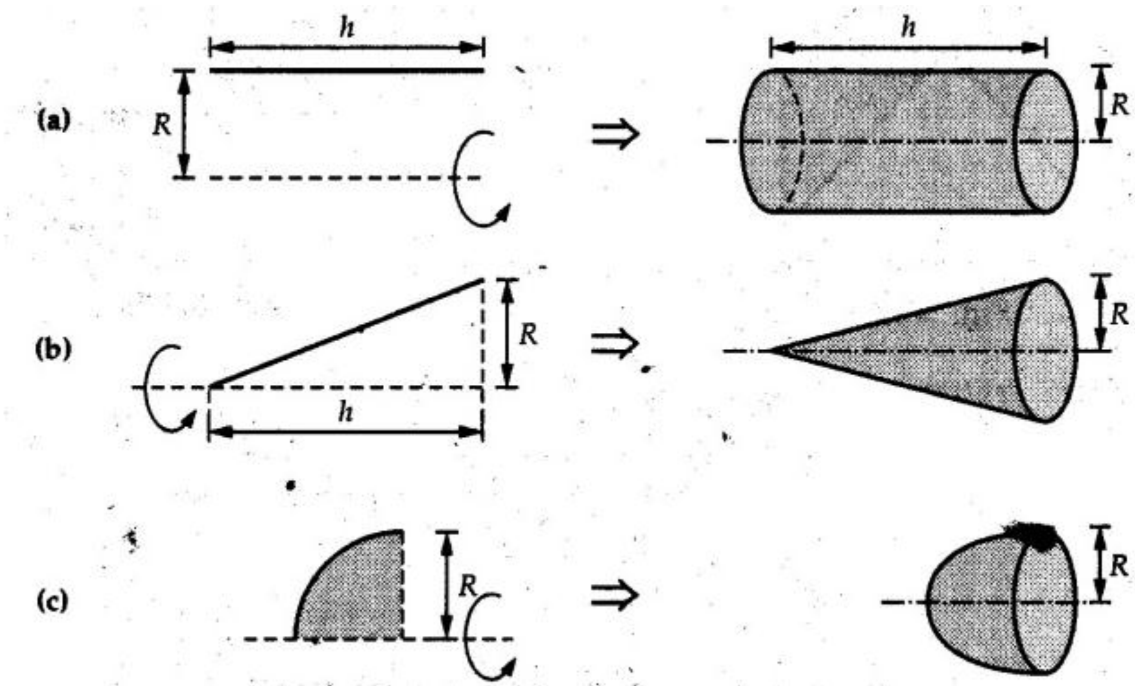
3.3 PAPPUS-GULDINUS THEOREMS

• The two theorems of Pappus and Guldinus are used to find the surface areas of surfaces of revolution and volumes of bodies of revolution.

• Surfaces of revolution can be generated by revolving any curve (or line) about any axis.

• For example, a straight line when rotated about a parallel line generates the surface of a cylinder as shown in Fig. An inclined line generates a cone as shown in Fig. and a quarter circle generates a hemisphere as shown in Fig.



Bodies of revolution are obtained by revolving areas about an axis. A few examples are shown in Fig.

• The first theorem is used to find the area of a surface of revolution.

Theorem I: *The area of surface of revolution is equal to the product of length of the generating curve and the distance travelled by the centroid of the generating curve while generating that surface.*

