Wear	Markings on the scale are not subjected to wear. Wear may occur on leading ends, which results in under sizing	Measuring surfaces are subjected to wear
Alignment	Alignment with the axis of measurement is not easy, as they do not contain a built-in datum	Alignment with the axis of measurement is easy, as they possess a built-in datum
Manufacture	Manufacturing process is simple	Manufacturing process is complex
Cost	Cost is low	Cost is high
Parallax effect	Subjected to parallax effect	No parallax error; their use depends on the feel of the operator
Wringing	Does not exist	Slip gauges are wrung together to build the required size
Examples	Scale (yard and metre)	Slip gauges, end bars, ends of micrometer anvils, and vernier callipers

1.12 Errors in Measurement

It is never possible to measure the true value of a dimension, there Is always some error. The error in measurement Is the difference between the measured value and the true value of the measured dimension.

Error In measurement =Measured value - True value.

The error in measurement may be expressed or evaluated either as an absolute error or as a relative error.

1.12.1 Absolute Error

True absolute error. It Is the algebraic difference between the result of measurement and the conventional true value of the quantity measured.

1.12.2 Apparent absolute error

If the series of measurement are made then the algebraic difference between one of the results of measurement and the arithmetical mean Is known as apparent absolute error.

1.12.3 Relative Error

It Is the quotient of the absolute error and the value of comparison used for calculation of that absolute error. This value of comparison may be the true value, the conventional true value or the arithmetic mean for series of measurement.

Table 1.4 Differences between systematic and random errors

Systematic error	Random error
Not easy to detect	Easy to detect
Cannot be eliminated by repeated measurements	Can be minimized by repeated measurements
Can be assessed easily	Statistical analysis required
Minimization of systematic errors increases the accuracy of measurement	Minimization of random errors increases repeatability and hence precision of the measurement
Calibration helps reduce systematic errors	Calibration has no effect on random errors
Characterization not necessary	Characterized by mean, standard deviation, and variance
Reproducible inaccuracies that are consistently in the same direction	Random in nature and can be both positive and negative

1.13 Types of Errors

Basically, three types of errors are studied: -

1. Gross Errors
2. Systematic Errors
3. Random Errors

1.13.1 Gross Errors

Gross Errors mainly covers the human mistakes in reading instruments and recording and calculating measurement results.

Gross Errors may be of any amount and then their mathematical analysis is impossible.

Then these are avoided by adopting two means: -

1. Great care is must in reading and recording the data.

2. Two, Three or even more reading should be taken for the quantity under measurement.

1.13.2 Systematic Errors

Systematic Errors classified into three categories: -

1. Instrumental Errors

2. Environmental Errors

3. Observational Errors

1.13.2.1 Instrumental Errors

These errors arise due to three main reasons.

1.Due to inherent shortcoming in the instrument.

2.Due to misuse of the instruments. 4. Due to Loading effects of instruments.

3.Due to misuse of the instruments. 5. Due to Loading effects of instruments.

1.13.2.2 Environmental Errors

These errors are due to conditions external to the measuring Device including conditions in the are surrounding the instrument. These may be effects of Temperature, Pressure, Humidity, Dust, Vibrations magnetic or of external or electrostatic fields.

1.13.2.3 Observational Errors

There are many sources of observational errors: -

Parallax, i.e. Apparent displacement when the line of vision is not normal to the scale. -- Inaccurate estimate of average reading. -- Wrong scale reading and wrong recording the data. -- Incorrect conversion of units between consecutive reading.

1.13.3 Random Errors

The quantity being measured is affected by many happenings in the universe. We are aware for some of the factor's measurement, but about influencing the rest we are unaware. The errors caused by happening or disturbances about which we are unaware are Random Errors. It's also known as residual Errors.

1.14 SOURCES / CAUSES OF ERROR

Errors due to deflection

Errors due to mis alignment

Errors due to contact pressure or stylus pressure

Error due to poor contact

Error due to vibrations

Error due to dirt

Error due to wear in gauges

Error due to looseness

Error due to parallax effect.

1.15 CONTROL OF ERRORS

There are many possibilities to control errors as follows.

1.Controlling or reducing of systematic error and

2. Controlling or reducing of random error.

1.15.1. Controlling or reducing of systematic error

1. Experimental design
2. calibration and accuracy
3. Repeatability and precision
4. Reproducibility
5. High gain feedback
6. Intelligent instruments

1.15. 2. Controlling or reducing of random error.

Random errors in measurements are triggered by unpredictable variations in the measurement system. It is also called precision errors.

We know that typical sources of random errors or us follows.

1.Measurements read by human observation of an analogue meter mainly these errors occur during interpolation between scale points

2.They may occur due to electrical noise

3.Random environmental changes may be the root cause for this type of error for example sudden draught of air.

1.16 MEASUREMENT UNCERTAINITY

An accurate measurement is the closeness to the true value read by the measuring instruments. As every measurement is prone to error, it is often stated that a measurement result is complete only when accompanied by a quantitative statement of its uncertainty.

SOURCES OF MEASUREMENT UNCERTAINTY:

Because real measurements are never made under perfect conditions, errors and uncertainties can come from the following sources:

1.**The measuring instrument:** Instruments can suffer from errors including bias, changes due to ageing, wear, or other kinds of drift, poor readability, noise (for electrical instruments) and many other problems.

2. **The item being measured:** Sometimes the measuring item may not be stable. For example, imagine trying to measure the size of an ice cube in a warm room.

3. **The measurement process:** The measurement itself may be difficult to make. For example, measuring the weight of small but lively animals presents particular difficulties in getting the subjects to co-operate.

4. **'Imported' uncertainties:** Calibration of instrument has an uncertainty which is then built into the uncertainty of the measurements made. (But remember that the uncertainty due to not calibrating would be much worse.)

5.**Operator skill:** Some measurements depend on the skill and judgement of the operator. One person may be better than another at the delicate work of setting up a measurement, or at reading fine detail by eye. The use of an instrument such as a stopwatch depends on the reaction time of the operator.

