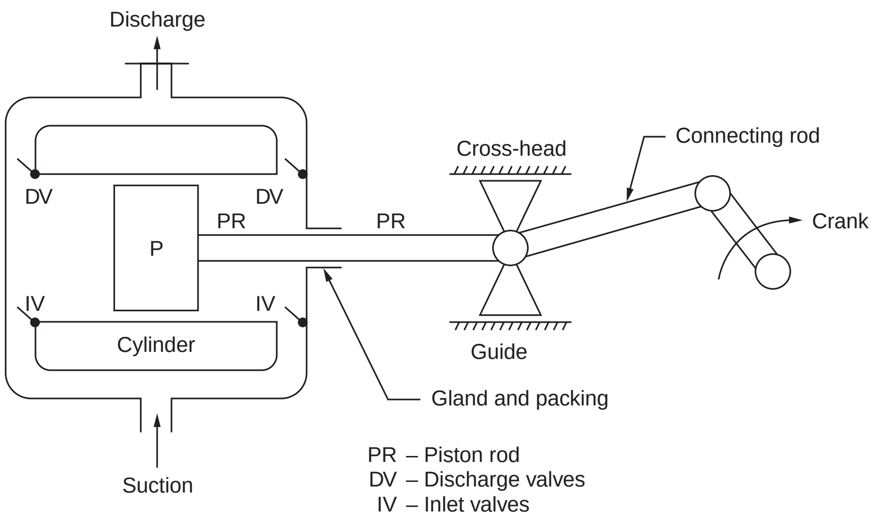



**Figure** Diagramatic view of a double action pump

## FLOW RATE AND POWER

Theoretical flow rate per second for single acting pump is given by, $Q_{SA} = \dfrac{L\,A\,N}{60}$ m$^3$/s

Where $L$ is the length of stroke, $A$ is the cylinder or piston area and $N$ is the revolution per minute. It is desirable to express the same in terms of crank radius and the angular velocity as simple harmonic motion is assumed.

$$\omega = \frac{2\pi N}{60},\ N = \frac{60\,\omega}{2\pi},\ r = \frac{L}{2}$$

$$Q_{SA} = \frac{2r\,.\,A \times 60\,\omega}{2\pi \times 60} = \frac{A\omega r}{\pi}\ \text{m}^3/\text{s} \tag{9.3.1a}$$

In double acting pumps, the flow will be nearly twice this value. If the piston rod area is taken into account, then

$$Q_{DA} = \frac{A\,L\,N}{60} + (A - A_{pr})\,\frac{L\,N}{60}\ \text{m}^3/\text{s} \tag{9.3.2}$$

Compared to the piston area, the piston rod area is very small and neglecting this will lead to an error less than 1%.

$$\therefore \qquad Q_{DA} = \frac{2A\,L\,N}{60} = \frac{2A\,w\,r}{\pi} \text{ m}^3\text{/s.} \tag{9.3.2a}$$

For multicylinder pumps, these expressions, (16.3.1), (16.3.1*a*), (16.3.2), and (16.3.2*a*) are to be multiplied by the number of cylinders.

## Slip

There can be leakage along the valves, piston rings, gland and packing which will reduce the discharge to some extent. This is accounted for by the term slip.

$$\text{Percentage of slip} = \frac{Q_{th} - Q_{ac}}{Q_{th}} \times 100$$

Where $Q_{th}$ is the theoretical discharge given by equation (16.3.1) and 2 and $Q_{ac}$ is the measured discharge.

$$\text{Coefficient of discharge, } C_d = \frac{Q_{ac}}{Q_{th}}$$

It has been found in some cases that $\mathbf{Q_{ac} > Q_{th}}$, due to operating conditions. In this case the slip is called **negative slip**. When the delivery pipe is short or the delivery head is small and the accelerating head in the suction side is high, the delivery valve is found to open before the end of suction stroke and the water passes directly into the delivery pipe. Such a situation leads to negative slip.

Theoretical power $= mg(h_s + h_d)$ W

where $m$ is given by $Q \times \delta$. (9.3.5)

**Example.1** *A single acting reciprocating pump has a bore of 200 mm and a stroke of 350 mm and runs at 45 rpm. The suction head is 8 m and the delivery head is 20 m. Determine the theoretical discharge of water and power required. If slip is 10%, what is the actual flow rate ?*

Theoretical flow volume $Q = \frac{L\,A\,N}{60} = \frac{0.35 \times \pi \times 0.2^2}{4} \times \frac{45}{60}$

$= 8.247 \times 10^{-3}$ m$^3$/s or 8.247 l/s or 8.247 kg/s

Theoretical power = (mass flow/s) × head in $m \times g$ Nm/s or $W$

$= 0.9 \times 8.247 \times (20 + 8) \times 9.81$

= **2039 W or 2.039 kW**

$$\text{Slip} = \frac{Q_{th} - Q_{ac}}{Q_{th}},\ 0.1 = \frac{8.247 - Q_{ac}}{8.247}$$

$\therefore$ $\mathbf{Q_{actual} = 7.422}$ **l/s**

The actual power will be higher than this value due to both solid and fluid friction.

**Example .2** *A double acting reciprocating pump has a bore of 150 mm and stroke of 250 mm and runs at 35 rpm. The piston rod diameter is 20 mm. The suction head is 6.5 m and the delivery head is 14.5 m. The discharge of water was 4.7 l/s. Determine the slip and the power required.*

$$\mathbf{Q} = \frac{L\,A_1\,N}{60} + \frac{L\,A_2\,N}{60} = \frac{L\,N}{60}\,[A_1 + A_2]$$

$$= \frac{0.25 \times 35}{60}\left[\frac{\pi \times 0.15^2}{4} + \frac{\pi}{4}\,(0.15^2 - 0.02^2)\right]$$

$$= \frac{0.25 \times 35 \times \pi}{60 \times 4}\,[2 \times 0.15^2 - 0.02^2]$$

$$= \mathbf{5.108 \times 10^{-3}\ m^3/s\ or\ 5.108\ l/s\ or\ 5.108\ kg/s}$$

It piston rod area is not taken into account

$$Q = 5.154 \text{ l/s.}$$

An error of 0.9% rather negligible.

$$\mathbf{Slip} = \frac{5.108 - 4.7}{5.108} \times 100 = \mathbf{7.99\%}$$

**Theoretical power** $= m\,g\,h = 4.7 \times 9.81 \times (14.5 + 6.5)\text{ W} = \mathbf{968\ W}$

The actual power will be higher than this value due to mechanical and fluid friction.

## INDICATOR DIAGRAM

The pressure variation in the cylinder during a cycle consisting of one revolution of the crank. When represented in a diagram is termed as indicator diagram. The same is shown in figure

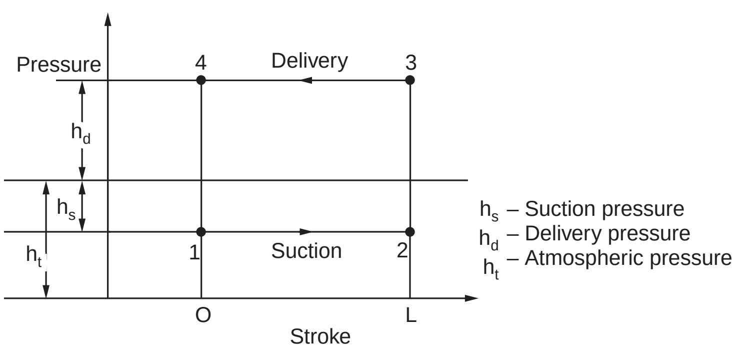



**Figure** Indicator diagram for a crank revolution

Figure represents an ideal diagram, assuming no other effects are involved except the suction and delivery pressures. Modifications due to other effects will be discussed later in the section.

