2.2 TWO RAY GROUND REFLECTION MODEL

Two ray model considers both the direct path and a ground reflected propagated path between transmitter and receiver.

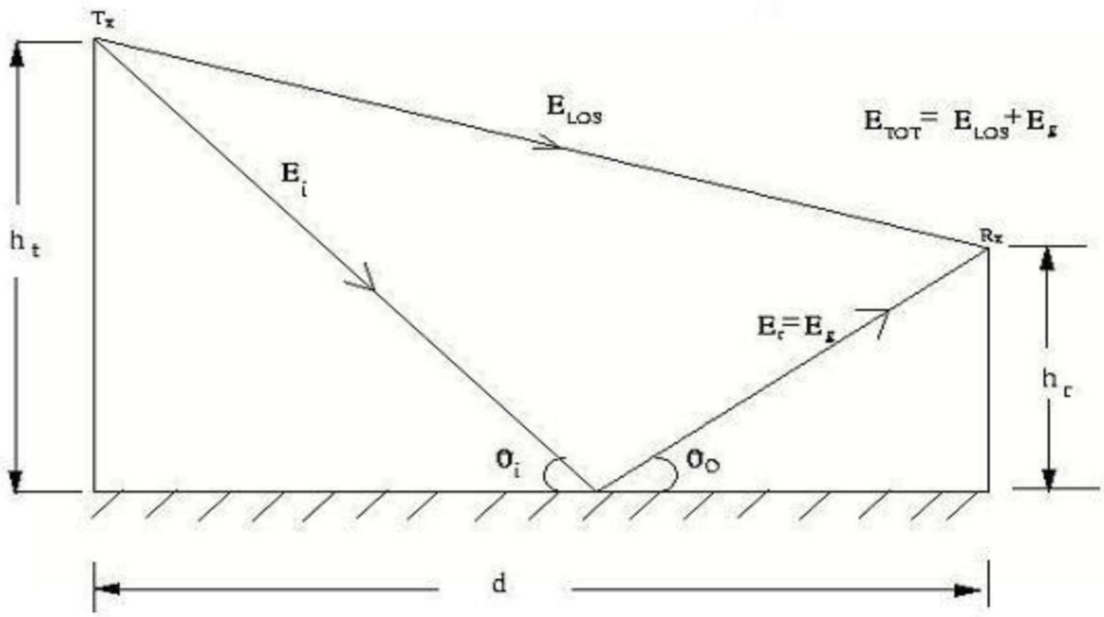

Fig1.2.1 : Two Ray Model

[Source : "Wireless communications "by Theodore S. Rappaport,Page- 86]

A two-ray model, which consists of two overlapping waves at the receiver, one direct path and one reflected wave from the ground.

The total received E-field E_{TOT} is the result of the direct line of sight component E**LOS** and the ground reflected component E**g**.

Referring to Figure 1.2.1, h**t** is the height of the transmitter and hr is the height of the receiver.

If E**0**is the free space electric field (in V/m) at a reference distance d**0** from the transmitter then for d>d0,

The free space propagating E-field is

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left(m_c\left(t - \frac{d}{c}\right)\right) \quad (d > d_0)$$

The envelop of the electric field at d meters from the transmitter at any time t is therefore

$$|E(d,t)| = \frac{E_0\ d_0}{d}$$

Two propagating waves arrive at the receiver, **one LOS wave** which travels a distance of d' and another **ground reflected wave**, that travels d''.

The E-field due to the line-of-sight component at the receiver can be expressed as

$$E_{LOS}(d',t) = \frac{E_0 d_0}{d'} \cos\left(\omega_c\left(t - \frac{d'}{c}\right)\right)$$

The E-fleld for the ground reflected wave, which has a propagation distance of d'', can be expressed as

$$E_g(d'',t) = \Gamma \frac{E_0 d_0}{d''} \cos\left(\omega_c\left(t - \frac{d''}{c}\right)\right)$$

According to the law of reflection in a dielectric,

$\theta_i = \theta_0$ and $E_g = \Gamma E_i$

$$E_t = (1 + \Gamma)E_i$$

where Γ is the reflection coefficient for ground.