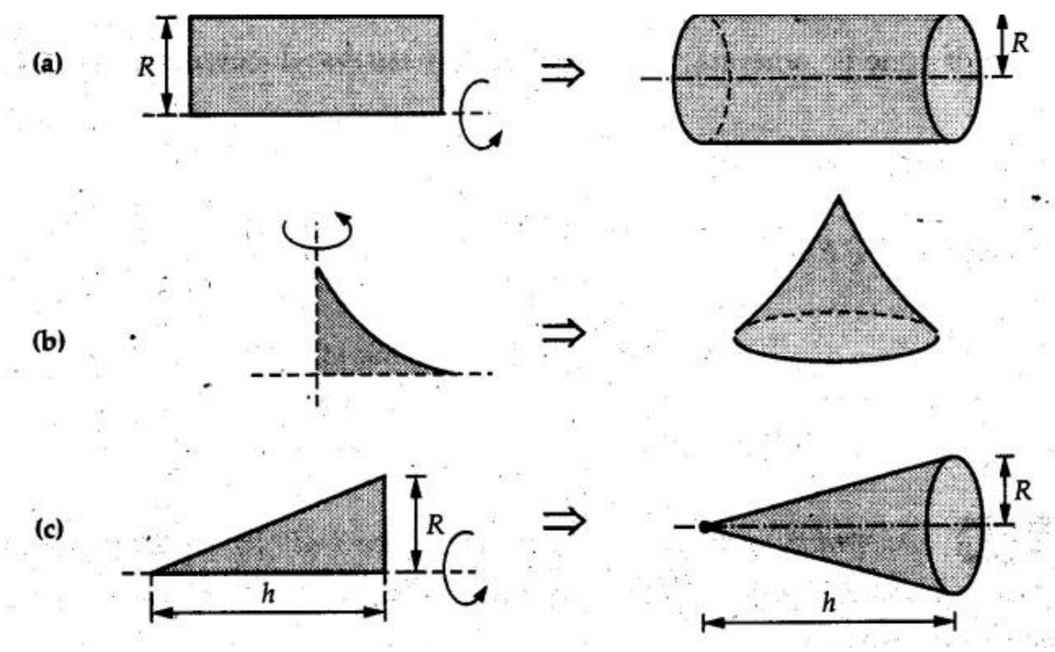
• If the centroid of a generating curve of length l is at distance $\bar{y}$ from the axis of rotation,

• Distance travelled by centroid = $2\pi\bar{y}$

$$A = 2\pi\bar{y}l$$

Theorem II: *The volume of a body of revolution is equal to the product of generating area and the distance travelled by centroid of the generating area while generating that volume.*

If A = Generating area



and $\bar{y}$= Distance of centroid of generating area from axis of rotation,

Distance travelled by centroid of generating area = $2\pi\bar{y}$

$$V = 2\pi\bar{y}A$$

Determine the surfaces area and volume of a right circular cone with radius of base R and height h using Pappus-Guldinus theorems.

Solution:

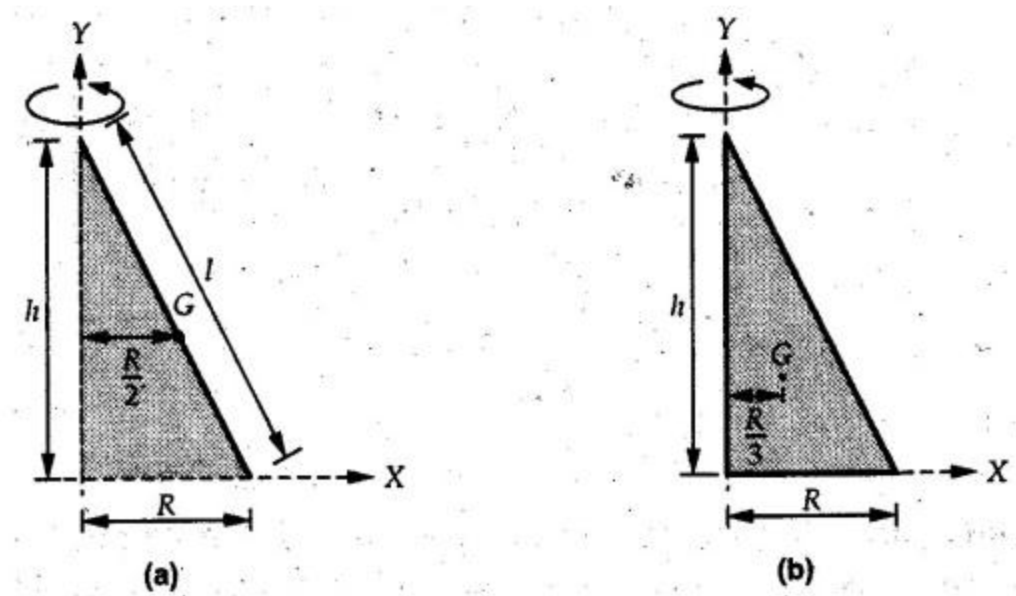
The surface of right circular cone can be generated by revolving a line as shown in Fig. about Y-axis. By similarity of triangles, centroid of the line is at a distance R/2 from axis of rotation.