6. **Sampling issues:** The measurements made must be properly representative of the process which are trying to assess. For example, if the temperature at the workbench to be measured, it should not be measured with a thermometer placed on the wall near an air conditioning outlet. If the samples from a production line is chosen for measurement, the first ten product made on a Monday morning should not be taken always.

7. **The environment:** Temperature, air pressure, humidity and many other conditions can affect the measuring instrument or the item being measured.

1.17. TYPES OF MEASUREMENT UNCERTAINTY

There are following four types of measurement uncertainty:

(a) Type A uncertainty

(b) Type B uncertainty

(c) Combined uncertainty

(d) Expanded uncertainty

1.17.1. Type A Evaluation of Measurement Uncertainty

Type A uncertainty is an evaluation of a component of measurement uncertainty calculated by a statistical analysis of measured data obtained under defined measurement conditions. Examples include:

(i) calculating the standard deviation of the mean of a series of independent observations
(ii) using the method of least squares to fit a curve to data to estimate the parameters of the curve and their standard deviations, and
(iii) carrying out an Analysis Of Variance (ANOVA) to identify and quantify random effects in certain kinds of measurements.

1.17.2. Type B Evaluation of Measurement Uncertainty

Type B uncertainty is an evaluation of a component of measurement Uncertainty determined by means other than a Type A evaluation of measurement uncertainty Type B evaluation of standard uncertainty is usually based on scientific judgment using all of the relevant information available, which may include:

(i) previous measurement data
(ii) experience with the behaviour and property of relevant materials and instruments
(iii) manufacturer's specifications
(iv) data provided in calibration and other reports and
(v) uncertainties assigned to reference data taken from handbooks.

Once Type B uncertainties are included in the error propagation, the equivalence of standard deviations and standard uncertainties is generally lost. The assessment of Type B uncertainties usually needs some degree of professional judgment and experience. Once the Type B uncertainty is decided, it is treated the same as a Type A uncertainty.

1.17.3 Combined Measurement Uncertainty

Combined Measurement Uncertainty combined uncertainty combines both type A and type B into one value of 68% confidence, combined standard uncertainty may contain relations whose components are derived from Type A and Type B evaluations without discrimination between types.

1.17.4 Expanded Measurement Uncertainty

It is the quantity defining an interval about the result of a measurement which may be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand.

1.18 ESTIMATION OF MEASUREMENT UNCERTAINTY

Measurement uncertainty is critical to risk assessment and decision making. organizations make decisions every day based on reports containing quantitative measurement data. If measurement results are not accurate, then decision risks will increase. If the ability to assess the quality of the measurement results were present, organizations and individuals could make decisions more confidently.

Improving quality is the key factor to reduce risks and associated costs. So, the measurement uncertainty is a parameter which is essential because it affects quality, costs, decisions and risks. Measurement uncertainty should be involved and acknowledged to assess the quality of the results stated to meet the established accuracy requirements.

1.18.1 Steps in Calculating Measurement Uncertainty

(i) Specify the measurement process and equation
(ii) Identify and characterize the uncertainty sources
(iii) Quantify the magnitude of uncertainty components
(iv) Characterize sources of uncertainty
(v) Convert uncertainty components to standard deviation equivalents
(vi) Calculate the combined uncertainty
(vii) Calculate the expanded uncertainty
(viii) Evaluate your uncertainty budget.

Step 1: Specifying the measurement process and equation

To specify the measurement process, the following instructions are considered as mentioned below.

1. The test or measurement function should be selected to evaluate.
2. The measurement method or procedure to be used should be selected.
3. The equipment should be selected that will be used.
4. The desired range of the measurement function should be selected.
5. The test-points to be evaluated should be determined.

Next, the mathematical equation should be found out wherever necessary to characterize the measurement function.

Step 2: Identifying and characterizing the uncertainty sources

The sources of uncertainty for the analysis should be identified as per the list below.

1. The test method, calibration procedure or measurement process should be evaluated.
2. The measurement equations should be evaluated.
3. The equipment, reference standards and reagents should be evaluated.
4. The minimum required sources of uncertainty should be identified.
5. Various sources of information should be analysed.
6. An expert should be consulted.

Below items will be more useful to find the sources of uncertainty.

1. Manufacturer manuals
2. Manufacturer datasheets
3. White papers
4. Technical notes and guides
5. Conference papers
6. Textbooks.

If the measurement function includes equations, then the process to estimate uncertainty is slightly different. Each variable in the equation and its influences on it should be identified.

Step 3: Quantifying the magnitude of uncertainty components

To quantify uncertainty, below steps are followed.

1. Information and data should be collected.
2. Right data should be evaluated and selected.
3. Data should be analysed.
4. Uncertainty components should be quantified.

(i) Collecting information and data:

To quantify the sources of uncertainty, the following items are needed.

1. Last three calibration reports
2. Repeatability and reproducibility studies
3. Method or procedure
4. Experiment results
5. Manufacture manuals and specifications
6. Technical documents and guides
7. Published papers, studies, journal articles, etc.

(ii) Evaluating information and selecting the right data:

Next, data related to your uncertainty analysis should be found and unwanted data should be eliminated. The selected right data should include the following information such as

1. Measurement function
2. Measurement range and
3. Test-point.

(iii) Analysing the data:

After evaluation, the data should be analysed using appropriate methods of analysis to find the magnitude of each uncertainty component.

(iv) Quantify uncertainty components:

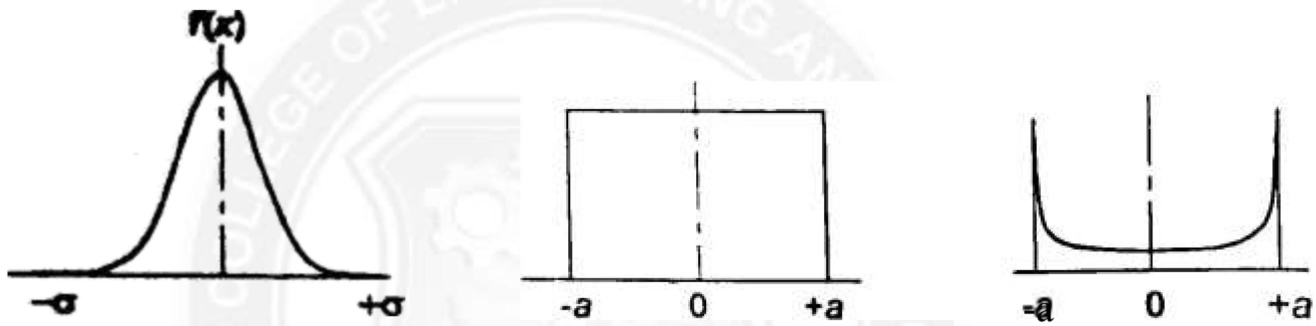
Finally. the results are used to quantify each uncertainty Component and the values added to uncertainty budget or uncertainty calculator.

