Design of RC members for combined Bending, Shear and Torsion

Problem: Design a reinforced concrete beam of rectangular cross-section for the following data

b = 300mm **d = 800mm**
D = 850mm **f_{ck}= 15 N/mm^2**
f_y= 250N/mm^2 **M_u= 200 kNm**
V = 100 kN **T_u=50kN.m**

Step1: Equivalent shear

$$V_e = V_u + 1.6\frac{T_u}{b}$$

$$= 100 + 1.6 \times \frac{50}{0.3} = 366.67\text{kN}$$

$$\tau_{ve} = \frac{366.67 \times 10^3}{300 \times 800} = 1.53\,\text{N/mm}^2$$

For M15 concrete, $\tau_{c,max} = 2.5$ MPa

Since tensile reinforcement is not known at the outset, therefore the minimum % of tension steel is

$$100\frac{A_{st}}{bd} = 100 \times \frac{0.85}{f_y} = 100 \times \frac{0.85}{250} = 0.34\%$$

$$\tau_c = 0.35 + \frac{(0.46 - 0.35)}{(0.5 - 0.25)} \times (0.34 - 0.25) = 0.39\text{MPa} < \tau_{ve}$$

Hence both the longitudinal and transverse reinforcement shall be provided

Equivalent Bending Moment

$$M_{el} = M_u + M_t$$

$$= 200 + T_u \cdot \frac{(1 + D/b)}{1.7}$$

$$= 200 + 50 \times \frac{(1 + 850/300)}{1.7}$$

$$= 200 + 112.75$$

$$= 312.75\text{kNm}$$

Since $M_u > M_t$, *no* longitudinal reinforcement will be required on compression flange.

Longitudinal Reinforcement

$$M_{e1} = 0.87 f_y A_{st} d\left(1 - \frac{A_{st} f_y}{b d f_{ck}}\right)$$

$$312.75 \times 10^6 = 0.87 \times 250 \times A_{st} \times 800\left(1 - \frac{A_{st} \times 250}{300 \times 800 \times 15}\right)$$

$$12.08 A_{st}^{\ 2} - 174000 A_{st} + 312.75 \times 10^6 = 0$$

$$A_{st} = 2105.06 mm^2$$

Provided $4\phi 28$

$$100 \times \frac{A_{st}}{bd}\% = 100 \times \frac{4 \times \frac{\pi}{4} \times 28^2}{300 \times 800}$$

$$= 1.03\% > 0.34\% \ (A_{st,min})$$

Now revised τ_c is given as

$$\tau_c = 0.6 + \frac{(0.64 - 0.6)}{(1.25 - 1.0)} \times (1.03 - 1.0) = 0.605 MPa$$

Transverse Reinforcement

$$A_{sv} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)}$$

Providing side and top cover of 30mm and $2\phi 10$ bars at the top

$$b_1 = 300 - 30 - 30 - \frac{28}{2} - \frac{28}{2} = 212 mm$$

$$d_1 = 800 - 30 - \frac{10}{2} = 765 mm$$

Assuming ϕ 8 two-legged stirrups

$$A_{sv} = 2 \times \frac{\pi}{4} \times 8^2 = 100.53 \text{ mm}^2$$

Substituting these values in the above equation

$$100.53 = \frac{50 \times 10^6 s_v}{212 \times 765 \times (0.87 \times 250)} + \frac{100 \times 1000 s_v}{2.5 \times 765 \times (0.87 \times 250)}$$

$$s_v = 60.64 \text{ mm}$$

Provided 2 ϕ 10 on each face.

The arrangement of reinforcements is shown in Figure