Distance travelled by

$$\text{centroid} = 2\pi \times \frac{R}{2} = \pi R$$

$$l = \sqrt{R^2 + h^2}$$

$$\boxed{A = \pi R l = \pi R\sqrt{R^2 + h^2}}$$



The volume of cone is generated by revolving a triangle as shown in Fig. about Y-axis.

Distance travelled by centroid = 2л R/3

Area of triangle = 1/2 Rh

$$V = \frac{2\pi R}{3} \times \frac{1}{2} Rh$$

$$\boxed{V = \frac{1}{3}\pi R^2 h}$$

Centroid of Composite Bodies

Composite areas can be divided into several basic areas for which the location of centroid is known. Each basic area is considered to be concentrated at its centroid. Replacing integration by summation in formulae for $\bar{x}$ *and* $\bar{y}$ of centroid,

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{A_1 x_1 + A_2 x_2 + \dots}{A_1 + A_2 + \dots}$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{A_1 y_1 + A_2 y_2 + \dots}{A_1 + A_2 + \dots}$$

General procedure to find centroid of composite areas

1) Choose an appropriate origin and a co-ordinate system.

2) Divide the body into basic shapes. If any of the basic shape has to be removed, treat its area to be negative.

3) Locate the centroid of each basic shape

4) Write the co-ordinates of centroids of basic shapes with respect to the chosen origin as (x_1, y_1), (x_2, y_2) etc.

5)Use formulae to find $\bar{x}$ and $\bar{y}$.

Note: If any horizontal or vertical line of symmetry can be easily obtained, centroid will lie on that line. This can be used to determine either directly $\bar{x}$ and $\bar{y}$ without any calculations.

6) When objects are suspended from some point, the centroid lies on the vertical line passing through the point of suspension. If the point of suspension is chosen as origin, the centroid lies on Y-axis for which $\bar{x}$= 0.

For plane lamina,

$$\frac{\sum Ax}{\sum A} = 0$$

$$\boxed{\sum Ax = 0}$$

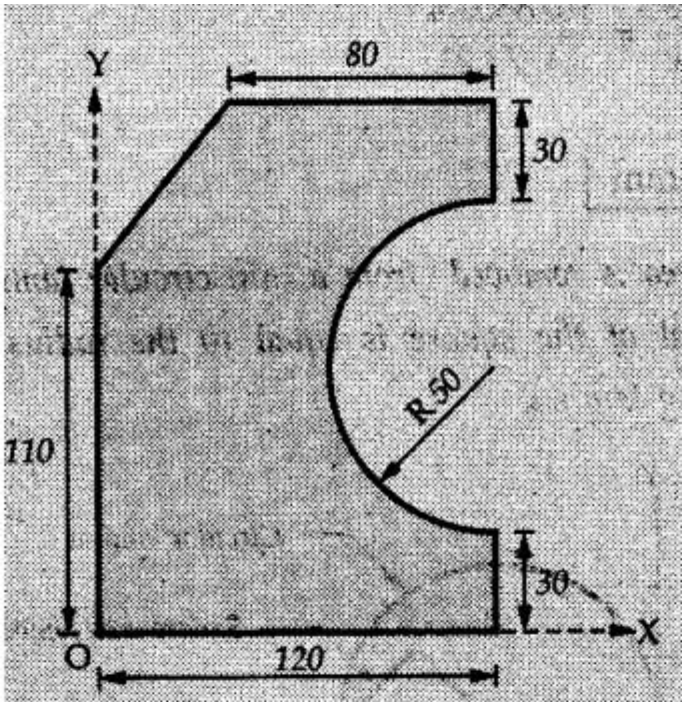
Such problems can also be solved by taking moment of area about the point of suspension. The total moment about the point of suspension will be zero for equilibrium, considering moments due to areas on left side of the point of suspension as positive and those on right side as negative. (The sign convention can be taken opposite to this).

