

### 2.3 Terzaghi equation:

Terzaghi (1943) was the first to propose a comprehensive theory for evaluating the safe bearing capacity of shallow foundation with rough base.

#### Assumptions:

1. Soil is homogeneous and Isotropic.
2. The shear strength of soil is represented by Mohr Coulombs Criteria.
3. The footing is of strip footing type with rough base. It is essentially a two dimensional plane strain problem.
4. Elastic zone has straight boundaries inclined at an angle equal to the horizontal.
5. Failure zone is not extended above, beyond the base of the footing. Shear resistance of soil above the base of footing is neglected.
6. Method of superposition is valid.
7. Passive pressure force has three components ( $P_{PC}$  produced by cohesion,  $P_{Pq}$  produced by surcharge and  $P_{P\gamma}$  produced by weight of shear zone).
8. Effect of water table is neglected.
9. Footing carries concentric and vertical loads.
10. Footing and ground are horizontal.
11. The properties of foundation soil do not change during the shear failure
12. Limit equilibrium is reached simultaneously at all points. Complete shear failure is mobilized at all points at the same time.

#### Limitations:

1. The theory is applicable to shallow foundations
2. As the soil compresses, increases which is not considered. Hence fully plastic zone may not develop at the assumed.
3. All points need not experience limit equilibrium condition at different loads.
4. Method of superposition is not acceptable in plastic conditions as the ground is near failure zone.

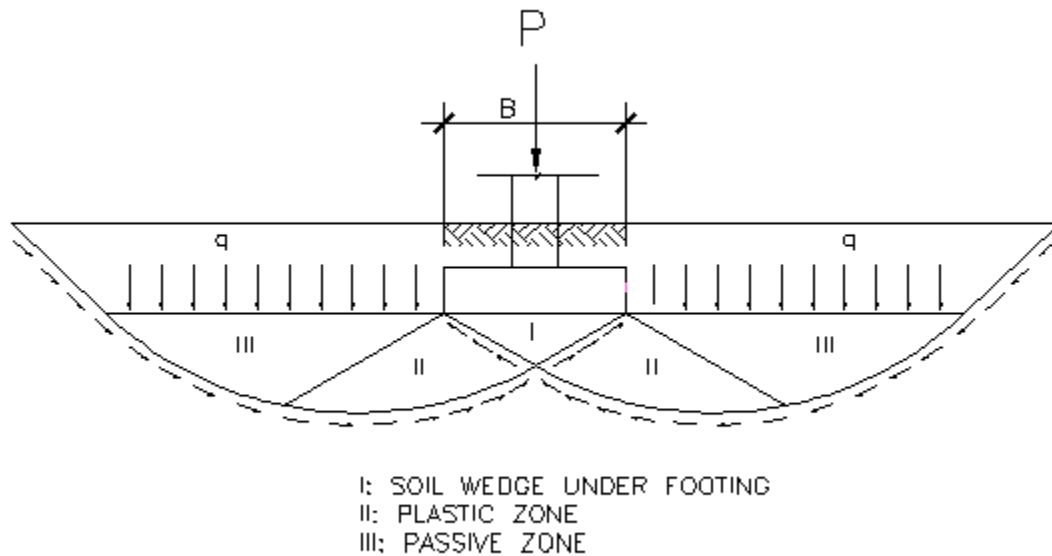


Fig 1 Shear stresses based on Terzaghi's soil bearing capacity theory.

[Fig 1 <https://civilengineeringbible.com/subtopics.php?i=1>]

Terzaghi's concept of Footing with five distinct failure zones in foundation soil

- The soil is semi-infinite, homogeneous and isotropic
- The problem is two-dimensional
- The base of the footing is rough
- The failure is by general shear
- the load is vertical and symmetrical
- The ground surface is horizontal
- the overburden pressure at foundation level is equivalent to a surcharge load
- the principle of superposition is valid,

Coulomb's law is strictly valid, that is

$$r = C + \sigma \tan \phi$$

### Mechanism of Failure:

- The shapes of the failure surfaces under ultimate loading conditions are given in Fig.
- The zones of plastic equilibrium represented in this figure by the area *gedcf* may be subdivided into three zones:
  - 1 . Zone I of elastic equilibrium
  2. Zones II of radial shear state

### 3. Zones III of Rankine passive state

- When load  $q_u$  per unit area acting on the base of the footing of width  $B$  with a rough base is transmitted into the soil, the tendency of the soil located within zone I is to spread but this is counteracted by friction and adhesion between the soil and the base of the footing.
- Due to the existence of this resistance against lateral spreading, the soil located immediately beneath the base remains permanently in a state of elastic equilibrium, and the soil located within this central Zone I behaves as if it were a part of the footing and sinks with the footing under the superimposed load.

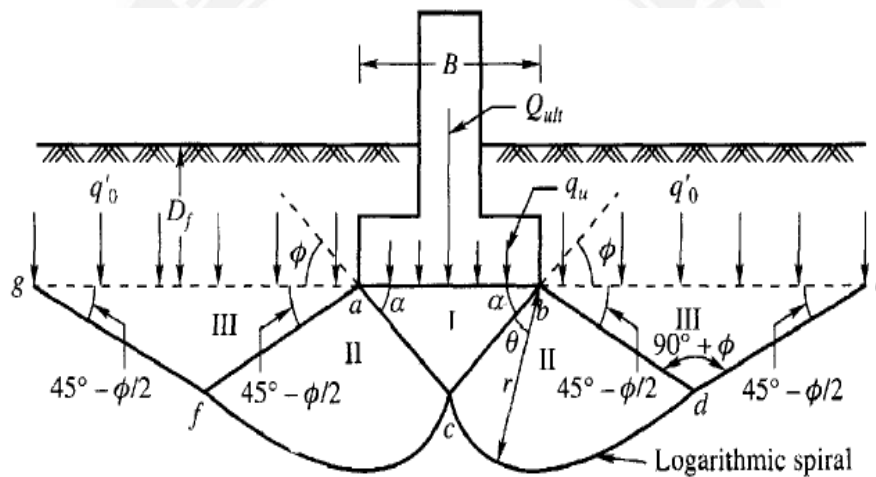
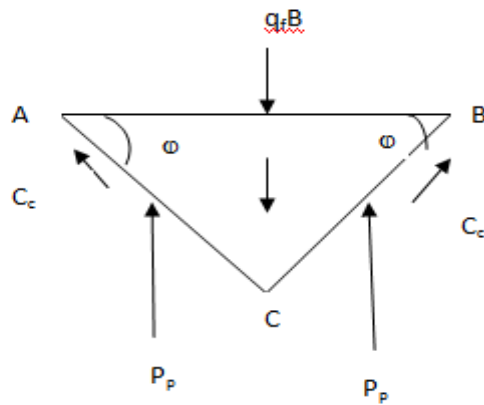


Fig 2 Shear stresses based on Terzaghi's soil bearing capacity theory

[Fig 2 <https://www.pinterest.com/pin/682013937310128083/>]

- The depth of this wedge shaped body of soil  $abc$  remains practically unchanged, yet the footing sinks.
- This process is only conceivable if the soil located just below point  $c$  moves vertically downwards. This type of movement requires that the surface of sliding  $cd$  (Fig.) through point  $c$  should start from a vertical tangent. The boundary  $be$  of the zone of radial shear bed (Zone II) is also the surface of sliding.
- As per the theory of plasticity, the potential surfaces of sliding in an ideal plastic material intersect each other in every point of the zone of plastic equilibrium at an angle  $(90^\circ - \phi)$ . Therefore, the boundary  $be$  must rise at an angle  $\phi$  to the horizontal provided the friction and adhesion between the soil and the base of the footing suffice to prevent a sliding motion at the base.

