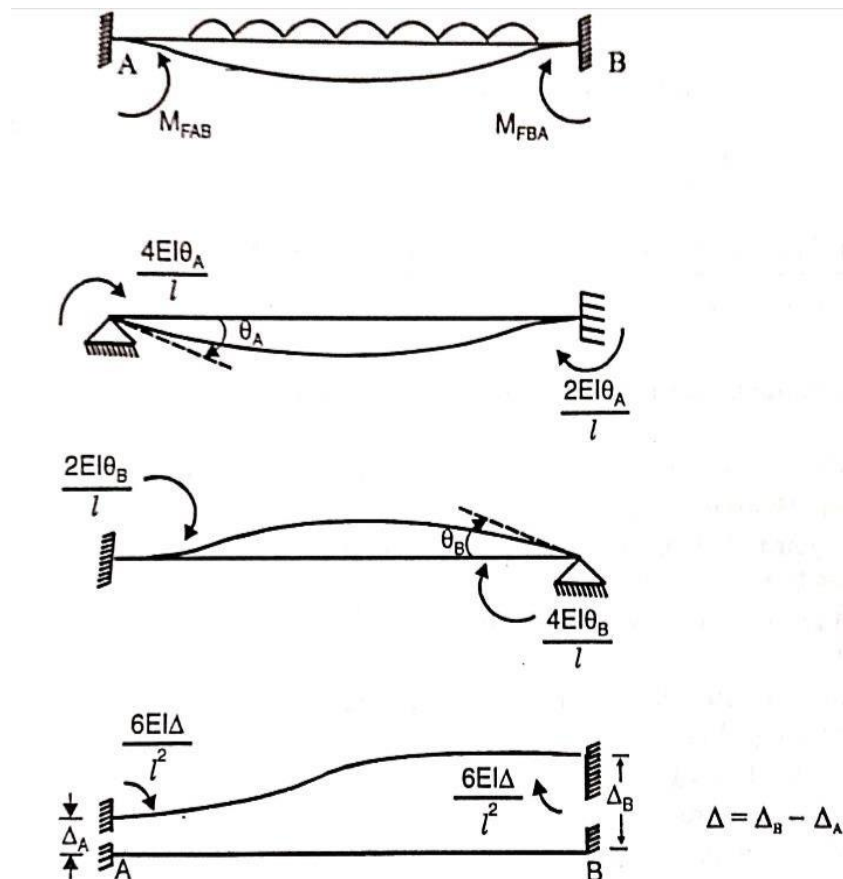


## 2.1 SLOPE DEFLECTION EQUATION

### Slope-Deflection Method of Analysis of Indeterminate Structures

#### 2.1.1 INTRODUCTION

In 1915, George A. Maney introduced the slope-deflection method as one of the classical methods of analysis of indeterminate beams and frames. The method accounts for flexural deformations, but ignores axial and shear deformations. Thus, the unknowns in the slope-deflection method of analysis are the rotations and the relative joint displacements. For the determination of the end moments of members at the joint, this method requires the solution of simultaneous equations consisting of rotations, joint displacements, stiffness, and lengths of members.



positive if its tangent turns in a clockwise direction. The rotation of the chord connecting the ends of a member ( $\Delta/L$ ) the displacement of one end of a member relative to the other, is positive if the member turns in a clockwise direction.

### 2.1.3 SLOPE DEFLECTION EQUATIONS

The slope-deflection equations, consider a beam of length  $L$  and of constant flexural rigidity  $EI$  loaded. The member experiences the end moments  $M_{FAB}$  &  $M_{FBA}$  at  $A$  and  $B$ , respectively. And undergoes the deformed shape with the assumption that the right end  $B$  of the member settles by an amount  $\Delta$ . The end moments are the summation of the moments caused by the rotations of the joints at the ends  $A$  and  $B$  ( $\theta_A$  and  $\theta_B$ ) of the beam, and it fixed at both ends referred to as fixed end moments ( $M_{FAB}$  &  $M_{FBA}$ )

**The slope equation is;**

**For member AB;**

$$M_{AB} = M_{FAB} + 2EI/L(2\theta_A + \theta_B + 3\delta/L)$$

$$M_{BA} = M_{FBA} + 2EI/L(2\theta_B + \theta_A + 3\delta/L)$$

Where,

$M_{AB}$  &  $M_{BA}$  – Final Moments at members  $AB$  and  $BA$ .

$M_{FAB}$  &  $M_{FBA}$  – Fixed end moments.

$E$  &  $I$  – young's modulus and moment of inertia.

$\theta_A$  &  $\theta_B$  – slope angles.

$\delta$  – Deflection

$L$  - Length

### 2.1.4 EQUILIBRIUM CONDITIONS

- Joint equilibrium conditions
- Shear equilibrium conditions

## JOINT EQUILIBRIUM

Joint equilibrium conditions imply that each joint with a degree of freedom should have no unbalanced moments i.e. be in equilibrium. Therefore,

Sum of (end moments + fixed end moments) = Sum of external moments directly applied at the joint.

$$M_{BA} + M_{BC} = 0;$$

## SHEAR EQUILIBRIUM

When there are chord rotations in a frame, additional equilibrium conditions, namely the shear equilibrium conditions need to be taken into account.