7) In problems of finding some length when location of centroid is given, take moment about an axis passing through the centroid. The total moment of all the areas will be zero. For moment about vertical axis, choose sign convention as in suspended objects. For moment about a horizontal axis, we can consider the moment due to area above the axis as positive and below the axis as negative.

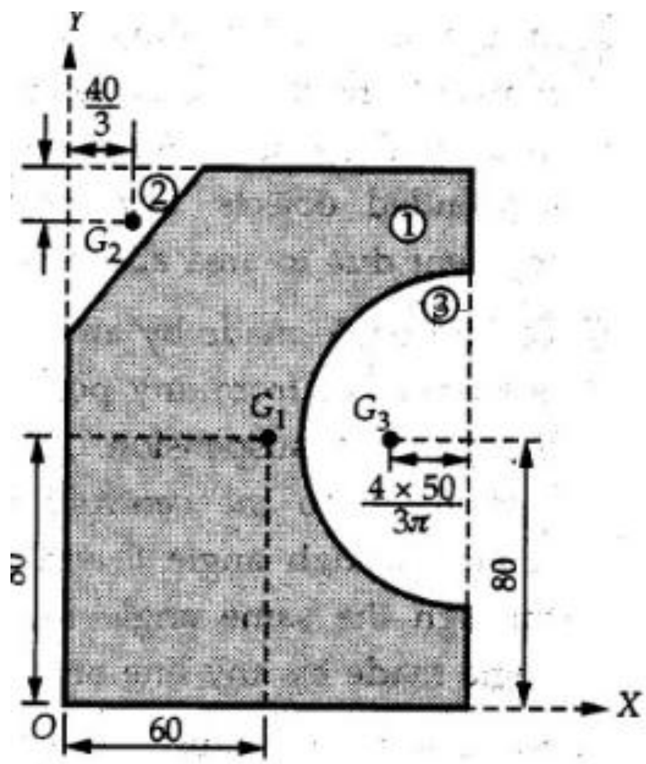
8) To find angle made by any line on an object with the horizontal or vertical when it is suspended from any point on it, locate the centroid co-ordinates with respect to the point of suspension. Find the angle @ made by the line joining the point of suspension to the centroid with vertical. When freely suspended, this line will rotate through angle @ and become vertical. Every line on the object will rotate through the same angle from its original position. This can be used to find the angle made by any line on the object with the horizontal or vertical.

Example

Locate the centroid of the area shown in Fig. The dimensions are in mm.



The calculations are tabulated as follows:



Component No.	Area A $(mm)^2$	x (mm)	y (mm)
1.	120×160	60	80
2.	$-\frac{1}{2} \times 40 \times 50$	$\frac{40}{3}$	$160 - \frac{50}{3}$
3.	$\frac{-\pi \times 50^2}{2}$	$120 - \frac{4 \times 50}{3\pi}$	80

$\sum A = 14273.01$ mm^2, $\sum Ax = 750761.1$ mm^3,

$\sum Ay = 1078507.4$ mm^3

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{750761.1}{14273.01}$$

$$\therefore \quad \boxed{\overline{X} = 52.6 \text{ mm}}$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{1078507.4}{14273.01}$$

$$\therefore \quad \boxed{\overline{Y} = 75.56 \text{ mm}}$$