- The sinking of Zone I creates two zones of plastic equilibrium, II and III, on either side of the footing. Zone II is the radial shear zone whose remote boundaries bd and af meet the horizontal surface at angles  $(45^\circ - \phi/2)$ , whereas Zone III is a passive Rankine zone. The boundaries de and fg of these zones are straight lines and they meet the surface at angles of  $(45^\circ - \phi/2)$ . The curved parts cd and cf in Zone II are parts of logarithmic spirals whose centers are located at b and a respectively.



Downward force:

i) weight of soil wedge ABC

$$w = \frac{1}{4} \gamma B^2 \tan \phi$$

ii) Total load on footing  $q_f B$

Upward force:

i) passive force

ii) cohesion(c)

Length of AC and CB

$$\cos \phi = \frac{\text{adj}}{\text{hypo}} = \frac{\frac{B}{2}}{AC}$$

$$AC = \frac{B}{2} \frac{1}{\cos \phi}$$

$$\text{Vertical component } C = \left( \frac{B/2}{\cos \phi} \cdot C \right) \sin \phi$$

$$C = \left(\frac{B}{2} \cdot C\right) \tan \phi$$

$$\text{vertical component of } C = \frac{B}{2} C \tan \phi$$

$$i) 2P_p$$

$$ii) \frac{B}{2} \gamma \tan \phi \times 2C$$

Upward = Downward

$$2P_p + BC \tan \phi = q_f B + \frac{1}{4} \gamma B^2 \tan \phi$$

$$q_f B = 2P_p + BC \tan \phi - \frac{1}{4} \gamma B^2 \tan \phi$$

The resultant passive earth pressure has 3 component

i)  $P_{p\gamma} \rightarrow$  Produced by weight of shear zone BCDE

ii)  $P_{pc} \rightarrow$  Produced by cohesion

iii)  $P_{pq} \rightarrow$  Produced by surcharge  $q$

$$q B = P_f + P_{pc} + P_{pq} + BC \tan \phi - \frac{1}{4} \gamma B^2 \tan \phi$$

$$q B = P_f + P_{pc} + 2P_{pq} + BC \tan \phi - \frac{1}{4} \gamma B^2 \tan \phi$$

$$q B = [2P_f - \frac{1}{4} \gamma B^2 \tan \phi] + P_{pc} + 2P_{pq}$$

$$\text{Let, } 2P_f - \frac{1}{4} \gamma B^2 \tan \phi = B \times \frac{1}{2} \gamma B N_\gamma$$

$$2P_{pc} + BC \tan \phi = BcN_c$$

$$2P_{pq} = B\gamma DN_q$$

Substitute in above equation

$$q_f B = B \times \frac{1}{2} \gamma B N_\gamma + BcN_c + B\gamma DN_q$$

$$q_f B = B \left[ \frac{1}{2} \gamma B N_\gamma + c N_c + \gamma D N_q \right]$$

$$q_f = \left[ \frac{1}{2} \gamma B N_\gamma + c N_c + \gamma D N_q \right]$$

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$N_c, N_q, N_\gamma$  = Bearing Capacity factor which are dimensionless depend on angle of shear resistance

$$N_q = \left[ \frac{a^2}{2 \cos^2 \left( 45 + \frac{\phi}{2} \right)} \right]$$

$$a = e^{\left( \frac{3\pi}{4} - \frac{\phi}{2} \right) \tan \phi}$$

$$N_c = (N_q - 1) \cos \phi$$

$$N_\gamma = \frac{1}{2} \left[ \frac{K_p}{\cos \phi} - 1 \right] \tan \phi$$

**Ultimate bearing capacity,**

$$q_f = cN_c + \gamma D N_q + 0.5 \gamma B N_\gamma$$

If the ground is subjected to additional surcharge load  $q$ , then

$$q_f = cN_c + (\gamma D + q) N_q + 0.5 \gamma B N_\gamma$$

Net ultimate bearing capacity,

$$q_n = cN_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma - \gamma D$$

$$q_n = cN_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma$$

Safe bearing capacity,

$$q_s = cN_c + \gamma D (N_q - 1) + 0.5 \gamma B N_\gamma / F + \gamma D$$

Here,  $F$  = Factor of safety (usually 3)

$c$  = cohesion

$\gamma$  = unit weight of soil

$D$  = Depth of foundation

$q$  = Surcharge at the ground level

$B$  = Width of foundation

$N_c, N_q, N_\gamma$  = Bearing Capacity factors

$$N_c = \cot \phi (N_q - 1)$$

$$N_q = e^{2(3\pi/4 - \phi/2) \tan \phi} / [2 \cos^2(45 + \phi/2)]$$

$$N_\gamma = (1/2) \tan \phi (K_{pr} / \cos^2 \phi - 1)$$

$K_p$ =passive pressure coefficient.

$$K_p = \frac{1 + \sin\phi}{1 - \sin\phi}$$

$K_p$ = coefficient of passive earth pressure.

Strip footings:  $q_f = c N_c + \gamma D N_q + 0.5 \gamma B N_\gamma$

Square footings:  $q_f = 1.3 c N_c + \gamma D N_q + 0.4 \gamma B N_\gamma$

Circular footings:  $q_f = 1.3 c N_c + \gamma D N_q + 0.3 \gamma B N_\gamma$

Rectangular footing:  $q_f = \left[1 + 0.3 \frac{B}{L}\right] c N_c + \gamma D N_q + \left[1 - 0.3 \frac{B}{L}\right] \gamma B N_\gamma$

**Note:**

Local shear failure( $\Phi < 28^\circ$ ) -----  $N'_c, N'_q, N'_\gamma$

General shear failure( $\Phi > 36^\circ$ ) -----  $N_c, N_q, N_\gamma$

### Terzaghi's Problems:

1. A square footing 2.5 m x 2.5 m is built in a homogeneous bed of sand of unit weight 20 KN/m<sup>3</sup> and having an angle of shearing resistance of 36°. The depth of the base of footing is 1.5 m below the ground surface. Find the safe load that can be carried by a footing with a factor of safety of 3 against complete shear failure. Use Terzaghi's analysis.

Given Data;

$$L=B=2.5\text{m}$$

$$D=1.5\text{m}$$

$$\text{Unit weight}(\gamma)=20\text{KN/m}^3$$

$$\Phi=36^\circ$$

$$C=0$$

$$\text{FOS}=3$$

From the graph

$$N'_c=60, N'_q=42, N'_\gamma=50$$

To find:

$$\text{Safe load}=?$$

Solution:

Here  $\Phi=36^\circ$  therefore it is general shear failure. The equation can be written as

Bearing capacity of soil,

$$q_f = [cN'_c + \gamma DN'_q + 0.4\gamma BN'_\gamma]$$

$$q_f = [(0 \times 60) + (20 \times 1.5 \times 42) + (0.4 \times 20 \times 2.5 \times 50)]$$

$$= 2260 \text{ KN/m}^2$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

$$q_{nf} = 2260 - (20 \times 1.5)$$

$$= 2230 \text{ KN/m}^2$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$q_s = \frac{q_{nf}}{F} + \gamma D$$

$$q_s = \frac{2230}{3} + 20 \times 1.5$$

$$= 743.33 + 30$$

$$= 773.3 \text{ KN/m}^2$$

$$q_s = \frac{\text{load}}{\text{Area}}$$

$$\text{safe load}(W) = q_s \times \text{Area}$$

$$\text{Area} = B^2 = (2.5)^2 = 6.25 \text{ m}^2$$

$$W = 773.3 \times 6.25 = 4833.3 \text{ KN}$$

2. A square footing located at a depth of 1.5 m below the ground surface in Cohesion less soil carries a column load of 1280 kN. The soil is submerged having an effective unit weight of 11.5 kN/m<sup>3</sup> and an angle of shearing resistance of 30°. Show and find the size of the footing for Fos = 3 by Terzaghi's theory of general shear failure.