Step 4: Characterizing sources of uncertainty

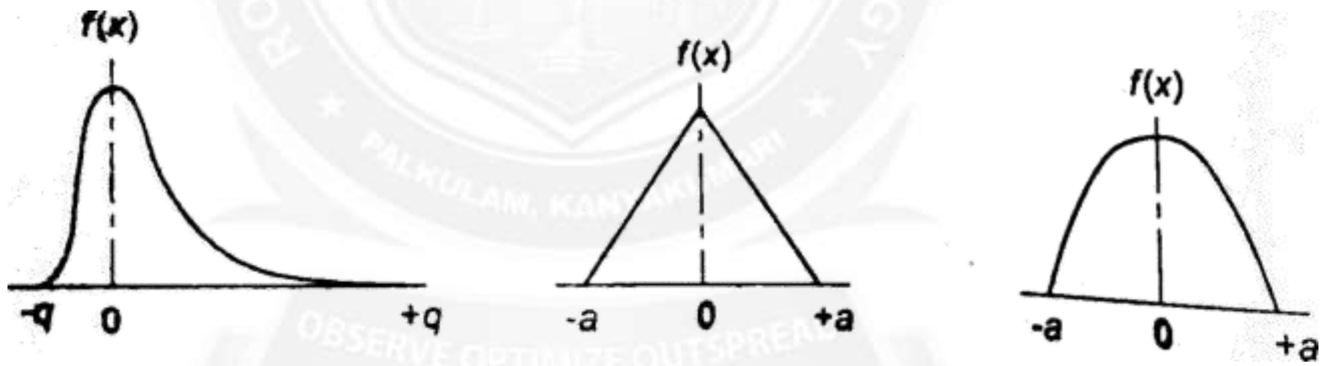
To characterize the sources of uncertainty, below steps are followed

1. Each source of uncertainty should be categorized as Type A or Type B.
2. A probability distribution should be assigned to each uncertainty Component.

It is an important step to determine how source of uncertainty is converted to a Standard deviation.



(a) Normal distribution (b) Uniform distribution(c) U-shaped distribution



(d) Log normal distribution (e) Triangle distribution (f)Quadratic distribution

Figure: Various types of probability distribution

Even though there are many different types of probability distributions, the normal (Gaussian) and rectangular (Uniform) distributions are the most commonly used.

The following most common probability distributions used to estimate Uncertainty are listed below.

1. Normal (i.e. Gaussian) distribution
2. Rectangular (i.e. Uniform) distribution

1. Triangular distribution
2. Log-normal distribution
3. Quadratic distribution
4. U-shaped distribution
5. Rayleigh distribution

Step 5: Converting uncertainty components to standard deviation equivalents

To convert uncertainty components to standard deviations, the following steps are listed

1. A probability distribution to each source of uncertainty should be assigned.
2. The divisor should be found for the selected probability distribution.
3. Each source of uncertainty should be divided by its respective divisor to convert them to a standard uncertainty.

Step 6: Calculating the combined uncertainty

To calculate the combined standard uncertainty, below instructions should be performed.

1. The value of each uncertainty component should be squared
2. All the results in step 1 should be added together.
3. The square root of the result ill step 2 should be calculated.

Step 7: Calculating the expanded uncertainty

To calculate the expanded measurement uncertainty; just follow these steps:

1. The combined uncertainty should be calculated.
2. The effective degrees of freedom should be calculated.
3. A coverage factor (k) should be found
4. The combined uncertainty should be multiplied by the coverage factor.

Step 8: Evaluating uncertainty for appropriateness

After calculating the expanded the uncertainty estimate for appropriateness shou be evaluated using some standard uncertainty models to make sure about the measurement uncertainty estimate for adequacy of the measurement process which should not be overestimated or underestimated.

1.19. STATISTICAL ANALYSIS OF MEASUREMENT DATA

Among different types of possible errors, the random errors cannot be determined. ordinary process of measurements but they can be treated mathematically. The mathematical analysis of the various measurements is called statistical analysis of the data.

Some statistical tools are also used to calculate the uncertainty of measurement as follows.

1. Analysis of data
2. Frequency distribution
3. Control chart
4. Acceptance sampling.

For statistical analysis, the same reading is taken number of times, generally different observers, different instruments and by different ways of measurement. The statistical analysis helps to determine analytically the uncertainty of the final test results.

1.19.1. Frequency Distribution

The distribution of a variable is the pattern of frequencies which means, the set of all possible values and the frequencies associated with these values. Frequency distributions represented as frequency tables or charts. Frequency distributions show either the number of observations falling in each range or the percentage of observations. Frequency distribution tables can be used for both categorical and numeric variables.

There are types of frequency distributions such as

i. Cumulative frequency distribution
ii. Relative frequency distribution
iii. Relative cumulative frequency distribution.

Some examples of frequency distribution in statistics are pie charts, bar charts, histograms.

1.19.2. Statistical Analysis of Measurement Data

Type A uncertainty is an evaluation of a component of measurement uncertainty determined by a statistical analysis of measured data obtained under defined conditions.

Arithmetic mean, median and mode of a set of measurement data are the parameters of central tendency. Their calculations are explained in detail below.

1. **Arithmetic mean or arithmetic average:**
   When the number of readings of the repeated measurement are taken, the most likely value from the set of measured value is the arithmetic mean of the number of readings taken.

The arithmetic mean value is mathematically obtained by

$$X_{mean} = \text{or } \overline{X} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

where - $X_1, X_2 \ldots X_n$ are the reading in each measurement

n is the total number of readings

This mean value is too close to true value if number of readings is very large.