### 2.1.5 ANALYSIS OF INDETERMINATE BEAMS

The procedure for the analysis of indeterminate beams by the slope-deflection method is summarized below.

Procedure for Analysis of Indeterminate Beams and Non-Sway Frames by the Slope-Deflection Method

- Determine the fixed-end moments for the members of the beam.
- Determine the rotations of the chord if there is any support settlement.
- Write the slope-deflection equation for the members' end moments in terms of unknown rotations.
- Write the equilibrium equations at each joint that is free to rotate in terms of the end moments of members connected at that joint.
- Solve the system of equations obtained simultaneously to determine the unknown joint rotations.
- Substitute the computed joint rotations into the equations obtained in step 3 to determine the members' end moments.

- Draw a free-body diagram of the indeterminate beams indicating the end moments at the joint.
- Draw the shearing force diagrams of the beam by considering the freebody diagram of each span of the beam in the case of a multi-span structure.

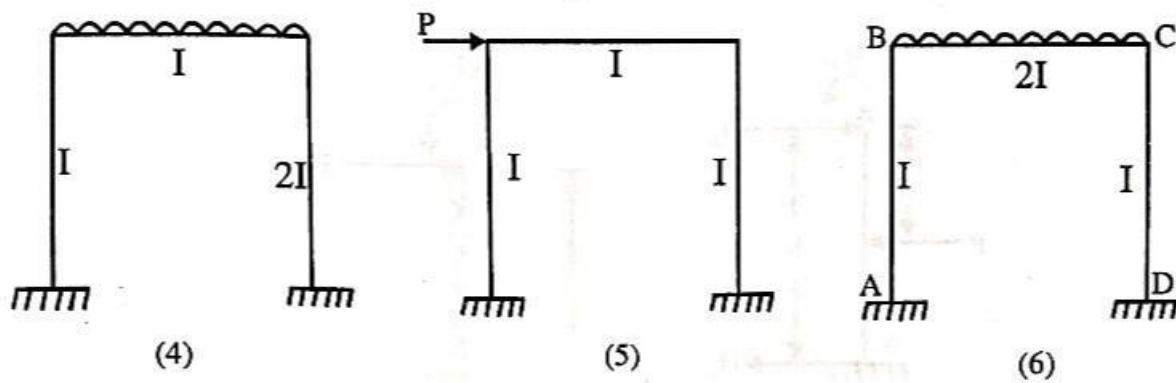
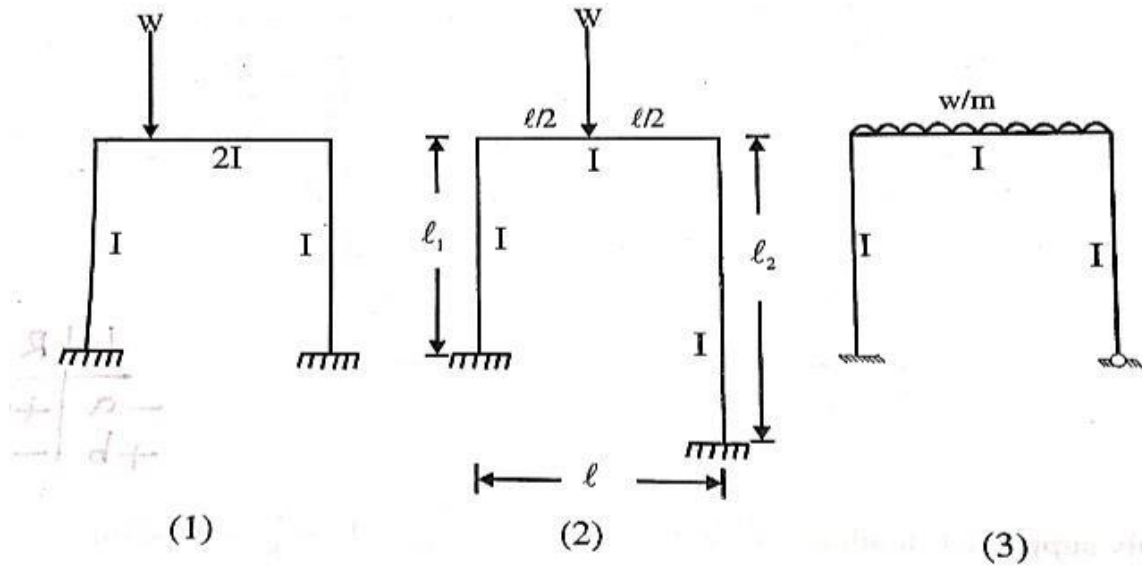
### **2.1.6 ANALYSIS OF INDETERMINATE FRAMES**

Indeterminate frames are categorized as frames with or without side-sway. A frame with side-sway is one that permits a lateral moment or a swaying to one side due to the asymmetrical nature of its structure or loading. The analysis of frames without side-sway is similar to the analysis of beams considered in the preceding section, while the analysis of frames with side-sway requires taking into consideration the effect of the lateral movement of the structure.

### **2.1.7. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.**

Portal frames may sway due to one of the following reasons:

- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.



D Settles down / Sinks by  $\delta$