Given Data;

B=?

D=1.5m

Unit weight( $\gamma$ )=11.5KN/m<sup>3</sup>

$\Phi=30^\circ$

Cohesion less,  $C=0$

Load=1280KN

From the graph

$N_c=37.2$  ,  $N_q=22.5$  ,  $N_\gamma=19.7$

To find:

Size of footing (B)=?

Solution:

Bearing capacity of soil,

$$q_f = [cN_c + \gamma DN_q + 0.4\gamma BN_\gamma]$$

$$q_f = [0 \times 37.2 + 11.5 \times 1.5 \times 22.5 + 0.4 \times 11.5 \times B \times 19.7]$$

$$q_f = 388.125 + 90.62B$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

$$q_{nf} = 388.125 + 90.62B - (11.5 \times 1.5)$$

$$q_{nf} = 388.125 + 90.62B - 17.25$$

$$q_{nf} = 370.875 + 90.62B$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$q_s = \frac{370.875 + 90.62B}{3} + (11.5 \times 1.5)$$

$$q_s = \frac{370.875 + 90.62B}{3} + 17.25$$

$$q_s = \frac{370.875 + 90.62B + (3 \times 17.25)}{3}$$

$$q_s = \frac{370.875 + 90.62B + 51.75}{3}$$

$$q_s = \frac{422.625 + 90.62B}{3}$$

$$q_s = \frac{\text{load}}{\text{Area}}$$

$$\text{safe load}(W) = q_s \times \text{Area}$$

$$\text{safe load}(W) = q_s \times B^2$$

$$1280 = \frac{422.625 + 90.62B}{3} \times B^2$$

$$3840 = (422.65 + 90.62B)B^2$$

$$3840 = 422.65B^2 + 90.62B^3$$

$$90.62B^3 + 422.65B^2 + 0B - 3840 = 0$$

$$B = 2.44\text{m}$$

$$\text{Size of footing} = 2.44 \times 2.44\text{m}$$

3. A rectangular footing (2x3m) rests on a C-φ soil which is base and 1.5m below the ground surface. Calculate the safe bearing capacity using FOS=3, C=10KN/m<sup>3</sup>, φ=30 degree. N<sub>c</sub> = 31.2, N<sub>q</sub> = 22.5 and N<sub>γ</sub> = 19.7 and also soil has following properties voids ratio=0.55, degree of saturation=50%, specific gravity=2.67.

Given data:

$$B = 2\text{m}$$

$$L = 3\text{m}$$

$$D = 1.5\text{m}$$

$$\text{FOS} = 3$$

$$C = 10\text{KN/m}^3$$

$$\phi = 30$$

$$N_c = 31.2, N_q = 22.5 \text{ and } N_\gamma = 19.7$$

$$\text{voids ratio}(e) = 0.55,$$

$$\text{degree of saturation}(S_r) = 50\% = 0.5$$

$$\text{specific gravity}(G) = 2.67$$

To find:

$$\text{Safe bearing capacity}(q_s) = ?$$

Solution:

$$\gamma = \frac{(G + eS_r)\gamma_w}{1 + e}$$

$$\gamma = \frac{(2.67 + 0.55 \times 0.5)9.81}{1 + 0.55}$$

$$\gamma = 18.639 \text{ KN/m}^3$$

Bearing capacity of soil,

$$q_f = [(1 + 0.3) \frac{B}{L} c N_c + \gamma D N_q + (1 - 0.3) \frac{B}{L} \times 0.5 \gamma B N_\gamma]$$

$$q_f = [(1 + 0.3) \frac{2}{3} 10 \times 31.2 + 18.639 \times 1.5 \times 22.5 + (1 - 0.3) \frac{2}{3} \times 0.5 \times 18.639 \times 2 \times 19.7]$$

$$q_f = 1158.82 \text{ KN/m}^2$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

$$q_{nf} = 1158.82 - (18.639 \times 1.5)$$

$$q_{nf} = 1130.86 \text{ KN/m}^2$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$q_s = \frac{1130.86}{3} + (18.639 \times 1.5)$$

$$q_s = 404.90 \text{ KN/m}^2$$

**Effect of water table:**

Ultimate bearing capacity with the effect of water table,

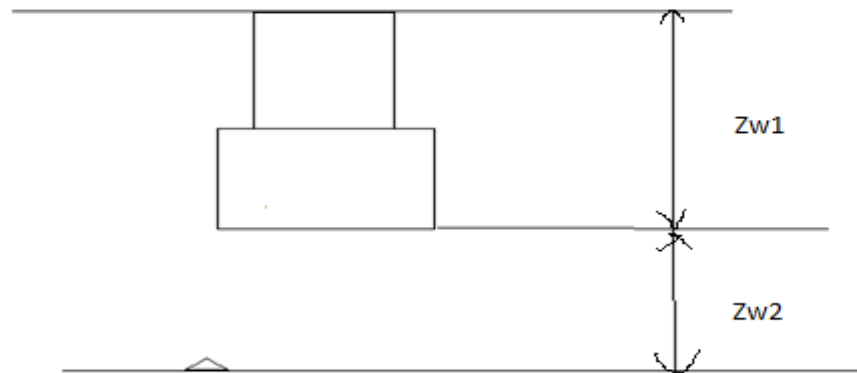
$$q_f = c N_c + \gamma D N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2}$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

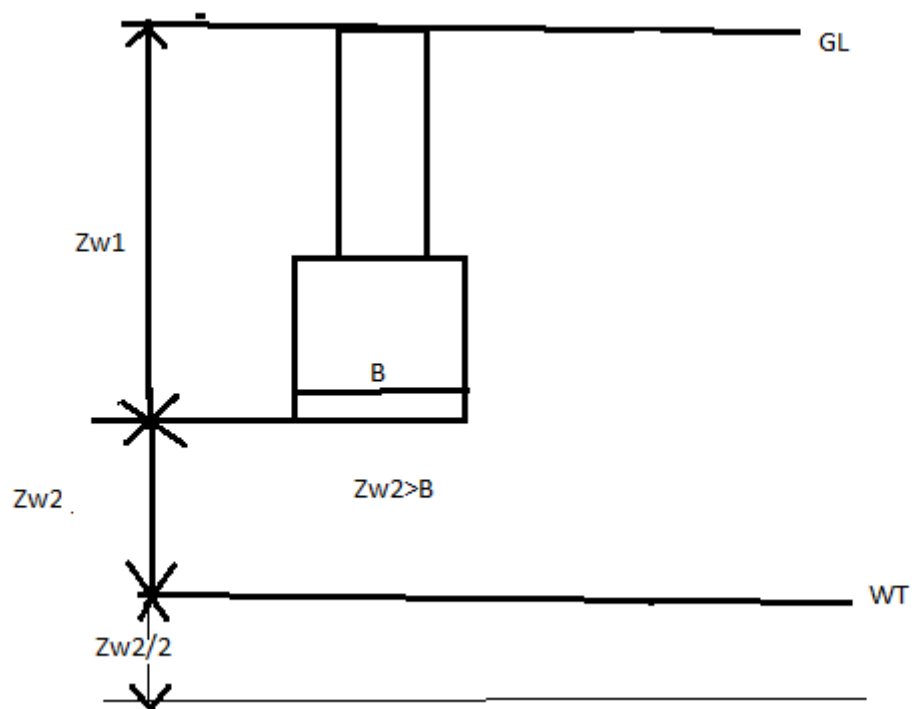
$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