2. **Median:**

At the same time, when the number of readings is large, the calculation of mean value is complicated. In such a case, a median value is calculated which will be a close approximation to the arithmetic mean value. For a set of measurements Xl, X2 . . .Xn, the measurements are arranged in the ascending order of magnitudes,

Case (i) If the total number of measurements is odd, the middle value is taken as median.

Case (ii) If the total number of measurements is even, the median is taken as the average of middle two number. For example, in a 10-number set of measurements, the median is calculated by

$$X_{median} = \frac{X_5 + X_6}{2}$$

where X5 = Reading of 5th reading

X6 = Reading of 6th reading.

3. Mode

The mode is a statistical term which refers to the most frequently occurring number found in a set of numbers.

For example, a set of measurements having the following values, l, l, 3, 5, 6, 6, 8, 8, 7, 8, the mode will be 8 because it occurs the most of all the values in the set.

4. Average deviation:

The deviation indicates the departure of a given reading from the arithmetic mean of the data set.

Deviation of i^{th} reading, $d_i = X_i - \overline{X}$

Average deviation, $d_{mean} = \frac{\Sigma(X_i - \overline{X})}{n}$

where X_i = Reading of i^{th} reading

n = Total number of readings

$\overline{X}$ = Arithmetic mean

The average deviation is defined as the sum of the absolute values of deviations divided by the number of readings. It is also called mean deviation.

5. Standard deviation:

Standard deviation of an infinite number of data is the square root of the sum of all the individual deviations squared and divided by the number of readings. It is expressed by

$$\text{Standard deviation, } \sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n}} = \sqrt{\frac{\sum d_i^2}{n}}$$

$$\text{Also, it can be written as } \sigma = \sqrt{\frac{\Sigma\left(X_i - \overline{X}\right)^2}{n}}$$

standard deviation is also known as root mean square deviation. It is the most important factor in the statistical analysis of measurement data. If the value of σ reduces, it wi indicate that there is an improvement in measurement process.

6. Skewness;

Skewness is a measure of the asymmetry of a distribution. A distribution is asymmetrical when it is left side and right side are not mirror images.

When a distribution has zero skew, it shows the distribution as symmetrical. It means both left and right sides are mirror images.

For example, normal distributions have zero skew because the distribution on will be equal from its peak,

A right-skewed distribution is longer on the right side of its peak than on its left' ' Called positive skew.

There are several formulas to measure skewness. One of the simplest is Pearson's median skewness. It takes advantage of the fact that the mean and median are unequal in a skewed distribution.

$$\text{Pearson's median skewness} = 3 \text{ x } \frac{(\text{Mean — Median})}{\text{Standard deviation}}$$

Pearson’s median skewness indicates that the amount of standard deviations separate the mean and median.

7. Kurtosis:

Kurtosis is a statistical measure which is used to describe a characteristic of a dataset. When normally distributed data is plotted on a graph, it generally forms the shape of an upside-down bell called bell curve. The plotted data from the mean of the data usually form the tails on each side of the curve. Kurtosis shows how much data resides in the tails.