For small values of θ_i (i.e., grazing incidence), the reflected wave is equal in magnitude and 180° out of phase with the incident wave.

The resultant total E-field envelope is given by

$$|E_{TOT}| = |E_{LOS} + E_g|$$

The electric field $E_{Tot}(d,t)$ can be expressed as

$$E_{TOT}(d,t) = \frac{E_0 d_0}{d'}\cos\left(\omega_c\left(t-\frac{d'}{c}\right)\right) + (-1)\frac{E_0 d_0}{d''}\cos\left(\omega_c\left(t-\frac{d''}{c}\right)\right)$$

Using the method of images, which is shown in Figure 1.2.2 , the path difference, Δ between the line-of-sight and the ground reflected paths can be expressed as

$$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

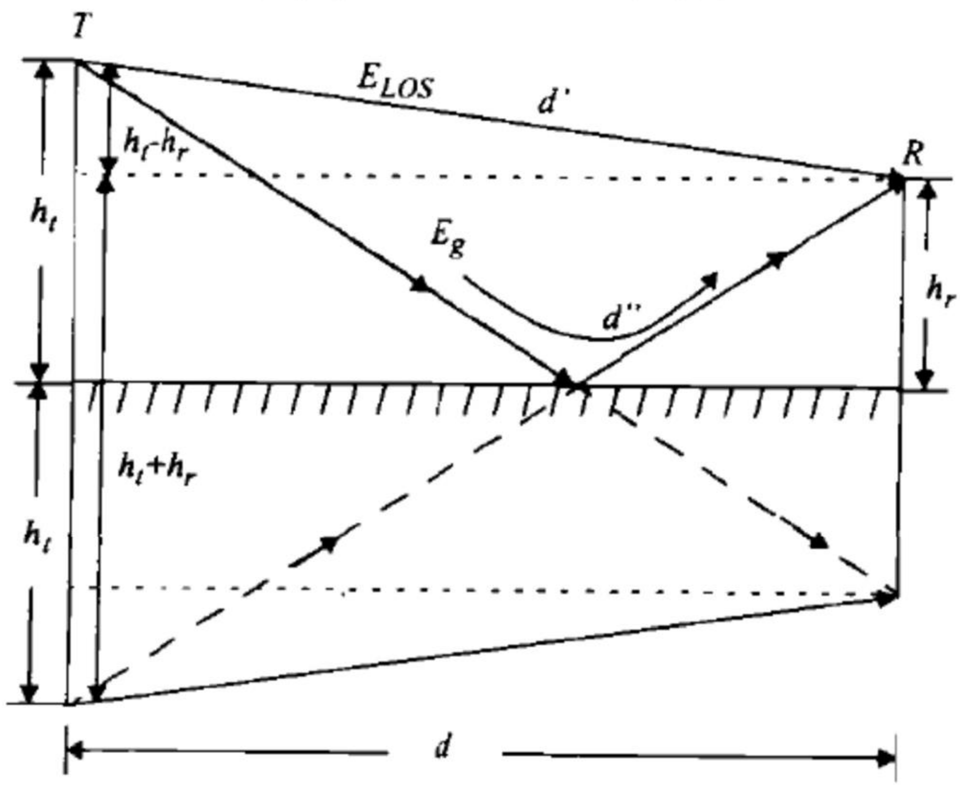

Fig 1.2.2: The Method of Images- Two Ray Model.