**Case1: water level Below the Footing:**

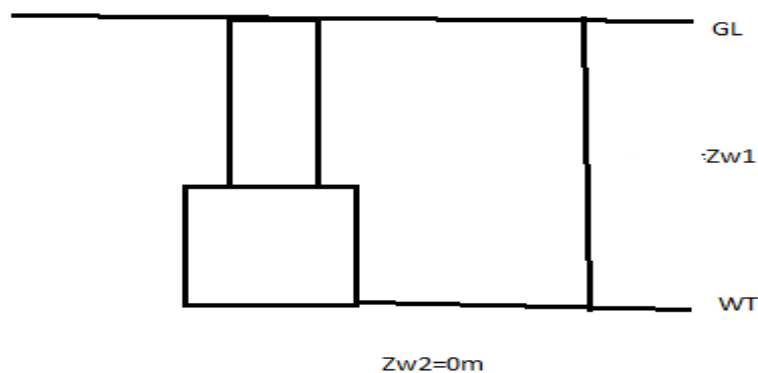
a)  $Z_{w2} < B$



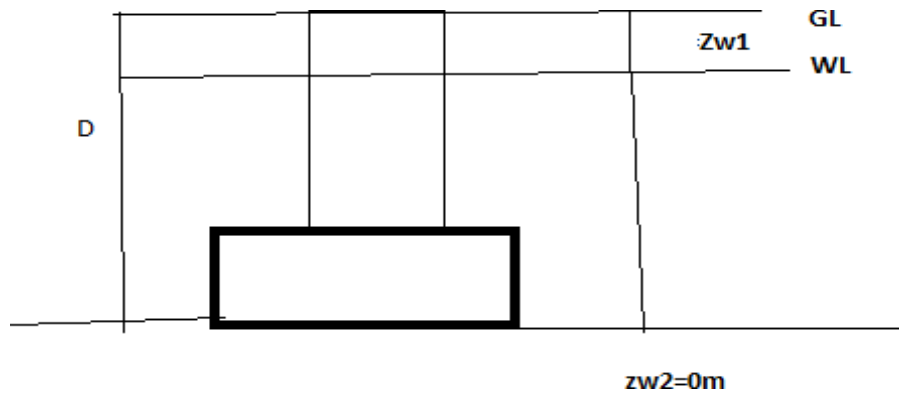
b)  $Z_{w2} > B$



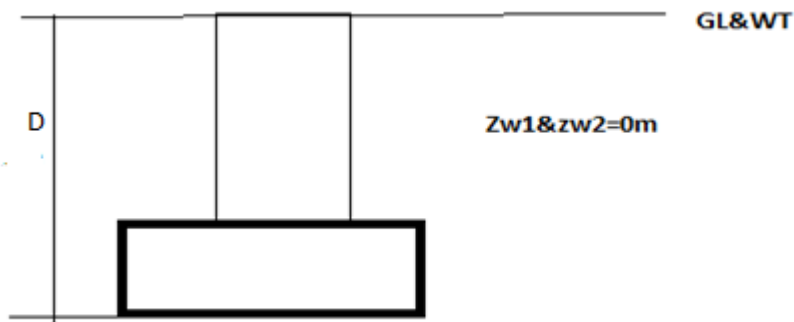
Case2: water level at the base of Footing:



Case3: water level above the Footing:



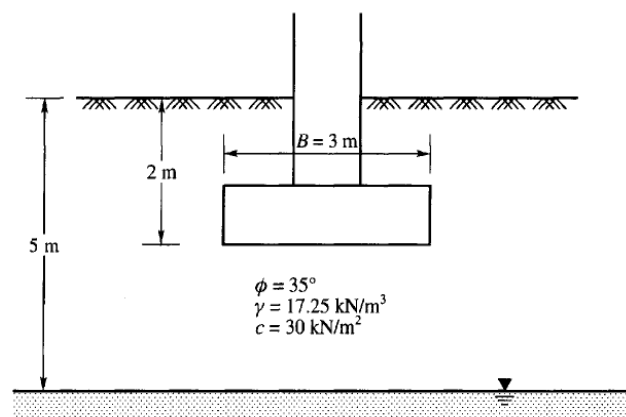
#### Case4: water level at Ground level:



#### Terzaghi's Problem with water table:

1. A strip footing of width 3 m is founded at a depth of 2 m below the ground surface in a  $(c - \phi)$  soil having a cohesion  $c = 30 \text{ kN/m}^2$  and angle of shearing resistance  $\phi = 35^\circ$ . The water table is at a depth of 5 m below ground level. The moist weight of soil above the water table is  $17.25 \text{ kN/m}^3$ .

Determine (a) the ultimate bearing capacity of the soil, (b) the net bearing capacity, and (c) the net allowable bearing pressure and the load/m for a factor of safety of 3. Use the general shear failure theory of Terzaghi.



**Given data:**

strip foundation

Width=3 m

Depth of foundation D =2m

$\phi=35^\circ$

$C=30\text{KN/m}^3$

$\gamma=17.25\text{KN/m}^3$

Fos=3

$N_c = 57.8, N_q = 41.4$  and  $N_\gamma = 42.4$

**To find:**

- (a) the ultimate bearing capacity of the soil,
- (b) the net bearing capacity, and
- (c) the net allowable bearing pressure and the load/m

Solution:

$$q_f = \left[ \frac{2}{3} c N_c + \gamma D N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = 1\text{m}$$

$Z_{w1}$  = depth of foundation from GL=2m

$Z_{w2}$  = depth of foundation to water level=5-2=3m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = 1\text{m}$$

$$q_f = \left[ \left( \frac{2}{3} \right) \times 30 \times 57.8 + (17.25 \times 2 \times 41.4 \times 1) + (0.5 \times 17.25 \times 3 \times 42.2 \times 1.25) \right]$$

$$q_f = [1156.6 + 1428.3 + 1097.1]$$

$$q_f = 3681 \text{KN/m}^2$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \sigma$$

$$q_{nf} = q_f - \gamma D$$

$$q_{nf} = 3681 - (17.25 \times 2)$$

$$q_{nf} = 3681 - 34.5 = 3646.5 \text{ KN/m}^2$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \sigma$$

$$q_s = \frac{3646.5}{3} + (17.25 \times 2)$$

$$q_s = 1250 \text{ KN/m}^2$$

2. A square foundation of size  $1.8\text{m} \times 1.8\text{m}$  is to be built at a depth of  $1.6\text{m}$  on a uniform clay strata having the following properties  $\phi = 0^\circ$ ,  $c = 30 \text{ KN/m}^2$  and  $\gamma = 18.2 \text{ KN/m}^3$ . Find the safe load that the foundation can carry with a factor of safety of 3. Use Terzaghi's bearing capacity theory. If the ground water table subsequently rises from depth of  $6\text{m}$  to the ground surface, find the load carrying capacity of the foundation. The submerged density of the soil is  $10.5 \text{ KN/m}^3$ .