Point 1 represents the condition as the piston has just started moving during the suction stroke. 1-2 represents the suction stroke and the pressure in the cylinder is the suction pressure below the atmospheric pressure. The point 3 represents the condition just as the piston has started moving when the pressure increases to the delivery pressure. Along 3-4 representing the delivery stroke the pressure remains constant. The area enclosed represents the work done during a crank revolution to some scale

$$\text{Power} = Q\,\rho\,g(h_s + h_d) = \rho\,g\,L\,A\,N\,(h_s + h_d)/60 \tag{9.4.1}$$

## Acceleration Head

The piston in the reciprocating pump has to move from rest when it starts the suction stroke. Hence it has to accelerate. The water in the suction pipe which is also not flowing at this point has to be accelerated. Such acceleration results in a force which when divided by area results as pressure. When the piston passes the mid point, the velocity gets reduced and so there is retardation of the piston together with the water in the cylinder and the pipe. This again results in a pressure. These pressures are called acceleration pressure and is denoted as head of fluid ($h = P/\rho g$) for convenience. Referring to the figure 9.4.2 shown below the following equations are written.

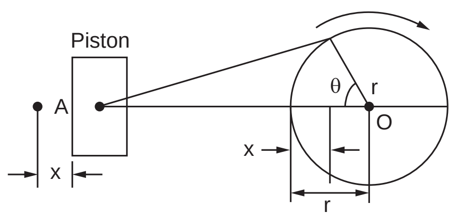



**Figure** Piston Crank Configuration

Let ω be the angular velocity.

Then at time $t$, the angle travelled $\theta = \omega t$

Distance $\quad x = r - r\cos\theta = r - r\cos\omega t$

Velocity at this point,

$$v = \frac{dx}{dt} = \omega r \sin wt \tag{9.4.2}$$

The acceleration at this condition

$$\ddot{x} = \frac{dv}{dt} = \omega^2 r \cos wt \tag{9.4.3}$$

This is the acceleration in the cylinder of area $A$. The acceleration in the pipe of area $a$ is

$$= \frac{A}{a}\omega^2\, r\cos\omega t. \tag{9.4.4}$$

Accelerating force = mass × acceleration

$$\text{mass in the pipe} = \rho\, al \text{ kg} = \frac{\gamma al}{g}$$

$$\therefore \quad \text{Acceleration force} = \frac{\gamma al}{g} \times \frac{A}{a} \omega^2 r \cos \omega t \tag{9.4.5}$$

$$\text{Pressure} = \text{force/area}$$

$$= \frac{ral}{g} \cdot \frac{1}{a} \cdot \frac{A}{a} \omega^2 r \cos \omega t$$

$$= \frac{rl}{g} \cdot \frac{A}{a} \omega^2 r \cos \theta$$

$$\text{Head} = \text{Pressure}/\gamma$$

$$h_a = \frac{l}{g} \cdot \frac{A}{a} \omega^2 r \cos \theta$$

This head is imposed on the piston in addition to the static head at that condition. This results in the modification of the indicator diagram as shown in figure .

(*i*) Beginning of suction stroke: $\theta = 0$, $\cos \theta = 1$

$$\therefore \quad h_{as} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r$$

This is over and above the static suction head. Hence the pressure is indicated by 1′ in the diagram.

(*ii*) Middle of stroke: $\theta = 90$ $\therefore$ $h_{as} = 0$. There is no additional acceleration head.

(*iii*) End of stroke: $\theta = 180$. $\cos \theta = -1$

$$\therefore \quad h_{as} = -\frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r$$

This reduces the suction head. Hence the pressure is indicated at 2′ in the diagram.

Similarly during the beginning of the delivery stroke

$$\theta = 0, \cos \theta = 1$$

$$h_{ad} = \frac{l_d}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r$$

This head is over and above the static delivery pressure. The pressure is indicated by point 3′ in the diagram. At the middle stroke $h_{ad} = 0$. At the end of the stroke $h_{ad} = -\frac{l_a}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r$.

This reduces the pressure at this condition and the same is indicated by 4′, in the diagram.

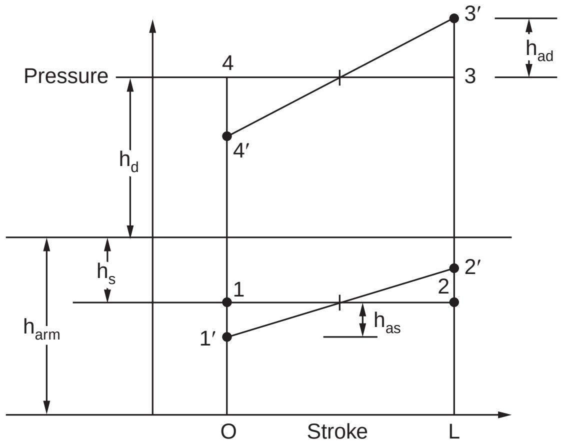



**Figure** Modified indicator diagram due to acceleration head

The effect of acceleration head are:

1. No change in the work done.

2. Suction head is reduced. This leads to the problem of separation in suction pipe in case the pressure at 1′ is around 2.5 m of head of water (absolute). As the value depends on ω which is directly related to speed, the speed of operation of reciprocating pumps is limited. Later it will be shown than the installation of an air vessel alleviates this problem to some extent.

**Example .3** *Calculate and tabulate the distance travelled as a fraction of stroke length as the crank rotates from 0 to 180°.*

$$\text{Distance travelled} = r - r\cos\theta$$

$$\text{Stroke} = 2r.$$

∴ Distance travelled as a fraction of, $L$

$$= \frac{r - r\cos\theta}{2r} = \frac{1-\cos\theta}{2}$$

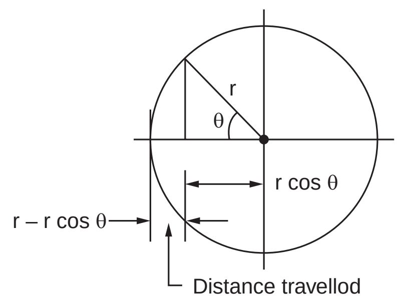



**Figure Ex. (a)**

Values are calculated for 0, 30, 60, 90, 120, 150, 180° and tabulated below.

| Angle | 0 | 30 | 60 | 90 | 120 | 150 | 180 |
|---|---|---|---|---|---|---|---|
| Distance moved | 0 | .0677 | 0.250 | 0.5 | 0.75 | 0.933 | 1 |

**Note:** The distances of piston movement is not uniform with crank angle.

For the data, speed = 40 rpm and $r$ = 0.15 m. calculate the velocity and acceleration as the crank moves from one dead centre to the next.

$$\text{Velocity} = \omega r \sin\theta = \frac{2\pi N}{60} \,.\, r\sin\theta$$

$$\text{Acceleration} = \omega^2 r\cos\theta = \left(\frac{2\pi N}{60}\right)^2 .\, r \,.\, \cos\theta$$

The values are calculated using the specified data and tabulated below.

| Angle | 0 | 30 | 60 | 90 | 120 | 150 | 180 |
|---|---|---|---|---|---|---|---|
| Velocity m/s | 0 | 0.314 | 0.544 | 0.628 | 0.544 | 0.314 | 0 |
| Acceleration $m/s^2$ | 2.632 | 2.279 | 1.316 | 0 | – 1.316 | – 2.279 | – 2.632 |

**Note:** The velocity follows sine curve and acceleration the cosine curve.

The acceleration is highest at start of stroke and decreases up to the middle of stroke and becomes zero and then decelerates at an increasing rate.

This can be illustrated as below.