[Source : "Wireless communications "by Theodore S. Rappaport,Page- 87]

When the T-R separation distance d is very large compared to $h_t + h_r$, the above equation can be simplified using a Taylor series approximation

$$\Delta = d'' - d' \approx \frac{2h_t h_r}{d}$$

Once the path difference is known, the phase difference θ_Δ between the two Electric field components and the time delay r_d between the arrival of the two components can be easily computed using the following relations.

$$\theta_\Delta = \frac{2\pi\Delta}{\lambda} = \frac{\Delta\omega_c}{c}$$

And

$$\tau_d = \frac{\Delta}{c} = \frac{\theta_\Delta}{2\pi f_c}$$

It should be noted that as d becomes large, the difference between the distances d' and d'' becomes very small, and the amplitudes of E_{LOS} and Eg are virtually identical and differ only in phase.

$$\left|\frac{E_0 d_0}{d}\right| \approx \left|\frac{E_0 d_0}{d'}\right| \approx \left|\frac{E_0 d_0}{d''}\right|.$$

If the received electric field is evaluated at $t = \frac{d''}{c}$, it can be expressed as

$$\begin{aligned} E_{TOT}\left(d, t = \frac{d''}{c}\right) &= \frac{E_0 d_0}{d'} \cos\left(\omega_c\left(\frac{d'' - d'}{c}\right)\right) - \frac{E_0 d_0}{d''} \cos 0^o \\ &= \frac{E_0 d_0}{d'} \cos\theta_\Delta - \frac{E_0 d_0}{d''} \\ &\approx \frac{E_o d_0}{d} [\cos\theta_\Delta - 1] \end{aligned}$$

Referring to the phasor diagram of Figure 1.3, which shows how the direct and ground reflected rays combine, the electric field (at the receiver) at a distance d from the transmitter can be written as

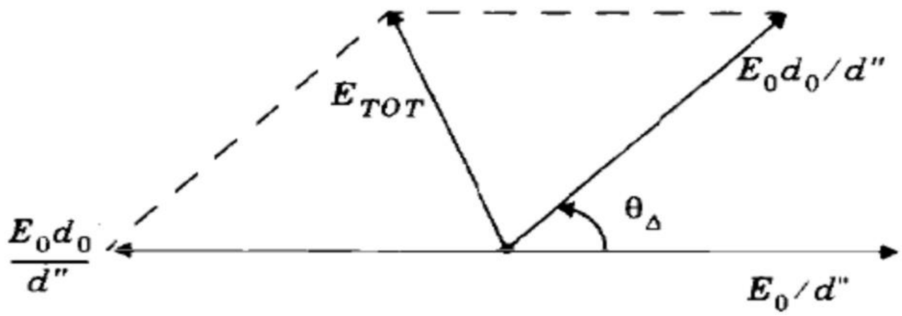

Fig 1.2.3: Phasor diagram

[Source : "Wireless communications" by Theodore S. Rappaport,Page- 89]

$$|E_{TOT}(d)| = \sqrt{\left(\frac{E_0 d_0}{d}\right)^2 (\cos\theta_\Delta - 1)^2 + \left(\frac{E_0 d_0}{d}\right)^2 \sin^2\theta_\Delta}$$

$$|E_{TOT}(d)| = \frac{E_0 d_0}{d}\sqrt{2 - 2\cos\theta_\Delta}$$

Using trigonometric identities, the above equation can be expressed as

$$|E_{TOT}(d)| = 2\frac{E_0 d_0}{d}\sin\left(\frac{\theta_\Delta}{2}\right)$$

Where d Implies that

$$d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$$

The received E-field can be approximated as

$$E_{TOT}(d) \approx \frac{2E_0 d_0}{d}\frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m}$$

where k is a constant related to E0, the antenna heights, and the wavelength.

The received power at a distance d from the transmitter can be expressed as

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

At large distances ($d \gg \sqrt{h_t h_r}$) the received power falls off with distance raised to the fourth power, or at a rate of 40 dB/ decade. This is a much more rapid path loss than is experienced in free space.

The path loss for the 2-ray model (with antenna gains) can be expressed in dB as

$$PL(\text{dB}) = 40\log d - (10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r)$$