Given data:

Square foundation size =  $1.8\text{m} \times 1.8\text{m}$

Depth of foundation =  $1.6\text{m}$

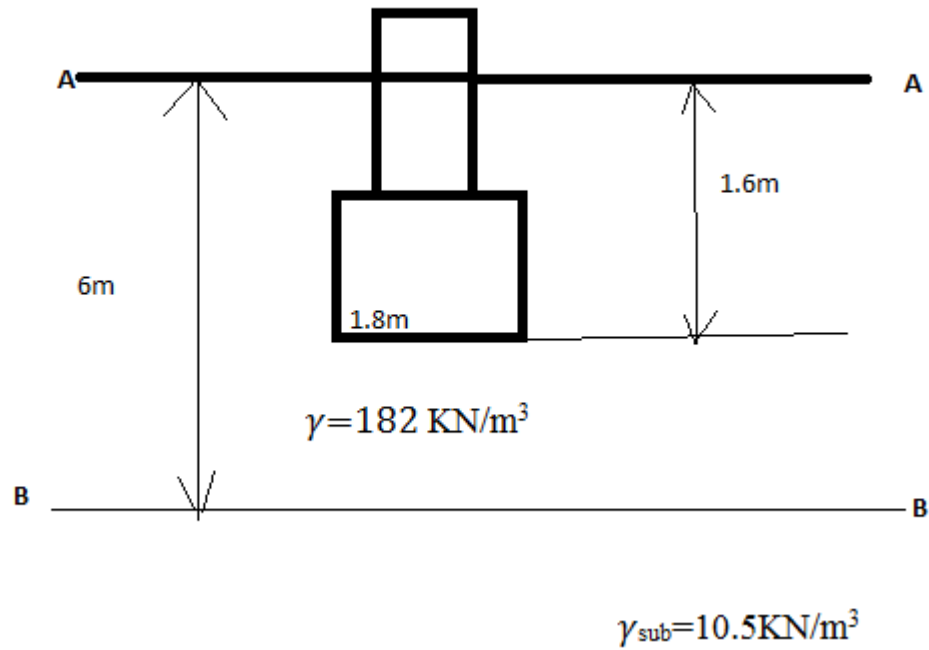
$\phi = 0$

$c = 30 \text{ KN/m}^2$

$\gamma = 18.2 \text{ KN/m}^3$

Fos = 3

$\gamma_{\text{sub}} = 10.5 \text{ KN/m}^3$



To find:

- i) case-i: water table at 6m from G.L. safe load=?
- ii) case-ii: water table at the ground surface safe load=?

Solution:

Case (1): water table at 6m from ground surface.

Bearing capacity of soil

$$q_f = [cN_c + \gamma D N_q R_{w1} + 0.4 \gamma_{\text{avg}} B N_\gamma R_{w2}]$$

Net Bearing capacity of soil,

$$q_{\text{nf}} = q_f - \bar{\sigma}$$

$$q_{\text{nf}} = q_f - \gamma D$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{\text{nf}}}{F} + \frac{\bar{\sigma}}{Z_{w1}}$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.6}{1.6} \right]$$

$$R_{w1} = 1$$

$Z_{w1}$  = depth of foundation from GL = 1.6m

$Z_{w2}$  = depth of foundation to water level = 6 - 1.6 = 4.4m,  $Z_{w2} > B$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{4.4}{1.8} \right]$$

$$R_{w2} = 1.72m$$

For  $\phi=0$  [Terzaghi's Bearing capacity factor]

$$N_c=5.7, N_q=1.0, N_r=0$$

$$\gamma_{avg} = \frac{(182 \times 4.4) + \left[ \frac{(4.4)}{2} \times 10.5 \right]}{\left[ 4.4 + \frac{4.4}{2} \right]}$$

$$\gamma_{avg} = 124.8 \text{ kN/m}^3$$

Bearing capacity of soil

$$q_f = [cN_c + \gamma D N_q R_{w1} + 0.4 \gamma_{avg} B N_\gamma R_{w2}]$$

$$q_f = [(30 \times 5.7) + (182 \times 1.6 \times 1 \times 1) + (0.4 \times 124.8 \times 1.8 \times 0 \times 1.875)]$$

$$q_f = 462.2 \text{ kN/m}^2$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = 462.2 - (182 \times 1.6)$$

$$q_{nf} = 171 \text{ kN/m}^2$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$171$$

$$q_s = \frac{171}{3} + (182 \times 1.6)$$

$$q_s = 348.2 \text{ kN/m}^2$$

$$q_s = \frac{\text{load}}{\text{Area}}$$

$$\text{safe load}(W) = q_s \times \text{Area}$$

$$\text{safe load}(W) = q_s \times B^2$$

$$\text{safe load}(W) = 348.2 \times (1.8)^2$$

$$W = 1128.2 \text{ kN}$$

Case (2) if water table at the ground surface

Bearing capacity of soil

$$q_f = [cN_c + \gamma DN_q R_{w1} + 0.4\gamma BN_\gamma R_{w2}]$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{0}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{0}{1.6} \right]$$

$$R_{w1} = 0.5m$$

Zw1= depth of foundation from GL=0m

Zw2= depth of foundation to water level=0m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{0}{1.8} \right]$$

$$R_{w2} = 0.5m$$

$$q_f = [cN_c + \gamma DN_q R_{w1} + 0.4\gamma BN_\gamma R_{w2}]$$

$$q_f = [(30 \times 5.7) + (182 \times 1.6 \times 1 \times 0.5) + (0.4 \times 182 \times 1.8 \times 0.5)]$$

$$q_f = [171 + 145.6 + 65.52]$$

$$q_f = 391.12 \text{ KN/m}^2$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

$$q_{nf} = 391.12 - (182 \times 1.6)$$

$$= 99.92 \text{ KN/m}^2$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$q_s = \frac{99.92}{3} + (182 \times 1.6)$$

$$q_s = 324.5 \text{ kN/m}^2$$

$$\text{safe load}(W) = q_s \times \text{Area}$$

$$\text{safe load}(W) = q_s \times B^2$$

$$\text{safe load}(W) = 324.5 \times (1.8)^2$$

$$W = 1051.38 \text{ kN}$$

3. A foundation, 2.0 m square of depth 1.2 m is installed 1.2 m above the water table and a submerged density of  $10 \text{ kN/m}^3$ . The strength parameters with respect to effective stress  $c' = 0$  and  $\phi = 30^\circ$ . Find the gross ultimate bearing capacity for the following conditions.

1. Water table is 1.2 m below the base of the foundation.
2. Water table raise to the level of the base of the foundation and
3. The water table rise to ground level. (For  $\phi = 30^\circ$ , Assume  $N_q = 22$  and  $N_\gamma = 20$ ).

