

COEFFICIENT QUANTIZATION ERROR

Co-efficient quantization error

- We know that the IIR Filter is characterized by the system function

$$H(Z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}$$

- After quantizing ,

$$[H(Z)]_q = \frac{\sum_{k=0}^M [b_k]_q z^{-k}}{1 + \sum_{k=1}^N [a_k]_q z^{-k}}$$

Where

$$[a_k]_q = a_k + \Delta a_k$$

$$[b_k]_q = b_k + \Delta b_k$$

- The quantization of filter coefficients alters the positions of the poles and zeros in z-plane.
 - If the poles of desired filter lie close to the unit circle, then the quantized filter poles may lie outside the unit circle leading into instability of filter.
 - Deviation in poles and zeros also lead to deviation in frequency response.

Consider a second order IIR filter with $H(z) = \frac{1.0}{(1 - 0.5z^{-1})(1 - 0.45z^{-1})}$ find the effect on quantization

on pole locations of the given system function in direct form and in cascade form. Take b=3bits.

[Apr/May-10] [Nov/Dec-11]

Solution:

Given that,

$$H(z) = \frac{1.0}{(1 - 0.5z^{-1})(1 - 0.45z^{-1})}$$

$$H(z) = \frac{1}{z^{-1}(z - 0.5z^{-1})z^{-1}(z - 0.5)}$$

$$= \frac{z^2}{(z - 0.5)(z - 0.45)}$$

The roots of the denominator of $H(z)$ are the original poles of $H(z)$. let the original poles of $H(z)$ be p_1 and p_2 .

Here $p_1=0.5$ and $p_2=0.45$

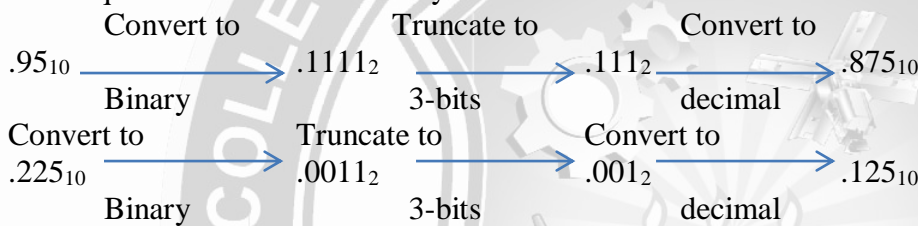
Direct form I:

$$H(z) = \frac{1.0}{(1 - 0.5z^{-1})(1 - 0.45z^{-1})}$$

$$H(z) = \frac{1}{1 - 0.5z^{-1} - 0.45z^{-1} + 0.225z^{-2}}$$

$$= \frac{1}{1 - 0.95z^{-1} + 0.225z^{-2}}$$

Let us quantize the coefficients by truncation.



Let $\bar{H}(z)$ be the transfer function of the IIR system after quantizing the coefficients.

$$\bar{H}(z) = \frac{1}{1 - 0.875z^{-1} + 0.125z^{-2}}$$

$$\text{let } \bar{H}(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - 0.875z^{-1} + 0.125z^{-2}}$$

On cross multiplying the above equation we get,

$$Y(z) - 0.875z^{-1}Y(z) + 0.125z^{-2}Y(z) = X(z)$$

$$Y(z) = X(z) + 0.875z^{-1}Y(z) - 0.125z^{-2}Y(z)$$

Cascade form:

Given that

$$H(z) = \frac{1.0}{(1 - 0.5z^{-1})(1 - 0.45z^{-1})}$$

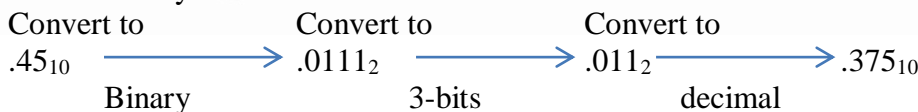
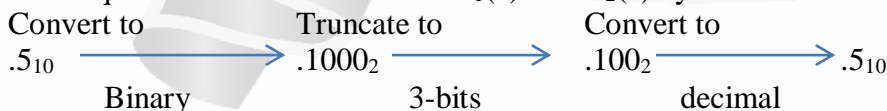
In cascade realization the system can be realized as cascade of first order sections.

$$H(z) = H_1(z)H_2(z)$$

Where,

$$H_1(z) = \frac{1}{1 - 0.5z^{-1}} \text{ and } H_2(z) = \frac{1}{1 - 0.45z^{-1}}$$

Let us quantize the coefficients of $H_1(z)$ and $H_2(z)$ by truncation.



let $\bar{H}_1(z)$ and $\bar{H}_2(z)$ be the transfer function of the first-order sections after quantizing the coefficients.

$$\overline{H}_1(z) = \frac{1}{1-0.5z^{-1}}$$

$$\overline{H}_2(z) = \frac{1}{1-0.375z^{-1}}$$

$$\text{let, } \overline{H}_1(z) = \frac{Y_1(z)}{X(z)} = \frac{1}{1-0.5z^{-1}}$$

$$Y_1(z) - 0.5z^{-1}Y_1(z) = X(z)$$

$$Y_1(z) = X(z) + 0.5z^{-1}Y_1(z)$$

$$\text{let, } \overline{H}_2(z) = \frac{Y(z)}{Y_1(z)} = \frac{1}{1-0.375z^{-1}}$$

on cross multiplying the above equation we get,

$$Y(z) - 0.375z^{-1}Y(z) = Y_1(z)$$

$$Y(z) = Y_1(z) + 0.375z^{-1}Y(z)$$