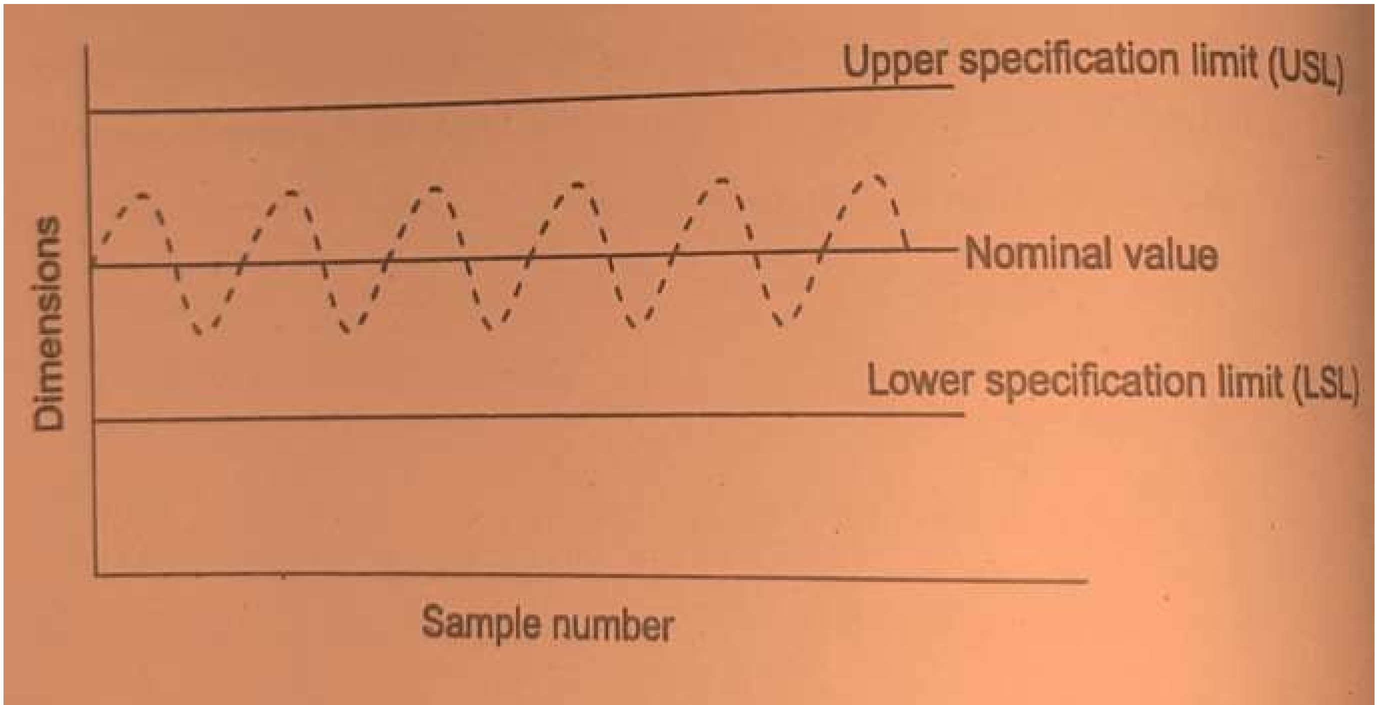


Figure 1.16 More capable process

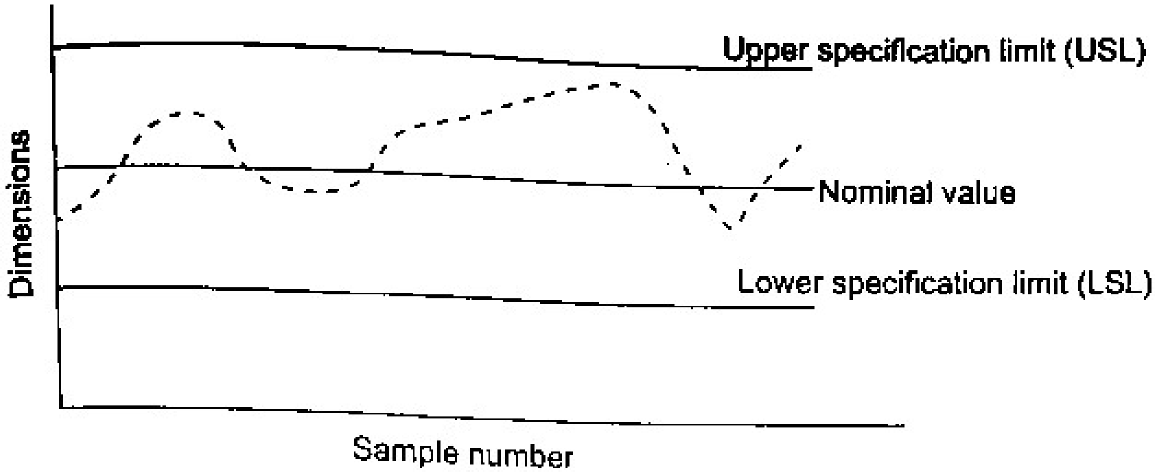
1.19.3. Control Chart

The process capability of a machine or a manufacturing process is the minimum spread or minimum tolerance to which machine or process is expected to work and produce defective parts or components under the specified condition. It is also stated that the capability is considered as a measure of spread of the process. process capability is to measure the ability of the process to meet the specified tolerance.

Case (a): More capable process

Most of the measurement data lie just above and below the nominal value. It indicates that there is no much deviation of measurement values from nominal value. This measurement is always preferred to obtain better assembly with negligible defective it is always suggested to for high precision parts where the cost minimisation is considered.

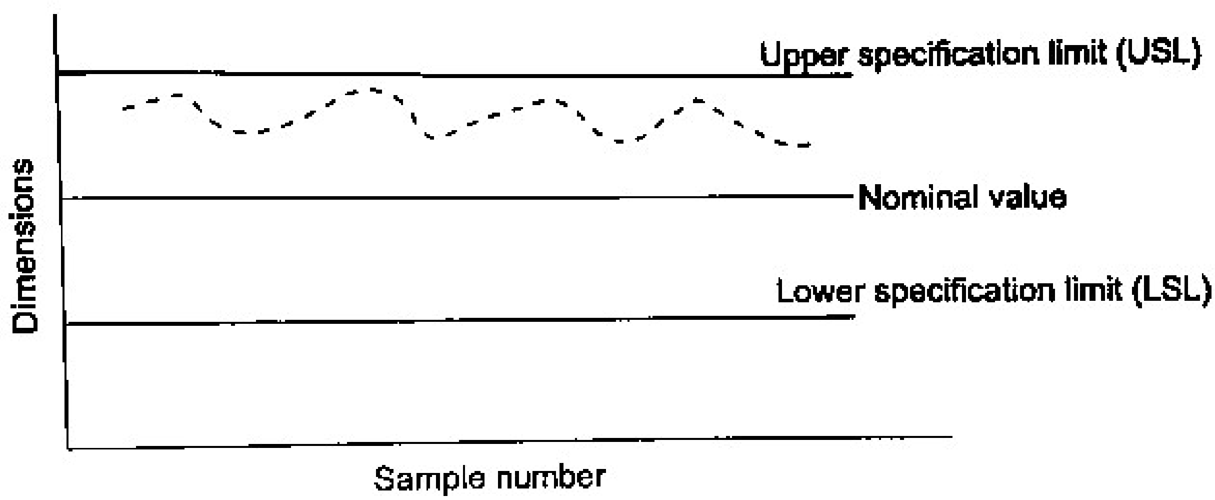
Case (b): Adequate process



Most of the measurement data lie above and below the nominal value within the deviation is slightly far from the normal value. At the same time, measurement.

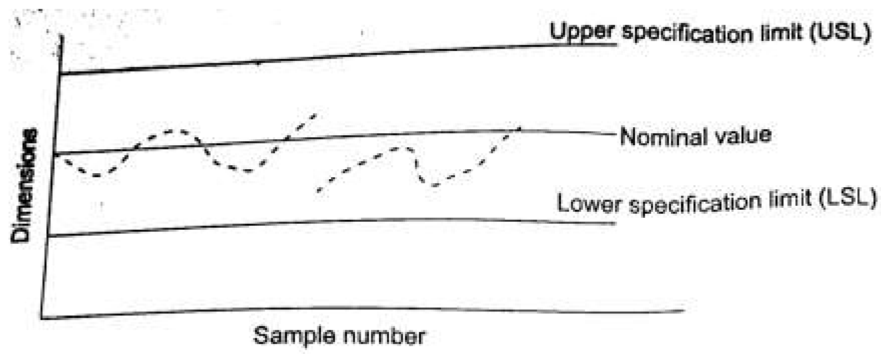
Case (c): Capable process with high USL

Most of the measurement data lie near USL line by indicating too far deviation from normal value but they are within 'the range. This type manufacturing process is mostly preferred for low cost and less precise components in mass production.