Solution:

Square footing ( $2\text{m} \times 2\text{m}$ )

$$C = 0, \phi = 30^\circ$$

$$N_q = 22, N_\gamma = 20$$

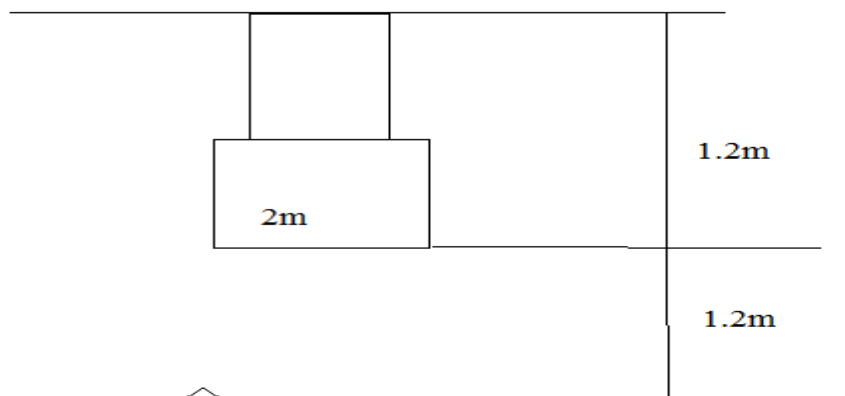
$$\gamma_{\text{sub}} = 10 \text{ kN/m}^3$$

$$\gamma_{\text{sub}} = \gamma_{\text{sat}} - \gamma_w$$

$$\gamma_{\text{sat}} = \gamma_{\text{sub}} + \gamma_w$$

$$\gamma_{\text{sat}} = 10 + 9.81 = 19.81 \text{ kN/m}^3$$

i) Water table is 1.2 m below the base of the foundation:



$$q_f = 1.3 c N_c + \gamma D N_q R_{w1} + 0.4 \gamma B N_r R_{w2}$$

Bearing capacity of soil

$$q_f = [c N_c + \gamma D N_q R_{w1} + 0.4 \gamma_{avg} B N_r R_{w2}]$$

Net Bearing capacity of soil,

$$q_{nf} = q_f - \bar{\sigma}$$

$$q_{nf} = q_f - \gamma D$$

Safe Bearing capacity of soil,

$$q_s = \frac{q_{nf}}{F} + \frac{\bar{\sigma}}{Z_{w1}}$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 1m$$

Z<sub>w1</sub> = depth of foundation from GL = 1.2 m

Z<sub>w2</sub> = depth of foundation to water level = 1.2

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{1.2}{2} \right]$$

$$R_{w2} = 0.8m$$

$$q_f = 1.3 c N_c + \gamma D N_q R_{w1} + 0.4 \gamma B N_r R_{w2}$$

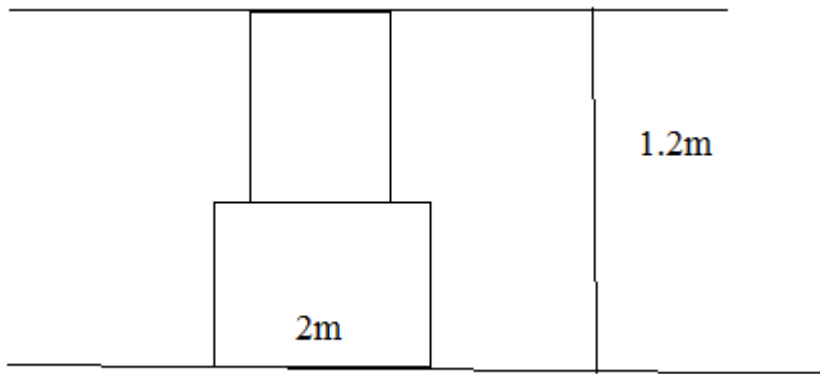
$$q_f = 0 + (19.8 \times 1.2 \times 22 \times 1) + (0.4 \times 19.8 \times 2 \times 20 \times 0.8)$$

$$q_f = 776.16 \text{ kN/m}^2$$

Net ultimate bearing capacity ( $q_{nf}$ )

$$q_{nf} = q_f - \gamma D = 776.16 - (19.8 \times 1.2) = 752.4 \text{ kN/m}^2$$

ii) Water table at base of the foundation:



$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 1\text{m}$$

Z<sub>w1</sub> = depth of foundation from GL = 1.2 m

Z<sub>w2</sub> = depth of foundation to water level = 0m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{0}{2} \right]$$

$$R_{w2} = 0.5\text{m}$$

$$R_{w2} = 0.5\text{m}$$

$$q_f = 1.3 c N_c + \gamma D N_q R_{w1} + 0.4 \gamma_{\text{sub}} B N_r R_{w2}$$

$$q_f = 0 + (19.8 \times 1.2 \times 22 \times 1) + (0.4 \times 10 \times 2 \times 20 \times 0.5)$$

$$q_f = 602.72 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D = 602.72 - (19.8 \times 1.2) = 578.96 \text{ kN/m}^2$$

iii) water table rises the ground level

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{0}{1.2} \right]$$

$$R_{w1} = 0.5\text{m}$$

Z<sub>w1</sub> = depth of foundation from GL = 0 m

Z<sub>w2</sub> = depth of foundation to water level = 0m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{0}{2} \right]$$

$$R_{w2} = 0.5$$

$$\gamma = \gamma_{\text{sub}}$$

$$q_f = 1.3 c N_c + \gamma_{\text{sub}} D N_q R_{w1} + 0.4 \gamma_{\text{sub}} B N_r R_{w2}$$

$$q_f = 0 + (10 \times 1.2 \times 22 \times 1) + (0.4 \times 10 \times 2 \times 20 \times 0.5)$$

$$q_f = 344 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma_{\text{sub}} D$$

$$= 344 - 10 \times 1.2 = 332 \text{ kN/m}^2$$

4. A footing 2.m square carries a gross pressure of 350kN/m<sup>2</sup> at a depth of 1.2m in sand. A saturated unit weight of sand is 20 kN/m<sup>3</sup> and the unit weight of sand above water table is 16 kN/m<sup>3</sup>. The shear strength parameters are C' =0,  $\phi = 30^\circ$  (for  $\phi = 30^\circ$ ,  $N_q=22$ ,  $N_\gamma=20$ ). Determine the factor of safety with respect to shear failure for the following cases

i) W.T is 5mbelowtheground level

ii) W.T is 1.2mbelow the ground level solution:

We will follow IS code method and terzaghi

For square footing in soil having  $c=0$   $q_f = \bar{\sigma} N_q + 0.4 \gamma B N_\gamma W'$   
 casei): W.T at 5 m below G.L

$$\bar{\sigma} = 16 \times 1.2$$

$$= 19.2 \text{ kN/m}^2$$

$$D_w = 5 \text{ m}$$

$$D+B = 3+1.2=4.2 \text{ m}$$

$$\text{Since } D_w > (D+B), W' = 1$$

$$\text{Also } \gamma = 16 \text{ kN/m}^3$$

$$q_f = \bar{\sigma} N_q + 0.4 B \gamma N_\gamma W''$$

$$=19.2 \times 22 + 0.4 \times 16 \times 3 \times 20 \times 1$$

$$=806.4 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D$$

$$=806.4 - 16 \times 1.2$$

$$=787.2 \text{ kN/m}^2$$

Safe bearing capacity,

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$=787.2/F + 16 \times 1.2$$

$$350 = 787.2/F + 19.2 \quad F = 2.38$$

Case ii): water table at 1.2m below the G.L

$$D_w = D \Rightarrow W'' = 0.5$$

$$\gamma = \gamma_{\text{sat}} = 20 \text{ kN/m}^3$$

$$\sigma = 16 \times 1.2 = 19.2 \text{ kN/m}^2$$

$$q_f = \bar{\sigma} N_q + 0.4 B \gamma N_\gamma W''$$

$$=19.2 \times 22 + 0.4 \times (20 - 9.81) \times 3 \times 20$$

$$q_f = 666.96 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D$$

$$=666.96 - 16 \times 1.2$$

$$=647.76 \text{ kN/m}^2$$

Safe bearing capacity

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$350 = 647.76/F + 19.2$$

$$F = 1.96$$

5. A circular footing is resting on a stiff saturated clay with unconfined compression strength of  $250 \text{ kN/m}^2$ . The depth of foundation is  $2\text{m}$ . Determine the diameter of the footing if the column load is  $700 \text{ KN}$ .