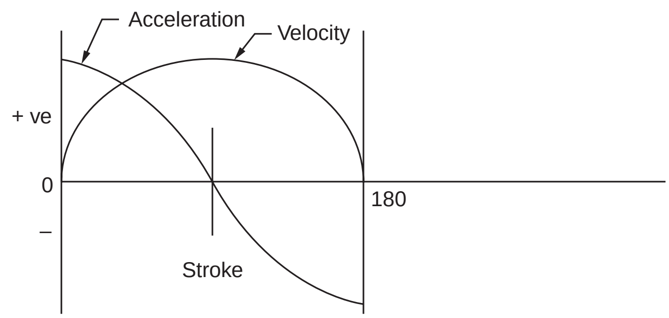



**Figure Ex. (b)** Velocity and acceleration plot during a stroke

**Example .4** *A single acting reciprocating pump of 200 mm bore and 300 mm stroke runs at 30 rpm. The suction head is 4 m and the delivery head is 15 m. Considering acceleration determine the pressure in the cylinder at the beginning and end of suction and delivery strokes. Take the value ofatmospheric pressure as 10.3 m of water head. The length of suction pipe is 8 m and that of deliverypipe is 20 m. The pipe diameters are 120 mm each.*

Acceleration head on the suction side, $h_{as} = \frac{l_s}{g}\frac{A}{a_s} . \omega^2 r$

$A$ – piston diameter, $a_s$ – pipe diameter, $r = L/2$

$$\frac{A}{a_s} = \frac{\pi \times 0.2^2}{4} \times \frac{4}{\pi \times 0.12^2} = 2.78,\ \omega = \frac{2\pi N}{60}$$

$$\therefore \qquad \mathbf{h_{as}} = \frac{8}{9.81} \times 2.78 \times \left(\frac{2 \times \pi \times 30}{30}\right)^2 \times 0.15 = \mathbf{3.35\ m}$$

At start of suction, $H_{BS} = h_{atm} - h_s - h_{as}$ = 10.3 – 4 – 3.35 = 2.95 m absolute or 7.05 m vacuum.

At end of suction $H_{es}$ = 10.3 – 4 + 3.35 = **9.65 m absolute or** 0.65 m vacuum.

$$h_{ad} = \frac{20}{9.81} \times 2.78 \times \left(\frac{2\pi \times 30}{60}\right)^2 \times 0.15 = \mathbf{8.38\ m\ of\ water\ column}$$

At starting of delivery, $\mathbf{H_{Bd}}$ = 10.3 + 15 +8.38 = **33.68 m absolute or 23.38 m gauge**

At end of delivery, $\mathbf{H_{ed}}$ = 10.3 + 15 – 8.38 = **16.92 m absolute or 6.62 m gauge**

This applies for both single acting and double acting pumps.

## Minimum Speed of Rotation of Crank

During the suction stroke, the head at the suction side is given by

$$h = h_{atm} - h_s - h_{as}$$

In case this head is below 2.5 m of head of water, water may vaporise at this point and the flow will be disrupted causing separation in the liquid column. Pumping will be discontinuous.

In order to avoid this, the acceleration head which can be changed should be limited. As this depends on the speed there is a limitation to the operating speed.

During delivery stroke also, there is a possibility of separation which may be caused by the layout of the delivery pipe. Two alternatives are shown in figure . The first method is to have a horizontal bend at the pump level and then to have the vertical line. In this case separation is avoided as at the bend the column of water above it exerts a pressure above 2.5 m (absolute). In the second arrangement the pressure at the bend is given by $(h_{atm} - h_{ad})$ and this may be below 2.5 m of water hence the preferred arrangement is to have a horizontal bend immediately after the pump.

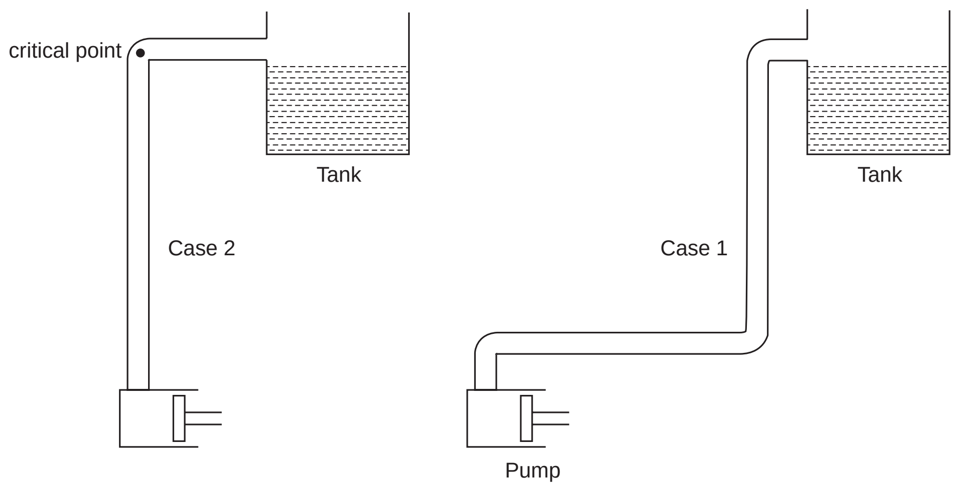



**Figure** Delivery pipe arrangement

**Example .5** *A single acting reciprocating pump of 200 mm plunger diameter and 300 mm stroke length has a suction head of 4 m. The suction pipe diameter is 110 mm and is 9 m long. The pressure at the beginning of suction should be above 2.5 m water column (absolute) to avoid separation. Determine the highest speed at which the pump can operate.*

At the beginning the pressure required is 2.5 m. This equals the difference between the absolute pressure and the sum of suction and accelerating head.

$$\therefore \quad 2.5 = 10.3 - 4 - h_{as} \quad \therefore \quad h_{as} = 3.8 \text{ m}$$

$$h_{as} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r \quad (\cos\theta = 1)$$

$$3.8 = \frac{9}{9.81} \times \frac{\pi \times 0.2^2}{4} \times \frac{4}{\pi \times .011^2} . \omega^2 \times 0.15$$

Solving $\omega^2 = 8.353$, $\omega = 2.89$ radians/second.

$$\omega = \frac{2\pi N}{60}, \quad \mathbf{N} = \frac{\omega \times 60}{2\pi} = \frac{2.89 \times 60}{2 \times \pi} = \mathbf{27.6\ rpm}$$

This will be the same for double acting pump also.

**Example .6** *The delivery pipe of a reciprocating pump is taken vertically up and then given a horizontal bend. The pump diameter is 180 mm and the stroke is 300 mm. The pipe diameter is 100 mm and the length is 18 m. The speed is 30 rpm. Check whether separation will occur at the bend. Separation is expected to take place if the absolute pressure is 2.5 m head. Atmospheric pressure may be taken as 10.3 m head of water.*

At the top of the pipe, the static head is zero. The only pressure is the accelerating head. To avoid separation $(P_{atm} - P_a) > 2.5$ m.

$$\mathbf{P_a} = \frac{l}{g} . \frac{A}{a} . \omega^2 r = \frac{18}{9.81} \times \left( \frac{\pi \times 0.18^2}{4} \times \frac{4}{\pi \times 0.1^2} \right) \times \left( \frac{2\pi \times 30}{60} \right)^2 \times 0.15$$

$$= \mathbf{8.8\ m}$$

**Pressure P at the bend = 10.3 – 8.8 = 1.5 m**

Hence separation will occur at the bend.

In the above problem if the pipe is taken first horizontally and then vertically, what will be the pressure at the bend. The delivery pressure is 15 m. The pressure at the bend will be the sum of the atmospheric pressure and the static pressure minus the acceleration head. The acceleration head itself is 8.8 m head.

$$P = 10.3 + 15 - 8.8 = 16.5 \text{ m head.}$$

Hence the arrangement is safe against separation.

## Friction Head

When air vessels are not fixed in a pump, the velocity variation of water in the cylinder is given by equation (16.4.2)

$$v = \omega r \sin \omega t = \omega r \sin \theta$$

In the pipe the velocity variation will be in the ratio of areas.

$$\therefore \qquad v_p = \frac{A}{a} . \omega r \sin\theta$$

Friction head $\qquad h_f = f\, l\, v^2 / 2gd$

$$= \frac{fl}{2gd} . \left( \frac{A}{a} . \omega r \sin\theta \right)^2 \tag{9.4.7}$$

The maximum value of friction heed

$$h_{f\,max} = \frac{fl}{2gd} . \left( \frac{A}{a} . \omega r \right)^2 \tag{9.4.8}$$

At the beginning of the stroke $\theta = 0°$ $\therefore$ $h_f = 0$

At middle of stroke, $\theta = 90°$ and $h_f = h_{f\,max}$

At end of stroke, $\theta = 180°$ and $h_f = 0$.