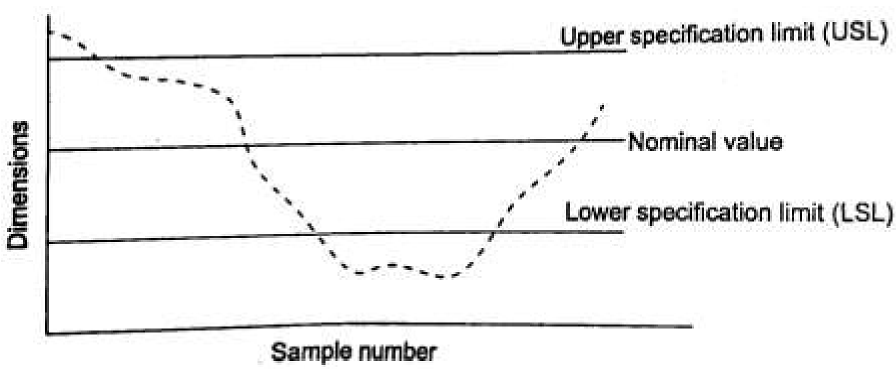
Case (d): Capable process with tool wear

Most of the measurement data lie near LSL line by indicating far deviation from normal value but they are within the range. So, this type of measurement data occurs only with tool having more wear. This 'type manufacturing process is also preferred for low cost and less precise components in mass production.



Case (e): Incapable process

The measurement data lie sometimes beyond USL and LSL line but some data lie within the range by indicating too far deviation from normal value. So, this type of measurement data may indicate about tool wear, improper measurement method used by observer' improper use of instruments for correct measurement, etc. This type manufacturing process is not suggested for manufacturing processes.



(i) Methods of variables:

When a measurement data is recorded of an actual measured quality characteristic, the quality characteristic is called variables. Variable data are measured on a continuous scale with individual measurement rounded to some desired number of decimal points.

Examples:

(i) A dimension of a part measured.

(ii) Hardness value of a part.

(iii) Temperature of a substance.

(iv) Tensile strength of some specimen.

(v) Weight of some parts.

(ii) Methods of attributes:

When a record displays only the number of parts conforming and number of parts not conforming to any of the specified requirement it means that the discrete data having integer values. In methods of attributes, the selection of manufacturing components is decided or justified for the assembly by using inspection gauges such as GO and NO-GO gauges. This method is mainly used to decide fitness of the mating components.

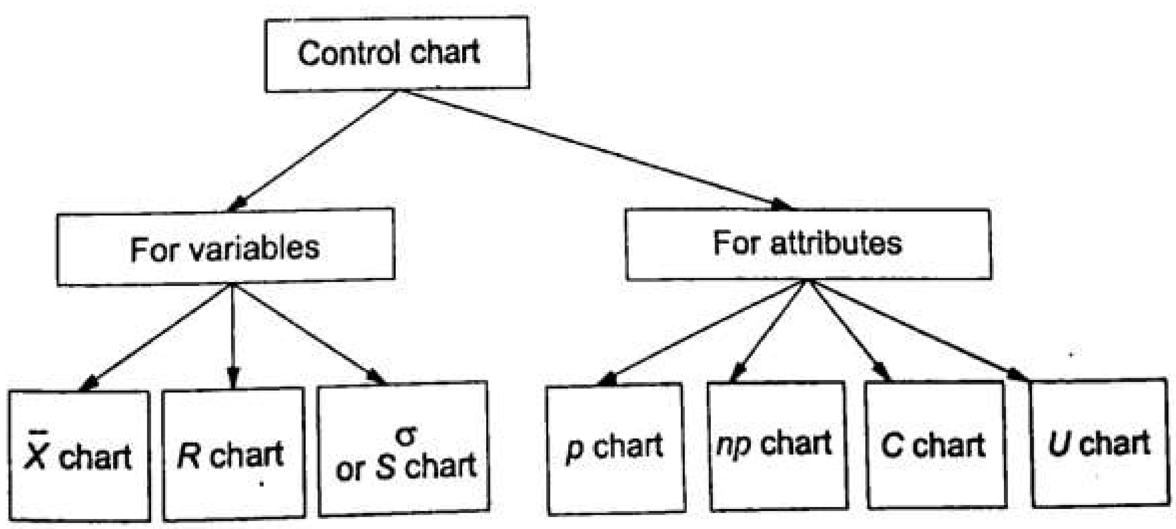
Examples:

(i) Number of defective pieces found in a sample.

(ii)Cracks in sheet by spot weld.

(iii) Number of casting defects in a casting.

One can actually measure the dimension variable whether the object is good or bad attribute. Sometimes, the variables are treated as attributes. Thus,



the measured quality of objects in dimension is called variable and growing of articles as per the dimension is called attribute (good/bad). Therefore, some control charts are used to analyse both variables and attributes.

1.19.3.1. Control charts to analyse variables

Mostly, X and R are used to analyse variables. X chart shows changes in pr average and erratic cyclic shift of the process and it is associated with R & (Change, ş/kutosis s) charts. But R chart is used to measure the spread.

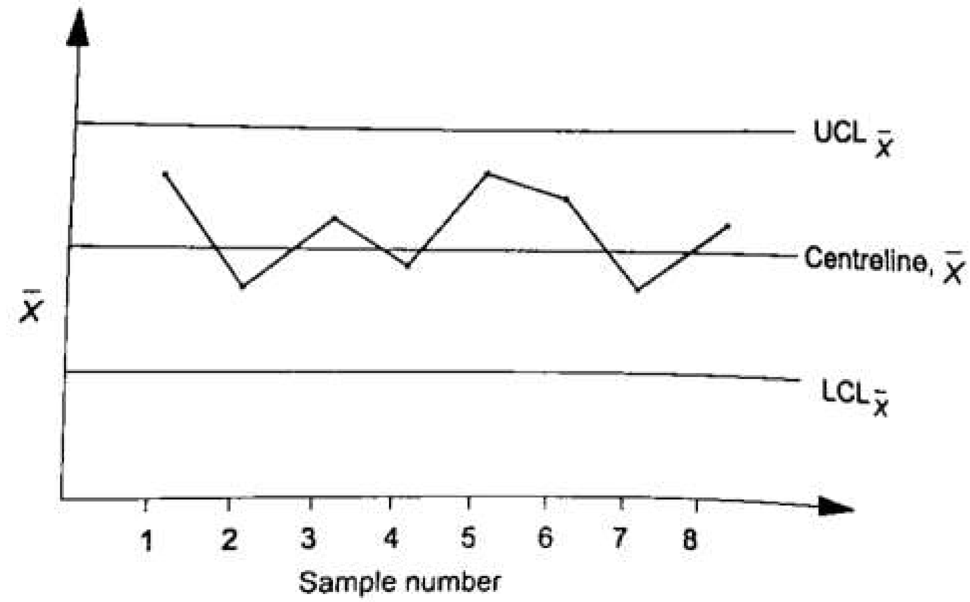
1. Construction of X chart:

