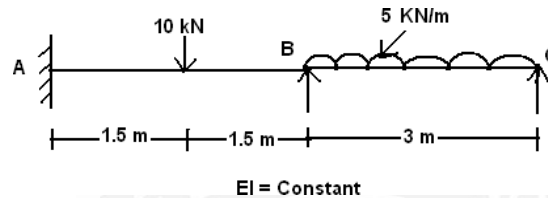


5.3. ANALYSIS OF CONTINUOUS BEAMS BY STIFFNESS METHOD

5.3.1. NUMERICAL PROBLEMS ON CONTINUOUS BEAMS;

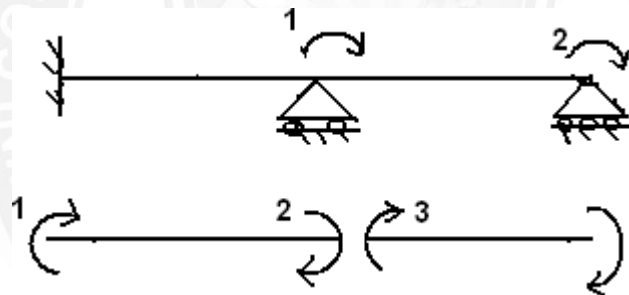
PROBLEM NO:01

Analysis the continuous beam by Stiffness Method. And find the final moments.



Solution:

- Assigned co-ordinates:



- Fixed End Moments:

$$MF_{AB} = -Wl/8 = -10 \times 3/8 = -3.75 \text{ kNm}$$

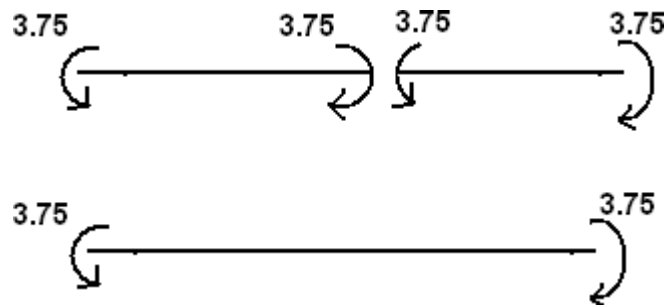
$$MF_{BA} = Wl/8 = 10 \times 3/8 = 3.75 \text{ kNm}$$

$$MF_{BC} = -Wl^2/12 = -5 \times 3^2/12 = -3.75 \text{ kNm}$$

$$MF_{CB} = Wl^2/12 = 5 \times 3^2/12 = 3.75 \text{ kNm}$$

- Fixed End Moments Diagrams:

$$W^0 = \begin{bmatrix} 0 \\ 3.75 \end{bmatrix}$$



- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L} \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

$$J = A^T \cdot K \cdot A$$

$$\begin{aligned}
 &= EI \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= EI \begin{bmatrix} 0.67 & 1.33 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 J &= EI \begin{bmatrix} 2.6 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}
 \end{aligned}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 0.431 & -0.217 \\ -0.217 & 0.861 \end{bmatrix}$$

- Displacement Matrix (Δ):**

$$\Delta = J^{-1} \cdot W$$

$$= J^{-1} [W^* - W^0]$$

$$\begin{aligned}
 &= \frac{1}{EI} \begin{bmatrix} 0.431 & -0.217 \\ -0.217 & 0.861 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 3.75 \end{Bmatrix} \right] \\
 \Delta &= \frac{1}{EI} \begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}
 \end{aligned}$$

- Element Force (P):**

$$P = K \cdot A \cdot \Delta$$

$$= \frac{EI}{EI} \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1 & 0.5 \\ 0 & 0 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= \frac{EI}{EI} \begin{bmatrix} 1.33 & 0.67 & 0 & 0 \\ 0.67 & 1.33 & 0 & 0 \\ 0 & 0 & 1.33 & 0.67 \\ 0 & 0 & 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & 0 \\ 1.33 & 0 \\ 1.33 & 0.67 \\ 0.67 & 1.33 \end{bmatrix} \begin{bmatrix} 0.814 \\ -3.228 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.545 \\ 1.082 \\ -1.081 \\ -3.75 \end{bmatrix}$$