The friction head leads to another modification of the indicator diagram shown in figure 9.4.5. With $h_s$ and $h_d$ as static suction and delivery heads, the pressure at the various locations are indicated below.

Suction stroke: At the start, head = $h_s + h_{as}$

At middle position, head = $h_s + h_{fs}$

At the end, $h = h_s - h_{as}$

Delivery stroke: At the start, head = $h_d + h_{ad}$

At middle position, head = $h_d + h_f$

At the end, head = $h_d - h_{ad}$.

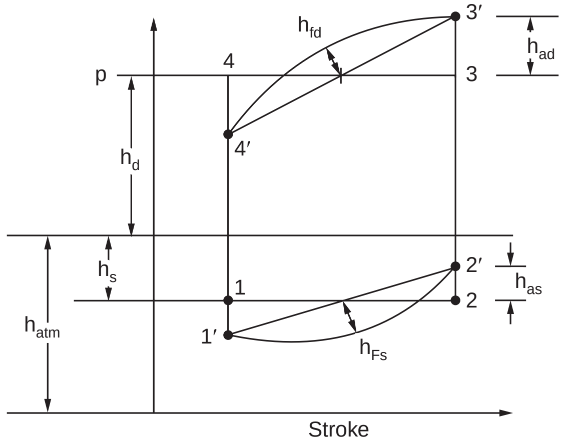



**Figure** Variation of indicator diagram taking pipe friction into account

It can be observed that friction head increases the work done as seen by the increased area enclosed in the indicator diagram. The introduction of air vessels will reduce the friction work considerably. In order to calculate the work done, it will be desirable to calculate the average friction head. As the variation is parabolic, the average is found to be 2/3 of the maximum.

$$h_{fav} = 2/3\ h_{f\,max} \tag{9.4.9}$$

The total head against which work is done equals

$$h_{to} = h_s + h_d + 2/3\ h_{f\,max\,s} + 2/3\ h_{f\,max\,d} \tag{9.4.10}$$

The addition work due to friction is given by

$$2/3\ Q\,\rho\,g\,(h_{fs} + h_{fd}) \tag{9.4.11}$$

Later it will be seen that the use of air vessels causes the velocity in the pipe to be constant without fluctuations and this reduces the work to be overcome by friction.

**Example.7** *A single acting pump with 200 mm bore diameter and 320 mm stroke runs at 30 rpm. The suction pipe diameter is 110 mm. The delivery pipe diameter is 100 mm. The suction and delivery pipes are 10 m and 22 m long. The friction factor is 0.01. Determine the frictional head at the suction and delivery.*

Assume no air vessels are fitted.

$$h_f = \frac{f\,lV^2}{2gd} = \frac{4f\,l}{2gd}\left(\frac{A}{a}\,.\,\omega r\sin\theta\right)^2$$

Maximum occurs at the middle of stroke.

$$\mathbf{h_{fs\ max}} = \frac{4\times 0.01\times 10}{2\times 9.81\times 0.11}\left[\frac{200^2}{110^2}\,.\,\frac{2\pi\times 30}{60}\times 0.15\,.\sin 90\right]^2$$

$$= \mathbf{0.45\ m\ \ head}$$

$$\mathbf{h_{fd\ max}} = \frac{4\times 0.01\times 22}{2\times 9.81\times 0.1}\left[\frac{200^2}{100^2}\times\frac{2\pi\times 30}{60}\times 0.15\,.\sin 90\right]^2$$

$$= \mathbf{1.59\ m\ head.}$$

**Note:** Two types of equations for frictional head are used:

$$\frac{4f\,lv^2}{2gd} \text{ and } \frac{f\,lv^2}{2gd}$$

When $4f$ is used, $f$ is the coefficient of friction commonly denoted as $c$ . When $f$ alone is used it is called Darcy friction factor and

Darcy friction factor = 4 $c_f$

Now the popular use is the second equation using $f$ from Moody diagram.

## AIR VESSELS

Air vessel is a strong closed vessel as shown in figure 9.5.1. The top half contains compressed air and the lower portion contains water or the fluid being pumped. Air and water are separated by a flexible diaphragm which can move up or down depending on the difference in pressure between the fluids. The air charged at near total delivery pressure/suction pressure from the top and sealed. The air vessel is connected to the pipe lines very near the pump, at nearly the pump level. On the delivery side, when at the beginning and up to the middle of the delivery stroke the head equals $h_s + h_f + h_a$ , higher than the static and friction heads. At this time part of the water from pump will flow into the air vessel and the remaining will flow through the delivery pipe. This will increase the compressed air pressure. At the middle stroke position the head will be sufficient to just cause flow. The whole of the flow from pump will flow to the delivery pipe. At the second half of the stroke the head will be equal to $h_s + h_f - h_a$. At the position the head will be not sufficient to cause flow. The compressed air pressure will act on the water and water charged earlier into the air vessel will now flow out. Similar situation prevails on the suction side. At the start and up to the middle of the suction stroke the head at the pump is higher than static suction head by the amount of acceleration head. The flow will be more and part will flow into the air vessel. The second half of the stroke water will flow out of the air vessel. In this process the velocity of water in the delivery pipe beyond the air vessel is uniform, and lower than the maximum velocity if air vessel is not fitted. Similar situation prevails in the suction side also. The effect is not only to give uniform flow but reduce the friction head to a considerable extent saving work. Without air vessel the friction head increases, reaches a maximum value at the mid stroke and then decreases to zero. With air vessel the friction head is lower and is constant throughout the stroke. This is due to the constant velocity in the pipe.

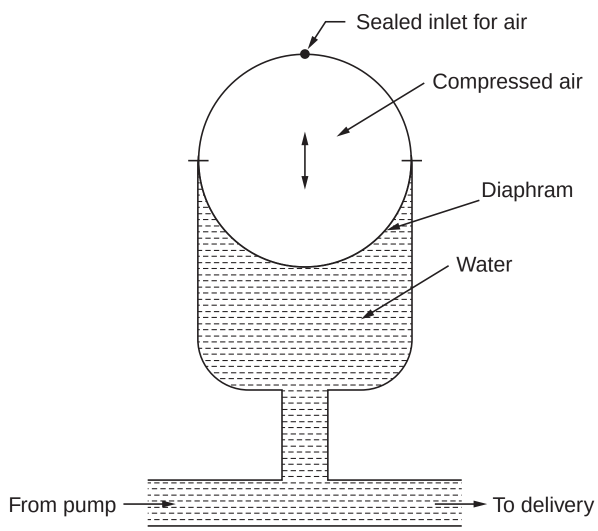



**Figure** Air vessel

The advantages of installing air vessels are:

(*i*) The flow fluctuation is reduced and a uniform flow is obtained.

(*ii*) The friction work is reduced.

(*iii*) The acceleration head is reduced considerably.

(*iv*) Enables the use of higher speeds.

The maximum friction head of water without air vessel, refer eqn. (9.4.8),

$$h_{f\,max} = \frac{4fl}{2gd}\left(\frac{A}{a}\,.\,\omega r\right)^2$$

The average friction head = 2/3 $h_{f\,max}$. (refer eqn. 9.4.9). When the air vessel is placed near the pump, the uniform velocity,

$$v = \frac{A}{a}\,.\,\frac{LN}{60} = \frac{A}{a}\,.\,2r\,.\,\frac{60\omega}{2\pi} = \frac{A}{a}\,.\,\frac{\omega r}{\pi} \tag{9.5.1}$$

$$h_f = \frac{4fl}{2gd}\,.\left(\frac{A}{a}\,.\,\frac{\omega r}{\pi}\right)^2 \tag{9.5.2}$$

$$\frac{h_f}{h_f a_v} = \frac{3}{2} \cdot \frac{1}{\pi^2} = 0.152 \qquad (9.5.3)$$

$\therefore$ Reduction is 84.8%

Naturally the work done due to friction will reduce by this percentage.