Assume a factor of safety as  $2.5$ . the bulk unit weight of soil is  $20 \text{ kN/m}^3$ . What will be the change in ultimate, net ultimate and safe bearing capacity if the water table is at ground level ?

For stiff saturated clay,  $\phi=0$

$N_c=5.7, N_q=1$  and  $N_\gamma=0$

$q_u=250 \text{ kN/m}^2$

$$c = \frac{q_u}{2}$$

$$\therefore c = 250/2 = 125 \text{ kN/m}^2$$

$$q_f = 1.3 c N_c + \gamma D N_q + 0.4 \gamma B N_\gamma$$

$$= 966 \text{ kN/m}^2$$

$$q_{nf} = q_f - \gamma D$$

$$= 966 - 20 \times 2 = 926 \text{ kN/m}^2$$

$$q_s = \frac{q_{nf}}{F} + \bar{\sigma}$$

$$= 926/2.5 + 40$$

$$= 410.4 \text{ kN/m}^2$$

$$W = q_s \times A$$

$$700 = 410.4 \times \frac{\pi \times d^2}{4}$$

$$d = \sqrt{\frac{4 \times 700}{\pi \times 410.4}} = 1.47 \text{ m}$$

$$q_{nf} = 1.3 c N_c + \gamma' D N_q$$

$$= 1.3 \times 125 \times 5.7 + 10 \times 2 \times 1$$

$$= 946.25 \text{ kN/m}^2$$

$$q_n = 946.25 - 20$$

$$= 926.25 \text{ KN/m}^2$$

$$q_s = 526.25 / 2.5 = 390.5 \text{ KN/m}^2$$

6. A strip footing 2 m wide carries a load intensity of  $400 \text{ KN/m}^2$  at a depth of 1.2 m on sand. A saturated unit weight of sand is  $19.5 \text{ KN/m}^3$  and unit weight above water table is  $16.8 \text{ KN/m}^3$ . The shear strength parameter  $C=0, \phi=36^\circ$ . Determine the factor of safety for a following condition.

1) WT below 4m from GL

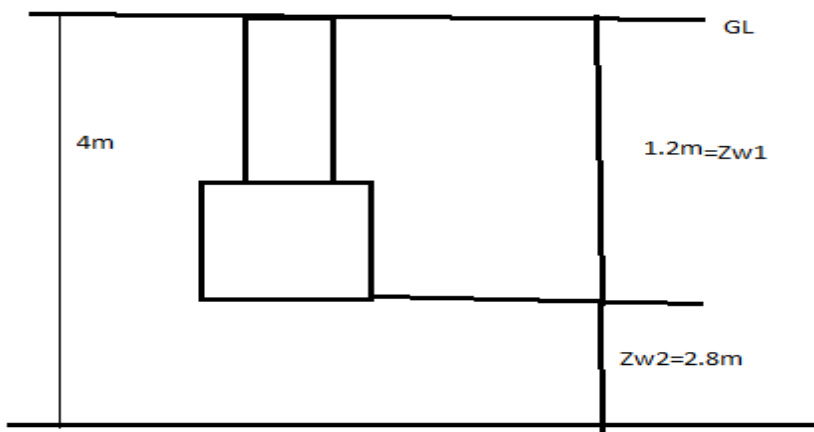
2) WT 1.2 m from GL

3) WT 2.5 m from GL

4) WT 0.5 m from GL

5) WT at GL

1) WT below 4m from GL



$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$1 \quad 1.2$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 1$$

$Z_{w1}$  = depth of foundation from GL = 1.2m

$Z_{w2}$  = depth of foundation to water level = 2.8m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{2.8}{2} \right]$$

$$R_{w2} = 1.2 \text{ m}$$

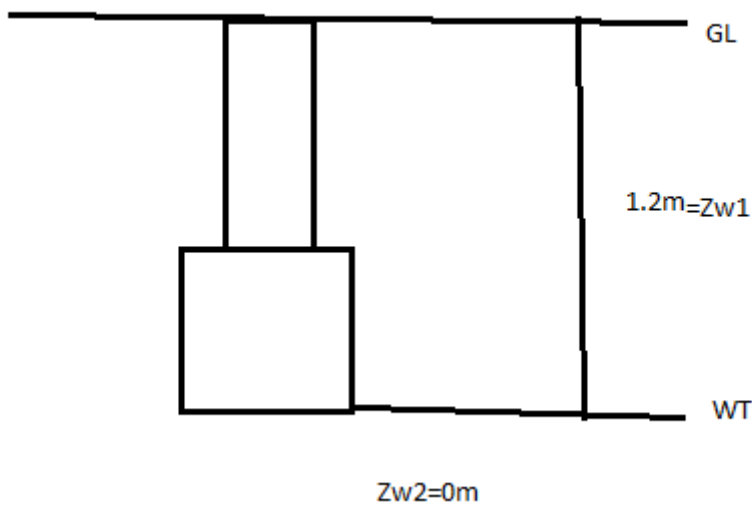
$$q_f = \left[ \frac{2}{3} c N_c + \gamma D N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2} \right]$$

$$\begin{aligned} q_f &= [16.8 \times 1.2 \times 40.4 \times 1 + 0.5 \times 16.8 \times 2 \times 33.4 \times 1.2] \\ &= 814.464 + 673.344 \\ &= 1487.8 \text{ kN/m}^2 \end{aligned}$$

$$F = \frac{q_f}{q_a}$$

$$F = \frac{1487.8}{400} = 3.7$$

2) WT 1.2 m from GL



$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 1 \text{ m}$$

$Z_{w1}$  = depth of foundation from GL = 1.2m

$Z_{w2}$  = depth of foundation to water level = 0m

$$R_{w2} = \frac{1}{2} \left[ 1 + \right]$$

$$\begin{matrix} Z_{w2} \\ B \end{matrix} \quad ]$$

—

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{0}{2} \right]$$

$$R_{w2} = 0.5 \text{ m}$$

$$q_f = \left[ \frac{2}{3} cN_c + \gamma DN_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2} \right]$$

$$q_f = \left[ \frac{2}{3} cN_c + 16.8 \times 1.2 \times 40.4 \times 1 + 0.5 \times 16.8 \times 2 \times 33.4 \times 0.5 \right]$$

