CRITICAL FLOW

The critical state of flow has been defined as the condition for which the Froude number is equal to unity. A more common definition is that it is the state of flow at which the specific energy is a minimum for a given discharge.

When the depth of flow of water over a certain reach of a given channel is equal to the critical depth yc, the flow is called critical flow.A

Critical slope is a slope such that normal flow occurs with Froude number, F=1. The smallest critical slope for a specified channel shape, discharge and roughness is termed as limiting slope. Furthermore, by adjusting the slope and discharge, critical uniform flow may be obtained at the given normal depth Scn.

The equation for specific energy in channel of small slope with α=1, may be written

$$E = y + \frac{Q^2}{2gA^2}$$

Differentiating with respect to y and noting that Q is constant,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\ \frac{dA}{dy} = 1 - \frac{v^2}{gA}\ \frac{dA}{dy}$$

dA can be written as Tdy. Therefore dA/dy = T and the hydraulic depth D=A/T so the above equation becomes

$$\frac{dE}{dy} = 1 - \frac{v^2T}{gA} = 1 - \frac{v^2}{gD}$$

$$\frac{v^2T}{gA}$$

At the critical state of flow the specific energy is a minimum or dE/dy=0. The above equation, therefore, gives

$$\frac{Q^2T}{gA^3} = 1$$

At critical state of flow, velocity head is equal to half hydraulic depth.¬ A flow at or near the critical state is unstable. This is because a minor change in specific energy at or close to critical state will cause a major change in depth.

SPECIFIC ENERGY AND SPECIFIC FORCE

The total energy of a channel flow referred to datum is given by,

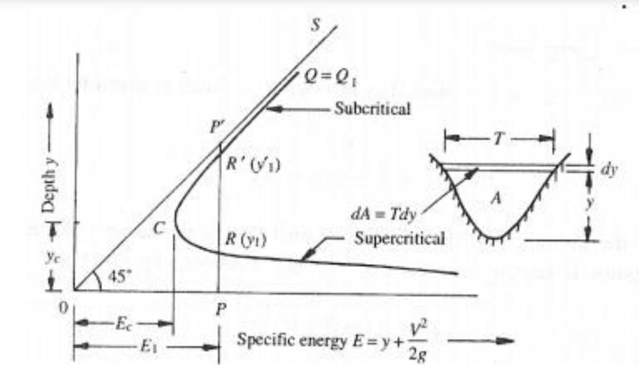
$$\mathbf{H = z + y + \frac{v^2}{2g}}$$

If the datum coincides with the channel bed at the cross-section, the resulting expression is know as specific energy and is denoted by E. Thus, specific energy is the energy at a cross-section of an open channel flow with respect to the channel bed. The concept of specific energy, introduced by Bakmeteff, is very useful in defining critical water depth and in the analysis of open channel flow. It may be noted that while the total energy in a real fluid flow always decreases in the downstream direction, the specific energy is constant for a uniform flow and can either decrease or increase in a varied flow, since the elevation of the bed of the channel relative to the elevation of the energy line, determines the specific energy.

Specific energy at a cross-section is,

$$\mathrm{E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}}$$

Here, cross-sectional area A depends on water depth y and can be defined as, A = A(y). show us that, there is a functional relation between the three variables as,



$$f(E, y, Q) = 0$$

In order to examine the functional relationship on the plane, two cases are introduced

1. Q = Constant = Q1 → E = f (y, Q1).

Variation of the specific energy with the water depth at a cross-section for a given discharge Q1.

2. E = Constant = E1 → E1 = f (y,Q) Variation of the discharge with the water depth at across-section for a given specific energy E1.

Constant Discharge Situation

Since the specific energy,

$$\mathbf{E = y + \frac{v^2}{2g} = y + \frac{Q^2}{2gA^2}}$$

For a channel of known geometry, E = f (y, Q). Keeping Q = constant = Q1, the variation of E with y is represented by a cubic parabola. (Figure 5.1). It is seen that there are two positive roots for the equation E indicating that any particular discharge Q1 can be passed in a given channel at two depths and still maintain the same specific energy E1. The depths of flow can be either PR = y1 or PR` = y`1. These two possible depths having the same specific energy are known as alternate depths. In Fig. (5.1), a line (OS) drawn such that E = y (i.e. at 450 to the abscissa) is the asymptote of the upper limb of the specific energy curve. It may be noticed that the intercept P`R` and P`R represents the velocity head. Of the two alternate depths, one (PR = y1) is smaller and has a large velocity head while the other (PR`= y`1) has a larger depth and consequently a smaller velocity head. For a given Q, as the specific energy is increased the difference between the two alternate depths increases. On the other hand, if E is decreased, the difference (y`1 – y1) will decrease and a certain value E = Ec, the two depths will merge with each other (point C in Fig. 5.1). No value for y can be obtained when E < Ec , denoting that the flow under the given conditions is not possible in this region. The condition of minimum specific energy is known as the critical flow condition and the corresponding depth yc is known as critical depth.

problem 1

Calculate the Specific energy, Critical depth and the velocity of the flow of 10 m^3 in a cement lined rectangular channel 2.5m wide with 2 m depth of water. Is the given flow is sub critical or super critical

Given Data

Q = 10 m3/s

b = 2.5 m

y = 2 m

To find

1. Specific Energy
2. Critical Depth
3. Velocity for the flow

SOLUTION:

STEP 1: Specific Energy:

$$E = y + \frac{v^2}{2g}$$

$V = \frac{Q}{A}$, $V = \frac{10}{2\times 2.5}$

v = 2m/s

$$E = 2 + \frac{2^2}{2 \times 9.81}$$

$E = 2.20\text{m}$

STEP 2 : Critical Depth

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$q = \frac{Q}{b}$

$q = \frac{10}{2.5}$ =5m²/sec

$$y_c = \left(\frac{5^2}{9.81}\right)^{1/3}$$

$y_c = 1.18m$

STEP 3: Velocity of flow

$$v_c = \sqrt{y_c \times g}$$

$$v_c = \sqrt{1.18 \times 9.81}$$

$v_c = 3.4\text{m/s}$

STEP 4: To find weather the flow is Sub critical of Super critical

$$F = \frac{v}{\sqrt{g \times D}}$$

$$= \frac{2}{\sqrt{9.81 \times 2}}$$

=0.45<1.0

Hence the flow is Sub critical