**Example .8** *In a single acting reciprocating pump with plunger diameter of 120 mm and stroke of 180 mm running at 60 rpm, an air vessel is fixed at the same level as the pump at a distance of 3 m. The diameter of the delivery pipe is 90 mm and the length is 25 m. Friction factor is 0.02. Determine the reduction in accelerating head and the friction head due to the fitting of air vessel.*

Without air vessel :

$$\mathbf{h_{ad}} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r = \frac{25}{9.81} \cdot \frac{0.12^2}{0.09^2} \cdot \left(\frac{2\pi \times 60}{60}\right)^2 \times 0.09$$

$$= \mathbf{16.097\ m}$$

With air vessel :

$$\mathbf{h'_{ad}} = \frac{3}{9.81} \cdot \frac{0.12^2}{0.09^2} \cdot \left(\frac{2\pi \times 60}{60}\right)^2 \times 0.09 = \mathbf{1.932\ m}$$

$$\textbf{Reduction} = 16.097 - 1.932 = \mathbf{14.165\ m}$$

Fitting air vessel reduces the acceleration head.

Without air vessel :

$$\text{Friction head} \quad h_f = \frac{4\,f\,l.V^2}{2gd} = \frac{4f\,l}{2gd}\left[\frac{A}{a} \cdot \omega r \sin\theta\right]^2$$

At $\theta = 90°$

$$\mathbf{h_{f\,max}} = \frac{4 \times 0.02 \times 25}{2 \times 9.81 \times 0.09}\left[\frac{0.12^2}{0.09^2} \cdot \frac{2\pi \times 60}{60} \times 0.09 \times 1\right]^2 = \mathbf{1.145\ m}$$

With air vessel, the velocity is constant in the pipe.

$$\text{Velocity} = \frac{A\,L\,N}{60} \times \frac{4}{\pi\,.\,d^2} = \frac{\pi \times 0.12^2}{4} \times \frac{0.18 \times 60 \times 4}{60 \times \pi \times 0.09^2}$$

$$= 0.102 \text{ m/s}$$

$$\text{Friction head} = \frac{4 \times 0.02 \times 25}{2 \times 9.81 \times 0.09} \times 0.102^2 = \mathbf{0.012\ m}$$

Percentage saving over maximum

$$= \frac{1.145 - 0.012}{1.145} \times 100 = 99\%$$

Air vessel reduces the frictional loss.

## Flow into and out of Air vessel

**Single acting pump:** The flow into the delivery side is only during half a revolution. This amount has to flow during the full revolution:

The average velocity in the pipe = $\left(\frac{L\,N}{60} \times \frac{A}{a}\right)$.

**Double acting pump:** There are two discharger per revolution. The average velocity in this case = $\frac{2L\,N}{60} \times \frac{A}{a}$.

Hence the frictional head will be different in single acting and double acting pumps. This is illustrated in figure

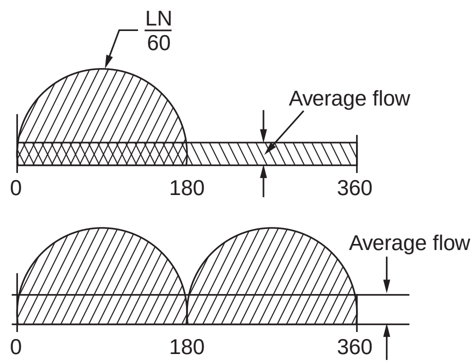



**Figure** Pipe flow in single acting and double acting pumps

Single acting pump :

The flow from cylinder, $Q = A\,\omega r \sin\theta$

With air vessel the average velocity (refer 16.5.1) = $\frac{A}{a} \cdot \frac{\omega r}{\pi}$

Flow through pipe $= a \cdot \frac{A}{a} \cdot \frac{\omega r}{\pi} = \frac{A\,\omega r}{\pi}$

Flow into or out of the air vessel with θ = 0 to 360 = $A\,\omega r\ \sin\theta - \frac{A\,\omega r}{\pi}$

$$= A\,\omega r\left(\sin\theta - \frac{1}{\pi}\right) \tag{9.5.4}$$

Double acting pump: Flow through the pump = $\frac{2A\,\omega r}{\pi}$

At any point of time flow from cylinder = $A\,\omega r \sin\theta$

Flow into air vessel = $A\,\omega r\left(\sin\theta - \frac{2}{\pi}\right)$ (9.5.5)

**Example .9** *Determine the rate of flow in and out of the air vessel on the delivery side in a single acting centrifugal pump of 200 mm bore and 300 mm stroke running at 60 rpm. Also find the angle of crank rotation at which there is no flow into or out of the air vessel.*

At any instant of time the flow rate from the pump cylinder is

$$Q = A\,\omega r \sin \theta$$

Beyond the air vessel the velocity in the pipe is constant

$$v = \frac{A}{a} \cdot \frac{\omega r}{\pi}$$

The flow rate $= a \times \dfrac{A}{a} \cdot \dfrac{\omega r}{\pi} = \dfrac{A\,\omega r}{\pi}$

Volume flow rate into the air vessel,

$q$ = Volume flow rate from cylinder – Volume flow rate beyond the air vessel

$$q = A\,\omega r \sin \theta - \frac{A\,\omega r}{\pi} = A\,\omega r \left[\sin \theta - \frac{1}{\pi}\right]$$

In this case

$$\mathbf{q} = \frac{\pi \times 0.2^2}{4} \times \frac{2\pi \times 60}{60} \times \frac{0.3}{2} \left[\sin \theta - \frac{1}{\pi}\right]$$

$$= \mathbf{0.0296} \left[\mathbf{\sin}\ \theta \mathbf{-} \frac{\mathbf{1}}{\pi}\right].$$

| | Delivery | | | | | | | | | | | Suction |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| θ | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | 300 | 330 | 360 |
| $q$ | – 0.318 | 0.182 | 0.548 | .0682 | 0.548 | 0.182 | – 0.318 | – 0.818 | – 1.184 | – 1.318 | – 1.118 | – 0.818 | – 0.318 |

At no flow condition, the quantity within the bracket showed be zero.

or $\boldsymbol{\theta} = \sin^{-1}\left(\dfrac{1}{\pi}\right) = \mathbf{18.56^o}$

Also $\theta = 161.44°$.

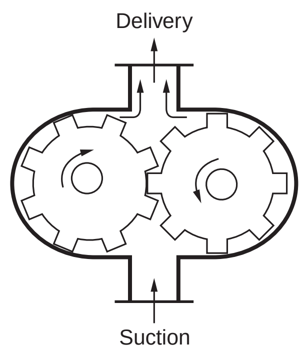
At two locations there is no flow into or out of the air vessel. Similar situation prevails on the suction side also.

## ROTARY POSITIVE DISPLACEMENT PUMPS

In order to avoid the complexity of construction and restriction on speed of the reciprocating pumps, rotary positive displacement pumps have been developed. These can run at higher speeds and produce moderately high pressures. These are very compact and can be made for very low delivery volumes also. These are extensively used for pumping lubricant to the engine parts and oil hydraulic control systems. These are not suited for water pumping. Some types described in this section are: (*i*) Gear pump, (*ii*) Lobe pump and Vane pump.

### Gear Pump

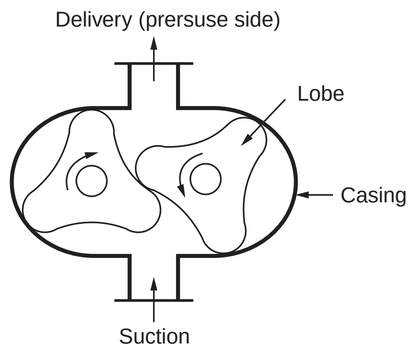
These are used more often for oil pumping. Gear pumps consist of two identical mating gears in a casing as shown in figure 16.6.1. The gears rotate as indicated in the sketch. Oil is trapped in the space between the gear teeth and the casing. The oil is then carried from the lower pressure or atmospheric pressure and is delivered at the pressure side. The two sides are sealed by the meshing teeth in the middle. The maximum pressure that can be developed depends on the clearance and viscosity of the oil. The operation is fairly simple. One of the gear is the driving gear directly coupled to an electric motor or other type of drives.