- **Final Moments (M):**

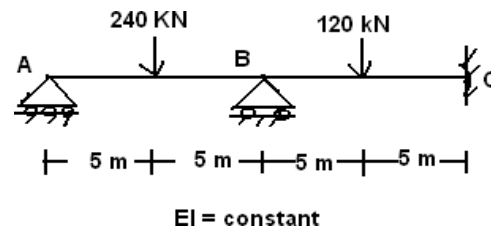
$$M = \mu + P$$

$$= \begin{bmatrix} -3.75 \\ 3.75 \\ -3.75 \\ 3.75 \end{bmatrix} + \begin{bmatrix} 0.545 \\ 1.082 \\ -1.081 \\ -3.75 \end{bmatrix}$$

$$M = \begin{bmatrix} -3.205 \\ 4.832 \\ -4.832 \\ 0 \end{bmatrix}$$

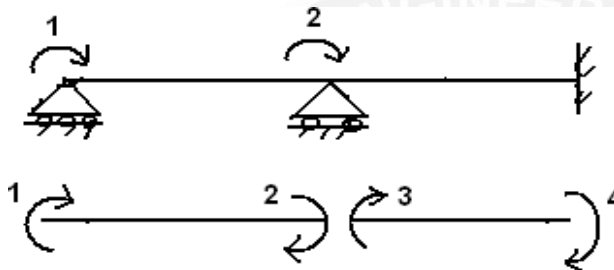
PROBLEM NO:02

Analysis the continuous beam by Stiffness Method. And find the final moments.



Solution:

- Assigned co-ordinates:



- Fixed End Moments:

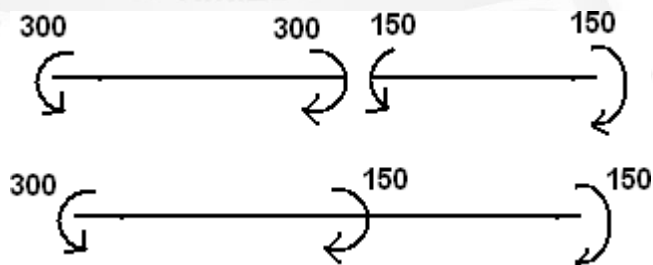
$$MF_{AB} = -Wl/8 = -240 \times 10/8 = -300 \text{ kNm}$$

$$MF_{BA} = Wl/8 = 240 \times 10/8 = 300 \text{ kNm}$$

$$MF_{BC} = -Wl/8 = -120 \times 10/8 = -150 \text{ kNm}$$

$$MF_{CB} = Wl/8 = 120 \times 10/8 = 150 \text{ kNm}$$

- Fixed End Moments Diagrams:



$$W^o = \begin{bmatrix} -300 \\ 150 \end{bmatrix}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L} \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

$$J = A^T \cdot K \cdot A$$

$$= EI \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$= EI \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0.4 & 0.2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$J = EI \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 2.86 & -0.71 \\ -0.71 & 1.43 \end{bmatrix}$$

- **Displacement Matrix (Δ):**

$$\Delta = J^{-1} \cdot W$$

$$= J^{-1} [W^* - W^0]$$

$$= \frac{1}{EI} \begin{bmatrix} 2.86 & -0.71 \\ -0.71 & 1.43 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -300 \\ 150 \end{Bmatrix} \right]$$

$$\Delta = \frac{1}{EI} \begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

- **Element Force (P):**

$$P = K \cdot A \cdot \Delta$$

$$= \frac{EI}{EI} \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.4 & 0.2 \\ 0 & 0 & 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.4 \\ 0 & 0.4 \\ 0 & 0.2 \end{bmatrix} \begin{bmatrix} 964.5 \\ -427.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 300 \\ 21.9 \\ -171 \\ -85.5 \end{bmatrix}$$