$$= 1095 \text{ KN/m}^2$$

$$F = \frac{q_f}{q_a}$$

$$F = \frac{1095}{400} = 2.7$$

3) WT 2.5 m from GL

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 1 \text{ m}$$

Zw1 = depth of foundation from GL = 1.2m

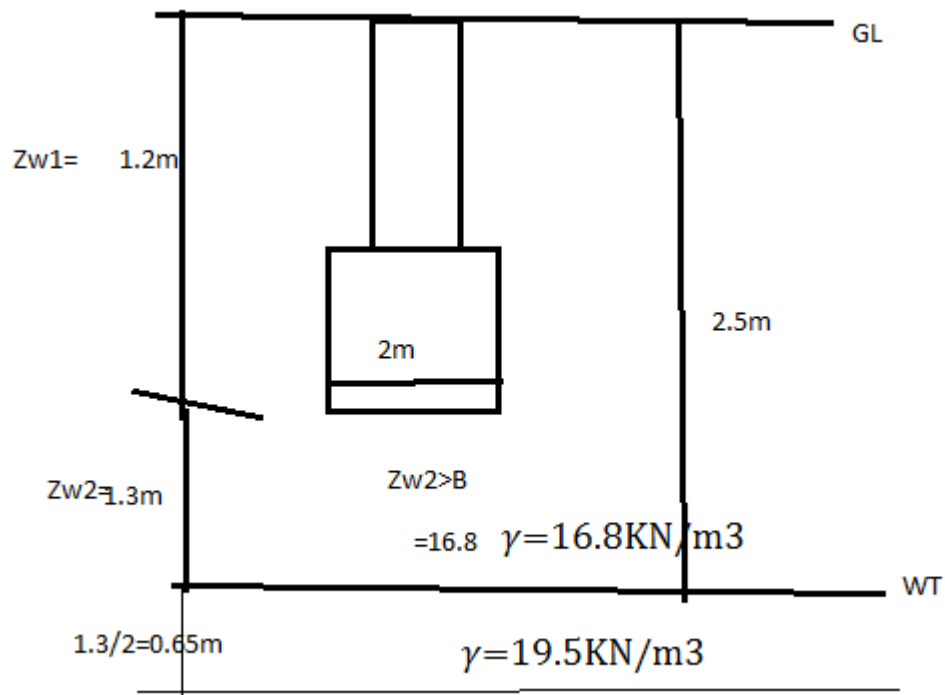
Zw2 = depth of foundation to water level = 1.3m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{1.3}{2} \right]$$

$$R_{w2} = 0.82 \text{ m}$$

$$q_f = \left[ \frac{2}{3} cN_c + \gamma DN_q R_{w1} + 0.5 \gamma_{avg} B N_\gamma R_{w2} \right]$$



$$\gamma_{avg} = \frac{(1.3 \times 16.8) + (0.65 \times 19.5)}{(1.3 + 0.65)} = 17.7 \text{ kN/m}^3$$

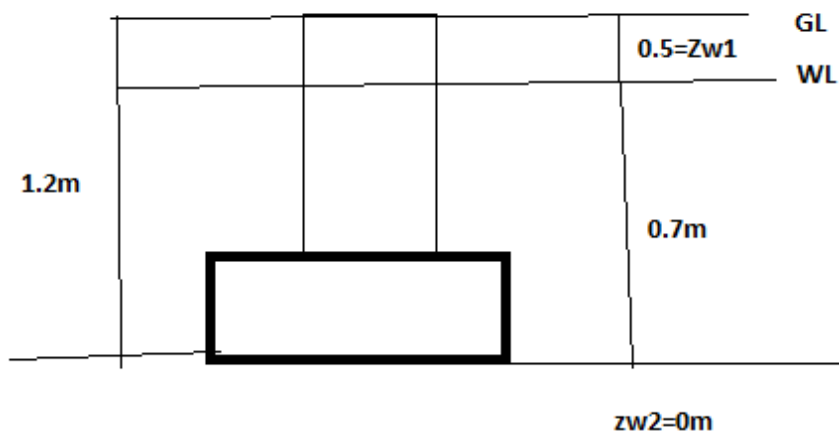
$$q_f = [(18.37 \times 1.2 \times 40.4 \times 1) + (0.5 \times 17.7 \times 2 \times 33.4 \times 0.82)]$$

$$q_f = 1375.3 \text{ kN/m}^2$$

$$F = \frac{q_f}{q_a}$$

$$F = \frac{1375.3}{400} = 3.4$$

4) WT 0.5 m from GL



$$R_{w1} = \frac{1}{1 + \frac{Z_{w1}}{0.5D}} \left[ 1 + \frac{Z_{w1}}{0.5D} \right]$$

$$R_{w1} = \frac{1}{1 + \frac{1.2}{1.2}} \left[ 1 + \frac{1.2}{1.2} \right]$$

$$R_{w1} = 0.708m$$

Zw1= depth of foundation from GL= 0.5m

Zw2= depth of foundation to water level=0m

$$R_{w2} = \frac{1}{1 + \frac{Z_{w2}}{B}} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{1 + \frac{0}{2}} \left[ 1 + \frac{0}{2} \right]$$

$$R_{w2} = 0.5 m$$

$$\gamma_{avg} = \frac{(0.5 \times 16.8) + (0.7 \times 19.5)}{(0.5 + 0.7)} = 18.37 \text{ KN/m}^3$$

$$q_f = \left[ \frac{2}{3} cN_c + \gamma_{avg} DN_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2} \right]$$

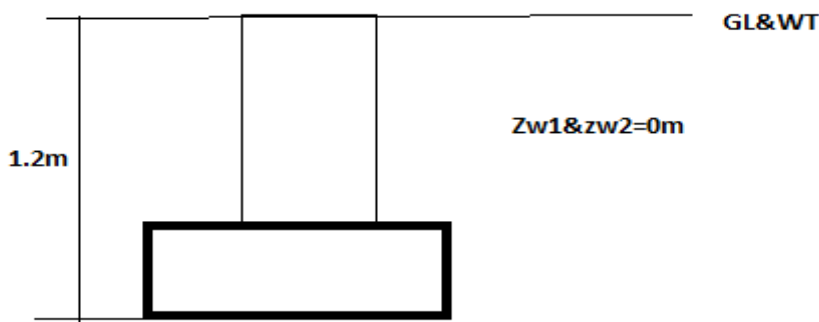
$$q_f = [ (18.37 \times 1.2 \times 40.4 \times 0.708) + (0.5 \times 19.5 \times 2 \times 33.4 \times 0.5) ]$$

$$q_f = 956.178 \text{ KN/m}^2$$

$$F = \frac{q_f}{q_a}$$

$$F = \frac{956.178}{400} = 2.39$$

5) WT At GL



$$R_{w1} = \frac{1}{1 + \frac{Z_{w1}}{D}} \left[ 1 + \frac{Z_{w1}}{D} \right]$$

$$R_{w1} = \frac{1}{2} \left[ 1 + \frac{0}{1.2} \right]$$

$$R_{w1} = 0.5m$$

Zw1= depth of foundation from GL= 0m

Zw2= depth of foundation to water level=0m

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{Z_{w2}}{B} \right]$$

$$R_{w2} = \frac{1}{2} \left[ 1 + \frac{0}{2} \right]$$

$$R_{w2} = 0.5 m$$

$$q_f = \left[ \frac{2}{3} c N_c + \gamma D N_q R_{w1} + 0.5 \gamma B N_\gamma R_{w2} \right]$$

$$q_f = [ (19.5 \times 1.2 \times 40.4 \times 0.5) + (0.5 \times 19.5 \times 2 \times 33.4 \times 0.5) ]$$

$$q_f = 798.33 \text{ KN/m}^2$$

$$F = \frac{q_f}{q_a}$$

$$F = \frac{798.33}{400} = 1.99$$