These pumps should be filled with oil before starting. The sketch shows an external gear pump. There is also another type of gear pump called internal gear pump. This is a little more compact but the construction is more complex and involved and hence used in special cases where space is a premium.

### Lobe Pump

This type is also popularly used with oil. The diagramatic sketch of a lobe pump is shown in figure 16.6.2. This is a three lobed pump. Two lobe pump is also possible. The gear teeth are replaced by lobes. Two lobes are arranged in a casing. As the rotor rotates, oil is trapped in the space between the lobe and the casing and is carried to the pressure side. Helical lobes along the axis are used for smooth operation. Oil has to be filled before starting the pump. Lobe type of compressors are also in use. The constant contact between the lobes makes a leak tight joint preventing oil leakage from the pressure side.




The maximum pressure of operation is controlled by the back leakage through the clearance. This type of pump has a higher capacity compared to the gear pump.

### Vane Pump

This is another popular type not only for oil but also for gases. A rotor is eccentrically placed in the casing as shown in figure 16.6.3. The rotor carries sliding vanes in slots along the length. Springs control the movement of the vanes and keep them pressed on the casing. Oil is trapped between the vanes and the casing. As the rotor rotates the trapped oil is carried to the pressure side. The maximum operating pressure is controlled by the back leakage.

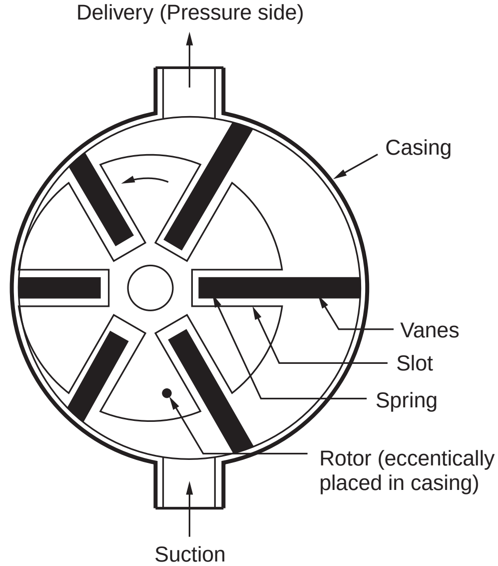



**Figure** Vane pump

## SOLVED PROBLEMS

**Problem .1** *A single acting reciprocating water pump of 180 mm bore and 240 mm stroke operates at 40 rpm. Determine the discharge if the slip is 8%. What is the value of coefficient of discharge. If the suction and delivery heads are 6 m and 20 m respectively determine the theoretical power. If the overall efficiency was 80%, what is the power requirement.*

$$\text{Theoretical discharge} = \frac{A\,L\,N}{60}$$

$$= \frac{\pi \times 0.18^2}{4} \times 0.24 \times \frac{40}{60} = 4.0775 \times 10^{-3}\ \text{m}^3/\text{s}$$

$$= 4.0715\ \text{l/s} = 4.0715\ \text{kg/s}$$

$$\text{Slip} = 8\%$$

$$\therefore \qquad \text{Actual flow} = 4.0715 \times \frac{92}{100} = 3.746\ \text{l/s or kg/s}$$

$$\text{Coefficient of discharge} = \frac{3.746}{4.0715} = 0.92$$

$$\text{Theoretical power} = mgH = 3.746 \times 9.81 \times 26$$

$$= 955.45\ \text{W}$$

$$\text{Actual Power} = 955.45/0.8$$

$$= \mathbf{1194.3\ W\ or\ 1.1943\ kW}$$

If it is a double acting pump, in case the piston rod diameter is neglected, the flow and power will be double this value. The slip and coefficient discharge and efficiency remaining the same.

**Problem.2** *It is desired to have a discharge of water of 10 l/min using a reciprocating pump running at 42 rpm. The bore to stroke ratio is to be 1 : 1.5. It is expected that the slip will be 12%. Determine the bore and stroke for (a) single acting pump, and (b) double acting pump. If the total head is 30 m and the overall efficiency is 82%, determine the power required in both cases.*

**Single acting pump :**

Theoretical discharge $= \dfrac{A\,L\,N}{60}$ m³/s

Actual discharge $= (1 - \text{slip})\,\dfrac{A\,L\,N}{60}$ m³/s

Actual per minute $= (1 - \text{slip}) \times A\,L\,N$

$$A = \frac{\pi D^2}{4}, \qquad L = 1.5\,D,\ (1 - 0.12)\,\frac{\pi D^2}{4} \times 1.5\,D \times 42 = 0.01$$

Solving $\quad D^3 = \dfrac{0.01 \times 4}{0.88 \times \pi \times 1.5 \times 42}$

Solving, $\quad$ **D = 62.7 mm** and **Stroke = 94 mm**

$$\textbf{Power} = \frac{mgh}{\eta} = \frac{10 \times 9.81 \times 30}{0.82 \times 60} = \textbf{60 W}$$

**Double acting :** (Neglecting piston rod diameter)

$$0.01 = (1 - 0.12)\left(\frac{2\pi D^2}{4} \times 1.5\,D \times 42\right)$$

Solving: $\quad$ **D = 48.6 mm, L = 97.2 mm**

The advantage of double acting pump is compactness and lower weight as can be seen form the values.

The power required will be double that of the single acting pump

**P = 120 W.**

**Problem .3** *A reciprocating pump with plunger diameter of 120 mm and 200 mm stroke has both delivery and suction pipes of 90 mm diameter. The suction length is 9 m and the delivery length is 18 m. The atmospheric head is 10.3 m of water head. Determine the suction head and the delivery head due to acceleration at speeds 30, 40, 50, 60 rpm operating speeds.*

**Delivery side:** $\mathbf{h_{a\,max\,d}} = \dfrac{l}{g} \cdot \dfrac{A}{a} \cdot \omega^2 r = \dfrac{18}{9.81} \times \dfrac{0.12^2}{0.09^2}\left(\dfrac{2\pi \times N}{60}\right)^2 \times 0.1$

$= \mathbf{3.577 \times 10^{-3}\,N^2}$

The values are tabulated below

| $N$ rpm | 30 | 40 | 50 | 60 |
|---|---|---|---|---|
| $h_{a\ max\ d}$ | 3.22 | 5.72 | 8.94 | 12.88 |

**Suction side:** $\mathbf{h_{a\ max\ s}} = \dfrac{l}{g} \cdot \dfrac{A}{a} \cdot \omega^2 r = \dfrac{9}{9.81} \times \dfrac{0.12^2}{0.09^2} \left(\dfrac{2\pi \times N}{60}\right)^2 \times 0.1$

$= \mathbf{1.7886 \times 10^{-3}\, N^2}$.

The value are tabulated below for various speeds.

| $N$ rpm | 30 | 40 | 50 | 60 |
|---|---|---|---|---|
| $h_{a\ max\ d}$ | 1.61 | 2.86 | 4.47 | 6.44 |

As the separation limit is 2.5 m absolute, the maximum suction head at 60 rpm (for example) will be equal to (10.3 = 2.5 – 6.44) = 1.36 m only. It can be seen that this speed itself is on the higher side.

**Problem .4** *In a reciprocating pump the bore is 180 mm and stroke is 280 mm. Water level is 5 m from the pump level. The suction pipe is 110 m diameter and 9 m long. The atmospheric pressure head is 10.3 m water. Determine the maximum speed if the head at pipe suction should not be less than 2.5 m head of water. If the suction pipe diameter is increased 125 mm and length reduced to 6 m, what will be the maximum speed ?*

Suction head = 5 m

$r = L/2 = 0.14$ m

Acceleration head available

$= 10.3 - 5 - 2.5 = 2.8$ m

$$h_{as} = \frac{l_s}{g} \cdot \frac{A}{a} \cdot \omega^2 r$$

$$2.8 = \frac{9}{9.81} \times \frac{0.18^2}{0.11^2}\, \omega^2 \times 0.14$$

$\therefore$ $\omega^2 = (2.8 \times 9.81 \times 0.11^2)/(9 \times 0.18^2 \times 0.14) = 8.413$

$$\omega = 2.8533 = \frac{2\pi N}{60}$$

$$\mathbf{N} = \frac{2.8533 \times 60}{2\pi} = \mathbf{27.25\ rpm}$$

Fairly low speed.