- **Final Moments (M):**

$$\mathbf{M} = \boldsymbol{\mu} + \mathbf{P}$$

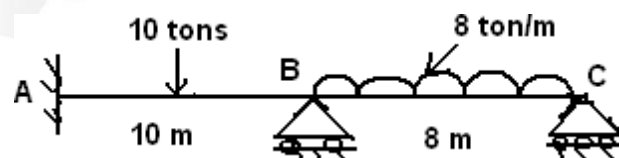
$$= \begin{bmatrix} -300 \\ 300 \\ -150 \\ 150 \end{bmatrix} + \begin{bmatrix} 300 \\ 21.9 \\ -171 \\ -85.5 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 0 \\ 321.9 \\ -321 \\ 64.5 \end{bmatrix}$$

PROBLEM NO:03

A two span continuous beam ABC is fixed at A and simply supported over the supports B and C. AB = 10 m and BC = 8 m. moment of inertia is constant throughout. A single central concentrated load of 10 tons acts on AB and a uniformly distributed load of 8 ton/m acts over BC. Analyse the beam by stiffness matrix method.

Solution:



- **Fixed End Moments:**

$$MF_{AB} = -Wl/8 = -10 \times 10/8 = -12.5 \text{ kNm}$$

$$MF_{BA} = Wl/8 = 10 \times 10/8 = 12.5 \text{ kNm}$$

$$MF_{BC} = -Wl^2/12 = -8 \times 8^2/12 = -42.67 \text{ kNm}$$

$$MF_{CB} = Wl^2/12 = 8 \times 8^2/12 = 42.67 \text{ kNm}$$

- **Formation of (A) Matrix:**

$$A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- **Stiffness Matrix (K):**

$$K = \frac{EI}{L} \begin{bmatrix} 4 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$$K = EI \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix}$$

- **System Stiffness Matrix (J):**

$$J = A^T \cdot K \cdot A$$

$$= EI \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$J^{-1} = \frac{1}{EI} \begin{bmatrix} 1.29 & -0.65 \\ -0.65 & 2.32 \end{bmatrix}$$

- Displacement Matrix (Δ):**

$$\Delta = J^{-1} \cdot W$$

$$= J^{-1} [W^* - W^0]$$

$$= \frac{1}{EI} \begin{bmatrix} 1.29 & -0.65 \\ -0.65 & 2.32 \end{bmatrix} \left[\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} -30.17 \\ 42.67 \end{Bmatrix} \right]$$

$$\Delta = \frac{1}{EI} \begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

- Element Force (P):**

$$P = K \cdot A \cdot \Delta$$

$$= \frac{EI}{EI} \begin{bmatrix} 0.4 & 0.2 & 0 & 0 \\ 0.2 & 0.4 & 0 & 0 \\ 0 & 0 & 0.5 & 0.25 \\ 0 & 0 & 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 0 \\ 0.4 & 0 \\ 0.5 & 0.25 \\ 0.25 & 0.5 \end{bmatrix} \begin{bmatrix} 66.65 \\ -118.60 \end{bmatrix}$$

$$P = \begin{bmatrix} 13.33 \\ 26.66 \\ 3.68 \\ -42.64 \end{bmatrix}$$

- **Final Moments (M):**

$$\mathbf{M} = \boldsymbol{\mu} + \mathbf{P}$$

$$= \begin{bmatrix} -12.5 \\ 12.5 \\ -42.67 \\ 42.67 \end{bmatrix} + \begin{bmatrix} 13.33 \\ 26.66 \\ 3.68 \\ -42.64 \end{bmatrix}$$

$$\mathbf{M} = \begin{bmatrix} 0.83 \\ 39.16 \\ -39 \\ 0 \end{bmatrix}$$

