2.2 TWO RAY GROUND REFLECTION MODEL

Two ray model considers both the direct path and a ground reflected propagated path between transmitter and receiver.

A two-ray model, which consists of two overlapping waves at the receiver, one direct path and one reflected wave from the ground.

The total received E-field $E_{TOT}$ is the result of the direct line of sight component $E_{LOS}$ and the ground reflected component $E_g$.

Referring to Figure 1.2.1, $h_t$ is the height of the transmitter and $h_r$ is the height of the receiver.

If $E_0$ is the free space electric field (in V/m) at a reference distance $d_0$ from the transmitter then for $d>d_0$,

The free space propagating E-field is

$$E(d, t) = \frac{E_0 d_0}{d} \cos\left(\frac{mc(t - \frac{d}{c})}{c}\right) \quad (d > d_0)$$

The envelop of the electric field at $d$ meters from the transmitter at any time $t$ is therefore
Two propagating waves arrive at the receiver, one LOS wave which travels a distance of $d'$ and another ground reflected wave, that travels $d^{||}$.

The E-field due to the line-of-sight component at the receiver can be expressed as

$$E_{LOS}(d', t) = E_0d_0 \frac{d'}{d} \cos(\frac{mc(t - \frac{d'}{c})}{})$$

The E-field for the ground reflected wave, which has a propagation distance of $d^{||}$, can be expressed as

$$E_g(d^{||}, t) = \Gamma E_0d_0 \frac{d^{||}}{d} \cos(\frac{mc(t - \frac{d^{||}}{c})}{})$$

According to the law of reflection in a dielectric,

$$\theta_i = \theta_0 \text{ and } E_g = \Gamma E_i$$

$$E_t = (1 + \Gamma)E_i$$

where $\Gamma$ is the reflection coefficient for ground.

For small values of $\theta_i$ (i.e., grazing incidence), the reflected wave is equal in magnitude and $180^\circ$ out of phase with the incident wave.

The resultant total E-field envelope is given by

$$|E_{TOT}| = |E_{LOS} + E_g|$$

The electric field $E_{Tot}(d, t)$ can be expressed as

$$|E(d, t)| = \frac{E_0d_0}{d}$$
\[ E_{TOT}(d, t) = \frac{E_0d_0}{d'} \cos\left( \omega_c \left( t - \frac{d'}{c} \right) \right) + (-1) \frac{E_0d_0}{d''} \cos\left( \omega_c \left( t - \frac{d''}{c} \right) \right) \]
Using the method of images, which is shown in Figure 1.2.2, the path difference, $\Delta$ between the line-of-sight and the ground reflected paths can be expressed as

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

Figure 1.2.2: The Method of Images—Two Ray Model.

[Source: "Wireless communications" by Theodore S. Rappaport, Page- 87]

When the T-R separation distance $d$ is very large compared to $h_t + h_r$, the above equation can be simplified using a Taylor series approximation

$$\Delta \approx \frac{2h_t h_r}{d}$$

Once the path difference is known, the phase difference $\theta_\Delta$ between the two Electric field
components and the time delay $r_d$ between the arrival of the two components can be easily computed using the following relations.

$$\theta_\Delta = \frac{2\pi \Delta}{\lambda} = \frac{\Delta \omega_c}{c}$$

And

$$r_d = \frac{\Delta}{c} = \frac{\theta_\Delta}{2\pi f_c}$$

It should be noted that as $d$ becomes large, the difference between the distances $d'$ and $d''$ becomes very small, and the amplitudes of $E_{LOS}$ and $E_g$ are virtually identical and differ only in phase.

If the received electric field is evaluated at $t = \frac{d''}{c}$, it can be expressed as

$$E_{TOT}(d, t = \frac{d''}{c}) = \frac{E_0d_0}{d''} \cos \left( \omega_c \left( \frac{d'' - d''}{c} \right) \right) - \frac{E_0d_0}{d''} \cos \theta$$

$$= \frac{E_0d_0}{d'} \cos \theta - \frac{E_0d_0}{d''}$$

$$\approx \frac{E_0d_0}{d} \left[ \cos \theta - 1 \right]$$

Referring to the phasor diagram of Figure 1.3, which shows how the direct and ground reflected rays combine, the electric field (at the receiver) at a distance $d$ from the transmitter can be written as
Using trigonometric identities, the above equation can be expressed as

$$|E_{TOT}(d)| = \frac{E_0 d_0}{d} \sqrt{2 - 2 \cos \theta_\Delta}$$

Where d Implies that

$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20h_t h_r}{\lambda}$$

The received E-field can be approximated as

$$E_{TOT}(d) \approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{V/m}$$

where k is a constant related to E0, the antenna heights, and the wavelength.

The received power at a distance d from the transmitter can be expressed as

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

At large distances \((d \gg \sqrt{h_t h_r})\) the received power falls off with distance raised to the fourth power, or at a rate of 40 dB/decade. This is a much more rapid path loss than is experienced in free space.
The path loss for the 2-ray model (with antenna gains) can be expressed in dB as

$$PL\ (\text{dB}) = 40 \log d - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$$