**At the changed condition,**

$$2.8 = \frac{6}{9.81} \times \frac{0.18^2}{0.125^2} \cdot \omega^2 \times 0.14$$

Solving, $\omega = 3.9711$

$\therefore$ **N = 37.92 rpm.**

**Problem.5** *In a single acting reciprocating pump the bore and stroke are 90 and 160 mm. The static head requirements are 4 m suction and 15 m delivery. If the pressure at the end of delivery is atmospheric determine operating speed. The diameter of the delivery pipe is 90 mm and the length of the delivery pipe is 22 m. Determine the acceleration head at θ = 30 from the start of delivery.*

In this case, the acceleration head equals the static delivery head.

$$\therefore \quad 15 = \frac{22}{9.81} \times \frac{0.09^2}{0.09^2} \cdot \omega^2 \times 0.08$$

Solving, $\omega = 9.1437$

$$\mathbf{N} = \frac{\omega \times 60}{2\pi} = \frac{9.1437 \times 60}{2 \times \pi}$$

$$= \mathbf{87.32\ rpm}$$

At the position 30° from start of delivery,

$$\mathbf{h_a} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r \cos\theta$$

$$= \frac{22}{9.81} \times \frac{0.09^2}{0.09^2} \cdot 9.1437^2 \times 0.08 \times \cos 30$$

$$\mathbf{= 12.99\ m.}$$

**Problem .6** *A reciprocating pump handling water with a bore of 115 mm and stroke of 210 mm runs at 35 rpm. The delivery pipe is of 90 mm diameter and 25 m long. An air vessel of sufficient volume is added at a distance of 2 m from the pump. Determine the acceleration head with and without the air vessel.*

Without air vessel:

$$\mathbf{h_a} = \frac{l}{g} \cdot \frac{A}{a} \cdot \omega^2 r$$

$$= \frac{25}{9.81} \times \frac{0.115^2}{0.09^2} \times \left(\frac{2\pi \times 35}{60}\right)^2 \times 0.105 = \mathbf{5.869\ m}$$

With air vessel $l$ reduces to 2 m.

$$\therefore \quad \mathbf{h'} = \frac{5.869 \times 2}{25} = \mathbf{0.47\ m}$$

A considerable reduction.

**Problem .7** *The bore and stroke of a reciprocating pump are 10 cm and 15 cm. The pump runs at 40 rpm. The suction pipe is 9 cm diameter and 12 m long. Determine the absolute pressure at suction if static suction is 3.5 m. Take $h_{atm}$ = 10.3 m. If an air vessel is fitted at 1.5 m from the pump determine the absolute pressure at suction.*

Without air vessel:

$$h_{as} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r = \frac{12}{9.81} \times \frac{0.1^2}{0.09^2} \cdot \left(\frac{2\pi \times 40}{60}\right)^2 \times 0.075$$

$= 1.987$ m

**Absolute pressure** $= 10.3 - 3.5 - 1.987 =$ **4.812.** Safe against separation

With air vessel :

$$h_a' = 1.987 \times 1.5/12 = 0.248 \text{ m}$$

**Absolute pressure** $= 10.3 - 35 - 0.248 =$ **6.55 m**

The pump can be run at a higher speed.

**Problem .8** *In a reciprocating pump delivering water the bore is 14 cm and the stroke is 21 cm. The suction lift is 4 m and delivery head is 12 m. The suction and delivery pipe are both 10 cm diameter, length of pipes are 9 m suction and 24 m delivery. Friction factor is 0.015. Determine the theoretical power required. Slip is 8 percent. The pump speed is 36 rpm.*

Volume delivered assuming single acting,

$$= A\,L\,N/60 = \frac{\pi \times 0.14^2}{4} \times 0.21 \times \frac{36}{60}$$

$= 1.9396 \times 10^{-3}$ m$^3$/s or 1.9396 kg/s

Slip is 8%

∴ Actual mass delivered = 1.9396 × 0.92 = 1.784 kg/s

Total static head = 4 + 12 = 16 m head

Friction head in the delivery pipe:

Maximum velocity, $v = \frac{A}{a}\omega r = \frac{0.14^2}{0.1^2} \times \frac{2\pi \times 36}{60} \times 0.105 = 0.7758$ m/s

$$h_{fd} = \frac{flv^2}{2gd} = \frac{0.015 \times 24}{2 \times 9.81 \times 0.1} \cdot [0.7758]^2 = 0.11 \text{ m}$$

Average is, 2/3 $\mathbf{h_{fd}}$ = **0.07363 m**

Friction head in the suction pipe ;

Velocity is the same as diameters are equal

$$h_{fs} = \frac{0.015 \times 9}{2 \times 9.81 \times 0.1} \times [0.7758]^2 = 0.0414 \text{ m}$$

Average $= 2/3\ h_{fs} = 0.0414 \times 2/3 = 0.02761$ m

Total head $= 16 + 0.07363 + 0.02761 = 16.10124$ m

**Theoretical Power** $= 1.784 \times 9.81 \times 16.1024 =$ **282 W.**

**Problem .9** *The bore and stroke of a single acting reciprocating water pump are 20 cm and 30 cm. The suction pipe is of 15 cm diameter and 10 m long. The delivery pipe is 12 cm diameter and 28 m long. The pump is driven at 32 rpm. Determine the acceleration heads and the friction head, f = 0.02. Sketch the indicator diagram. The suction and delivery heads from atmosphere are 4 m and 16 m respectively.*

$$\mathbf{h_{as\,max}} = \frac{l_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 r = \frac{10}{9.81} \times \frac{0.2^2}{0.15^2} \left(\frac{2\pi \times 32}{60}\right)^2 \times 0.15 = \mathbf{3.05\ m}$$

$$\mathbf{h_{ad\,max}} = \frac{l_d}{g} \cdot \frac{A}{a_d} \cdot \omega^2 r = \frac{28}{9.81} \times \frac{0.2^2}{0.12^2} \left(\frac{2\pi \times 32}{60}\right)^2 \times 0.15 = \mathbf{13.35\ \ m}$$

$$V_{s\,max} = \frac{A}{a} \omega r = \frac{0.2^2}{0.15^2} \times \frac{2\pi \times 32}{60} \times 0.15 = 0.8936$$

$$\mathbf{h_{fs}} = \frac{fl_s}{2gd_s} \cdot v_s^2 = \frac{0.2 \times 10 \times 0.8436^2}{2 \times 9.81 \times 0.15} = \mathbf{0.5427\ \ m}$$

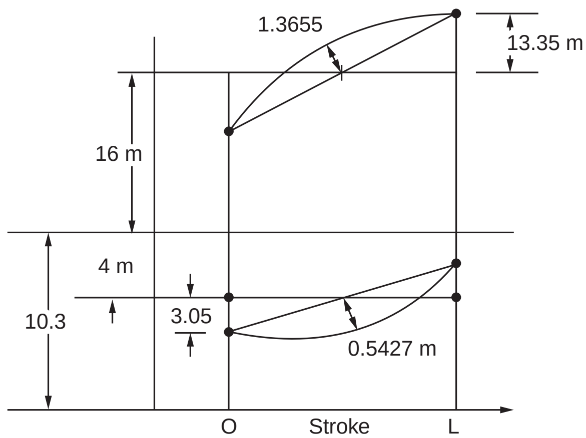



**Figure P.16.9** Problem model

$$\mathbf{h_{fd}} = \frac{fl_d}{2gd_d} \cdot V_d^2,\ V_{d\,max} = \frac{A}{a} \cdot \omega r = \frac{0.2^2}{0.12^2} \times \frac{2\pi \times 32}{60} \times 0.15$$