A set of samples of items or products are collected at random at different intervals of time Let m samples are taken each containing n observations on the quality Characteristic. For example, n may be small either 4, 5 or 6.

If X1, X2, X3, . . . , X_n, be the dimensions of a sample of size n.

Then, the average dimension is calculated by

$$\overline{X} = \frac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$$

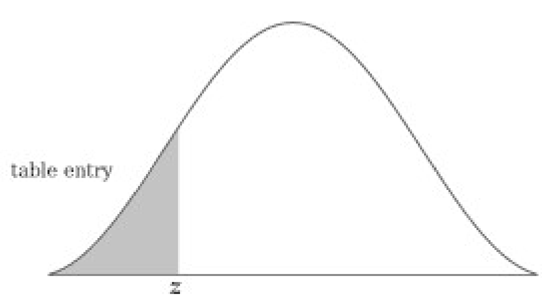


Again, if Xl, X2, X3, . xn be the average of each dimension value, then the estimate of process average is calculated by

$$\overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \overline{X}_3 + \ldots + \overline{X}_n}{m}$$

where m be the total number samples.

Negative Z score table

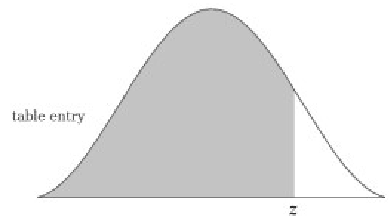


Use the negative Z score table below to find values on the left of the mean as can be seen in the graph alongside. Corresponding values which are less than the mean is marked with a negative score in the z-table and represent the area under the bell curve to the left of z.

Negative Z score table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414
-0.1	.46017	.45620	.45224	.44828	.44433	.44034	.43640	.43251	.42858	.42465
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-1	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08692	.08534	.08379	.08226
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-2	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-3	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-4	.00003	.00003	.00003	.00003	.00003	.00003	.00002	.00002	.00002	.00002

Positive Z score table



Use the positive Z score table below to find values on the right of the mean as can be seen in the graph alongside. Corresponding values which are greater than the mean is marked with a positive score in the z-table and represent the area under the bell curve to the left of z.

Positive Z score table

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
+0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
+0.1	.53983	.54380	.54776	.55172	.55567	.55966	.56360	.56749	.57142	.57535
+0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
+0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
+0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
+0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
+0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
+0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
+0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
+0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
+1	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
+1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
+1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
+1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
+1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
+1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
+1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
+1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
+1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
+1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
+2	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
+2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
+2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
+2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
+2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
+2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
+2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
+2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
+2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
+2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
+3	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
+3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
+3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
+3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
+3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
+3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
+3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
+3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
+3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
+3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997
+4	.99997	.99997	.99997	.99997	.99997	.99997	.99998	.99998	.99998	.99998

1.19.3.2. control charts to analyse attributes

To analyse attributes, many types charts are used as shown below. There are two types control charts for attributes such as

1. **Nonconforming defectives**
   i. p chart r— It is used for subgroup size as variable
   ii. np chart It is used for subgroup size as constant.
2. **Nonconforming defects**
   i. C chart — It is used for subgroup size as variable
   ii. U chart — It is used for subgroup size as constant.

1. **p chart:**

If the true fraction nonconforming p in the production process is known or if the standard value is mentioned by the manufacturer, then control limits are calculated by

If the standard value is specified, the control limits for p chart are calculated by

$$\text{Upper control limit, } UCL_p = p + 3\sqrt{\frac{p(1-p)}{n}}$$

$$\text{Centreline} = p \text{ and}$$

$$\text{Lower control limit, } LCL_S = p - 3\sqrt{\frac{p(1-p)}{n}}$$

2.Plotting np chart:

When the subgroup size is variable, p chart is used. Though the subgroup size is constant np chart is used due to the following reasons.

1. np chart saves the calculation for each subgroup.

2. Some people may easily understand the details of np chart. So, it avoids confusion even for constant subgroup

The average fraction defective $\bar{p}$ is used as the best available estimate of p calculated

$$\bar{p} = \frac{\Sigma p}{\Sigma n}$$

Therefore, upper control limit, $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

Centreline $= n\bar{p}$ and

Lower control limit, $LCL_S = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

3. Plotting C chart:

C chart is used economically in the following situations.

1. Number of surface defect in a role of coated paper or a sheet of photographic film.
2. Number of defective rivets in an aircraft wing.
3. Number of surface defects noticed in a galvanized sheet or a painted plate.
4. Number of small air holes in glass holes in glass bottles etc.

Similar to above charts, control limits for different cases to C chart are calculated as follows.

Case (i) If the standard value is known,

Upper control limit, $UCL_C = C + 3\sqrt{C}$

Centreline $= C$ and

Lower control limit, $LCL_C = C - 3\sqrt{C}$

Case (ii) If the standard value is not known,

Upper control limit, $UCL_{\bar{C}} = \bar{C} + 3\sqrt{\bar{C}}$

Centreline $= C$ and

Lower control limit, $LCL_{\bar{C}} = \bar{C} - 3\sqrt{\bar{C}}$

4.Plotting U chart:

The average fraction defective U is used as the best available estimate of U calculated by

$$\overline{U} = \frac{\sum U_i}{\sum n_i}$$

where $$U_i = \frac{C_i}{n_i}$$

C_i is the number of nonconformities

n_i is the sample size

$\therefore$ Upper control limit, $UCL_{\overline{U}} = \overline{U} + 3\sqrt{\overline{U}}$

Centreline $= \overline{U}$ and

Lower control limit, $LCL_{\overline{U}} = \overline{U} - 3\sqrt{\overline{U}}$

1.19.4 Sampling Acceptance

Inspection is the act of checking the manufactured components about whether it performs the necessary function or satisfies the required specification. Inspection is not carried out in a single stage of manufacturing but it is done in many stages.