$= \mathbf{1.396\ m/s}$

$$\mathbf{h_{fd\,max}} = \frac{0.02 \times 28}{2 \times 9.81 \times 0.12} \times 2.396^2 = \mathbf{1.3655\ m.}$$

**Problem.10** *Using the data from problem 16.9 determine the theoretical power required.*

Flow rate $= \dfrac{A\,L\,N}{60} = 0.2^2 \times 0.3 \times 32/60$

$= 6.4 \times 10^{-3}$ m$^3$/s or 6.4 kg/s

Total head $= 16 + 4 + 2/3(1.3655 + 0.5427) = 21.272$ m

**Power** $= 21.272 \times 9.81 \times 6.4 =$ **1336 W.**

**Problem .11** *A single acting reciprocating of pump handles water. The bore and stroke of the unit are 20 cm and 30 cm. The suction pipe diameter is 12 cm and length is 8 m. The delivery pipe diameter is 12 cm and length is 24 m. f = 0.02. The speed of operation is 32 rpm. Determine the friction power with and without air vessels.*

**Without air vessels** $V = \frac{A}{a}\omega r = \frac{0.2^2}{0.12^2} \times \frac{2\pi \times 32}{60} \times 0.15$

$= 1.3963$ m/s

$$\mathbf{h_{fs\,max}} = \frac{fl_s}{2gd_s} \times v^2 = \frac{0.02 \times 8}{2 \times 9.81 \times 0.12} \times (1.3963)^2 = \mathbf{0.1325\ m}$$

$$\mathbf{h_{fd\,max}} = \frac{0.02 \times 24}{2 \times 9.81 \times 0.12} \times \left(\frac{0.2^2}{0.12^2} \cdot \frac{2\pi \times 32}{60} \times 0.15\right)^2 = \mathbf{0.3975\ m}$$

Total average friction head

$$= \frac{2}{3}[0.3975 + 0.1325] = 0.3533 \text{ m}$$

Flow rate $= \frac{A\,L\,N}{60} = \frac{\pi \times 0.2^2}{4} \times 0.3 \times \frac{32}{60} = 5.0265 \times 10^{-3}\ \text{m}^3/\text{s}$

or $= 5.0265$ kg/s

**Friction power** $= 5.0265 \times 9.81 \times 0.3533 =$ **17.42 W**

**When air vessels are installed,**

Average velocity in suction pipe

$$= \frac{A}{a}\frac{L\,N}{60} = \frac{0.02^2}{0.12^2} \times 0.3 \times \frac{32}{60} = 0.4444 \text{ m/s}$$

$$h_{fs} = \frac{fl_s\,v^2}{2g\,d_s} = \frac{0.02 \times 8 \times 0.4444^2}{2 \times 9.81 \times 0.12} = 0.013424 \text{ m}$$

As diameters are equal velocity are equal

$$h_{fd} = \frac{0.02 \times 24 \times 0.4444^2}{2 \times 9.81 \times 0.12} = 0.040271 \text{ m}$$

Friction power $= 5.0265 \times 9.81 \times (0.013424 + 0.040271) =$ **2.65 W**

The percentage reduction is $\frac{17.42 - 2.65}{17.42} \times 100 =$ **84.8%**

By use of air vessels there is a saving of 84.8% in friction power.

**Problem.12** *Show that in a double acting pump the work saved by fitting air vessels is about 39.2%.*

In a double acting pump during a revolution, the discharge

$$Q = \frac{2A\,L\,N}{60}$$

Velocity in the pipe with air vessel

$$= \frac{2\,A\,L\,N}{a \times 60} = \frac{2A \times 2r}{a \times 60} \times \frac{60 \times \omega}{2\pi} = \frac{2A}{a} \times \frac{\omega r}{\pi}$$

Friction head $= \dfrac{fl}{2gd} \,.\, v^2 = \dfrac{fl}{2gd} \,.\, \left(\dfrac{2A}{a} \times \dfrac{\omega r}{\pi}\right)^2$

Without air vessel,

Maximum friction head $= \dfrac{fl}{2gd} \left(\dfrac{A}{a} \,.\, \omega r\right)^2$

Average value is $= \dfrac{2}{3} \times \dfrac{fl}{2gd} \left(\dfrac{A}{a}\, \omega r\right)^2$

The ratio of effective friction head is also the ratio of power as power = $mgH$, and $mg$ are constant for a pump.

$$\frac{h_f \text{ with air vessel}}{h_f \text{ without air vessel}} = \frac{\dfrac{f\,l}{2gd}\left(\dfrac{2A}{a} \times \dfrac{\omega r}{\pi}\right)^2}{\dfrac{2}{3} \,.\left(\dfrac{f\,l}{2gd}\right)\left(\dfrac{A}{a}\,\omega r\right)^2} = \frac{6}{\pi^2} = 0.608$$

$\therefore$ Reduction is **39.2%.**

**Problem .13** *In a double acting pump, determine the angle at which there will be no flow in or out of the air vessel.*

Refer equation (16.5.5). Flow to or from air vessel

$$Q = A\omega r\left(\sin\theta - \frac{2}{\pi}\right)$$

When there is no flow in or out of the air vessel,

$$Q = 0,$$

$\therefore$ $\sin\theta = \dfrac{2}{\pi}$ $\quad\therefore\quad \theta = \sin^{-1}\left(\dfrac{2}{\pi}\right) =$ **39.54°**

and $(180 - 39.54) =$ **140.46°**

This is as against about 18° in the case of single acting pump.

**Problem.14** *Tabulate the flow rate as a product of Aωr for various angles of θ in the case of double acting pump.* $\left(\sin\theta - \frac{2}{\pi}\right)$ *is calculated and tabulated.*

| Crank Angle θ | 0 | 30 | 60 | 90 | 120 | 150 | 180 |
|---|---|---|---|---|---|---|---|
| Flow rate Aωr times | – 0.637 | – 0.137 | 0.2294 | 0.3634 | 0.2294 | – 0.137 | – 0.637 |

Note that this is different from example 16.9

For the following data speed is 30 rpm, and $r$ = 0.15 m, bore = 20 cm.

at 30°, flow is $\left(\sin\theta - \frac{2}{\pi}\right) - 0.137$

$$\therefore \qquad \mathbf{q} = -0.137 \times \frac{\pi \times 0.2^2}{4} \times \frac{2 \times \pi \times 30}{60} \times 0.15$$

$$= -2.028 \times 10^{-3} \text{ m}^3/\text{s or } 2.028 \text{ l/s out of the air vessel.}$$

At the starting of delivery stroke, flow is zero, but there is flow in the pipe. This should come out of the air vessel.

**Problem.15** *In a single acting pump of 16 cm bore and 24 cm stroke, the delivery pipe is 20 m long. f = 0.02. The speed 45 rpm. Determine the friction head on the delivery side, with and without air vessel for pipe diameters of 8 cm and 12 cm.*

**Without air vessel** : Effective friction head

$$\mathbf{h_f} = \frac{2}{3} \cdot \frac{fl}{2gd}\left(\frac{A}{a}\,\omega r\right)^2$$

for 8 cm pipe diameter,

$$= \frac{2}{3} \times \frac{0.02 \times 20}{2 \times 9.81 \times 0.08}\left(\frac{0.16^2}{0.08^2} \times 2\pi \times \frac{45}{60} \times 0.12\right)^2$$

$$= \mathbf{0.8692\ m}$$

**For 12 cm dia, $h_f$ = 0.1145 m** (obviously, larger the pipe diameter lower the friction head)

**With air vessel :**

$$\mathbf{h_f} = \frac{fl}{2gd}\left(\frac{A}{a} \cdot \frac{\omega r}{\pi}\right)^2$$

$$= \frac{0.02 \times 20}{2 \times 9.81 \times 0.08}\left(\frac{0.16^2}{0.08^2} \times \frac{2\pi \times 45}{60} \times \frac{0.12}{\pi}\right)^2 = \mathbf{0.1321\ m}$$

For $\mathbf{d = 0.12\ m,\ h_f = 0.0174\ m.}$