Usually, inspection is carried for two different reasons such as

(i) to segregate defective pans

(ii) to locate defects in raw material.

Methods of inspection is done in two ways as follows.

(i) Hundred percent inspection in which all parts are inspected without any omission

(ii) Sampling inspection in which random inspection is carried out.

Sampling inspection is defined as a technique to determine the acceptance /rejection of a bunch of parts based on the number of defective parts found in a random sample drawn from a lot.

Advantage of sampling inspection over 100% inspection:

I. Parts are subjected to destructive test which must be inspected by sampling inspection only

2. Cost and time are less when compared to 100% inspection.
3. Inspection fatigue is elimination taken place.
4. Less staff for inspection is required.
5. The problem of monitory and inspection error can be easily eliminated.
6. The charts are with more effective in a quality improvement due to the rejection of entire lot based on the quality at sample.

Limitations of sampling inspection over 100% inspection:

I. There is a risk of making wrong decision.

2. It provides less information about the product.

The success of sampling plan depends on

(i) Randomness of sample.

(ii) Sample size (Sample size = Batch size in this case).

1.20 Problems on Errors and Estimation of uncertainty:

Problem 1:

Three different sources of systematic errors are identified in a measurement system such as system loading error is ± 1.3%, environmental error is 0.89% and calibration error is 0.53%. Calculate the maximum possible total error the likely system error by root-sum-squares method.

Given data:

System loading error, $x_1 = \pm 1.3\%$

Environmental error, $x_2 = 0.89\%$

Calibration error, $x_3 = 0.53\%$

Solution:

Maximum possible error $= x_1 + x_2 + x_3$

$= 1.3 + 0.89 + 0.53$

$= \pm 2.72\%$ **Ans.**

Likely error by root-sum-squares method,

$$= \pm\sqrt{x_1^2 + x_2^2 + x_3^2}$$

$$= \pm\sqrt{1.3^2 + 0.89^2 + 0.53^2} = \pm 1.66\%$$

Problem 2:

The mass and the length of one side of a cube are measured to calculate its density. If the percentage errors in the measurement of mass and length are 2% and 4% respectively, then what is the percentage error in the density?

Given data:

Percentage error in mass, $s_{r_m} = \frac{\Delta m}{m} = 2\% = 0.02$

Percentage error in length, $s_{r_l} = \frac{\Delta l}{l} = 4\% = 0.04$

Solution:

Density, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{l^3}$ $\left[\because \text{Volume of a cube} = l^3\right]$

Relative error in density, ρ is calculated by

$$\frac{\Delta\rho}{\rho} = \left(\frac{\Delta m}{m}\right) + 3\times\left(\frac{\Delta l}{l}\right) = 0.02 + 3\times 0.04 = 0.14$$

∴ Percentage error in density = 0.14 × 100 = **14%** **Ans.**

Problem 3:

25 samples of headphones are inspected for different sample size and the number of defects in each headset is noted in the following table below. Construct a suitable control chart for the number of defects per carpet.

Part No.	*No. of headset*	*No. of defects*	*Part No.*	*No. of headset*	*No. of defects*	*Part No.*	*No. of headset*	*No. of defects*
1.	12	15	11.	17	16	21.	21	15
2.	16	05	12.	24	03	22.	32	05
3.	36	11	13.	29	06	23.	24	14
4.	33	08	14.	15	18	24.	18	06
5.	14	04	15.	22	12	25.	34	13
6.	24	12	16.	31	06	26.	27	07
7.	18	10	17.	19	17	27.	35	05
8.	22	17	18.	30	04	28.	22	13
9.	16	04	19.	15	13	29.	31	09
10.	21	09	20.	26	06	30.	17	04

Given data:

No. of parts, $N = 30$

☺ ***Solution:***

Part No.	*No. of headset, n*	*No. of defects, C*	$U_i = C_i/n_i$	*Part No.*	*No. of headset, n*	*No. of defects, C*	$U_i = C_i/n_i$
1.	12	15	1.250	16.	31	06	0.194
2.	16	05	0.313	17.	19	17	0.895
3.	36	11	0.306	18.	30	04	0.133
4.	33	08	0.242	19.	15	13	0.867
5.	14	04	0.286	20.	26	06	0.231
6.	24	12	0.500	21.	21	15	0.714

7.	18	10	0.556	22.	32	05	0.156
8.	22	17	0.773	23.	24	14	0.583
9.	16	04	0.250	24.	18	06	0.333
10.	21	09	0.429	25.	34	13	0.382
11.	17	16	0.941	26.	27	07	0.259
12.	24	03	0.125	27.	35	05	0.143
13.	29	06	0.207	28.	22	13	0.591
14.	15	18	1.200	29.	31	09	0.290
15.	22	12	0.545	30.	17	04	0.235
				Total, Σ	**701**		**13.929**

Average, $\overline{U} = \frac{\sum U_i}{\sum n_i} = \frac{13.929}{701} = 0.0199$

Average n, $\bar{n} = \frac{\sum n_i}{N} = \frac{701}{30} = 23.367$

Upper control limit, $UCL_{\overline{U}} = \overline{U} + 3\sqrt{\frac{\overline{U}}{\bar{n}}} = 0.0199 + 3\sqrt{\frac{0.0199}{23.367}} = \mathbf{0.107}$ **Ans.**

Centreline $= \overline{U} = \mathbf{0.0199}$ **Ans.**

Lower control limit, $LCL_{\overline{U}} = \overline{U} - 3\sqrt{\frac{\overline{U}}{\bar{n}}} = 0.0199 - 3\sqrt{\frac{0.0199}{23.367}} = \mathbf{0.068}$ **Ans.**