Sampling distribution of the Difference between two means

The difference between the population means,

$$\mu_{\overline{x_1}-\overline{x_2}} = \mu_1 - \mu_2$$

The standard deviation of the sampling distribution of $\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}$

1. Strength of wire were produced by company A has a mean of 4500kg and a S.D of 200kg, company B has a mean of 4000kg and a S.D of 300kg. If 50 wires of company A and 100 wires of company B are selected at random and tested for strength. What is the probability that the sample mean strength of A will be atleast 600kg more than that of B.

   Solution:

$$n_1 = 50, n_2 = 100$$

$$\mu_1 = 4500, \qquad \mu_2 = 4000$$

$$\sigma_1 = 200, \qquad \sigma_2 = 300$$

Sampling distribution of the Difference between two means

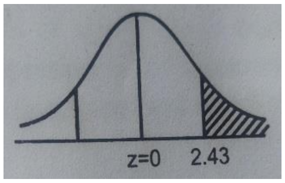
$$\mu_{\overline{x_1}-\overline{x_2}} = \mu_1 - \mu_2 = 4500 - 4000 = 500$$

$$\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} = \sqrt{\frac{40000}{50} + \frac{90000}{100}}$$

$$= \sqrt{800 + 900} = \sqrt{1700} = 41.231$$

$$p[(\overline{x_1} - \overline{x_2}) > 600] = p\left[\frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}} > \frac{600 - 500}{41.23}\right]$$

$$= p[Z > 2.425]$$



$$= 0.5 - 0.4925 = 0.0075$$

2. Car stereo manufacturer of A have mean life time of 1400 hours with a S.D of 200hrs while those of manufacturer B have mean lifetime of 1200hrs with a S.D of 100hrs. If a random sample of 120 stereos of each manufacturer are

tested. (i) What is the probability that the manufacturer of A'S stereo's will have a mean life time of atleast 160hrs more than the manufacturer B'S stereo's (ii) and 250hrs more than the manufacturer B stereos.

Solution:

$$n_1 = 120, n_2 = 120$$

$$\mu_1 = 1400, \qquad \mu_2 = 1200$$

$$\sigma_1 = 200, \qquad \sigma_2 = 300$$

Sampling distribution of the Difference between two means

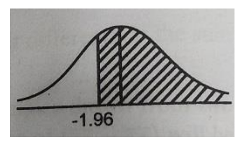
$$\mu_{\overline{x_1}-\overline{x_2}} = \mu_1 - \mu_2 = 1400 - 1200 = 200$$

$$\sigma_{\overline{x_1}-\overline{x_2}} = \sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}} = \sqrt{\frac{40000}{120} + \frac{90000}{120}}$$

$$= 20.41$$

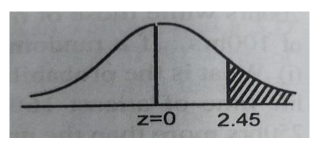
(i)

$$p[(\overline{x_1} - \overline{x_2}) > 160] = p\left[\frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}} > \frac{160 - 200}{20.41}\right]$$



$$= p[Z > 1.95] = 0.5 + 0.4750 = 0.9750$$

$$(ii) p[(\overline{x_1} - \overline{x_2}) > 250] = p\left[\frac{(\overline{x_1} - \overline{x_2}) - (\mu_1 - \mu_2)}{\sqrt{\frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}}} > \frac{250 - 200}{20.41}\right]$$



$$= p[Z > 2.45] = 0.5 - 0.4929 = 0.0071$$

Sampling Distribution of Proportions

A sampling distribution of proportions whose mean and sampling distribution are given by

$$\mu_p = P \text{ } and \text{ } \sigma_p = \sqrt{\frac{PQ}{n}}$$

Where σ_p is the standard error of proportion.

Note

For large values of n ($n \geq 30$) the sampling distribution of proportion is very closely normally distributed.

Sampling Distribution of the Difference of Two Proportions

The mean and standard deviation of this sampling distribution is given below,

$$\mu_{p_1 - p_2} = P_1 - P_2$$

$$\sigma_{p_1 - p_2} = \sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}$$

Note:

If n_1 and n_2 are large ($n_1, n_2 \geq 30$) the sampling distributions of difference of proportion ($P_1 - P_2$) are very closely normally distributed.

PROBLEMS:

1. In a quality department of Manufacturing paints at the time of dispatch of decorators 30% of the containers are found to be defective. If a random sample of 500 is drawn with replacement from the population. What is the probability that the sample proportion will be less than 25% defective.

   Solution:

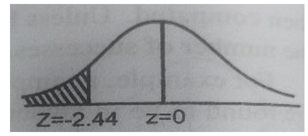
$$\mu_p = P = 0.3,$$

$$Q = 1 - P = 1 - 0.3 = 0.7,$$

$$n = 700$$

$$and \text{ } \sigma_p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.3 * 0.7}{500}} = 0.0205$$

The required probability is $p(p \leq 0.25) = p\left(\frac{P-p}{\sqrt{\frac{PQ}{n}}} \leq \frac{0.25-0.3}{0.0205}\right.$



$$= p(Z \le -2.44) = 0.5 - p(0 \le Z \le 2.44)$$
$$= 0.5 - 0.4927 = 0.0073$$

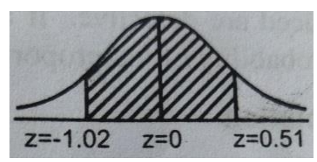
2. A manufacturer of watches has determined from experience that 3% of the watches he produces are defective. If a random sample of 300 watches is examined, what is the probability that, the proportion defective is between 0.02 and 0.035

   Solution:

$$\mu_p = P = 0.03,$$
$$Q = 1 - P = 1 - 0.03 = 0.97,$$
$$n = 300$$
$$and\ \sigma_p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.03 * 0.97}{300}} = 0.0098$$

   The required probability is $p(0.02 \le Z \le 0.25)$

$$= p\left(\frac{0.02 - 0.03}{0.0098} \le \frac{P - p}{\sqrt{\frac{PQ}{n}}} \le \frac{0.035 - 0.03}{0.0098}\right)$$
$$= p(-1.02 \le Z \le 0.51)$$



$$= 0.3451 + 0.1950 = 0.5411$$

3. In two proportions 10% of machine produced by a company A are defective and 5% of machine produced by a company B are defective. A random sample of 250 machines are taken from company A and has the random sample of 300 machines from company B. What is the probability that the difference in sample proportion is ≤ 0.02.

   Solution

$$P_1 = 0.10, P_2 = 0.05, n_1 = 250, n_2 = 300$$
$$Q_1 = 0.90, Q_2 = 0.95$$

$$\mu_{p_1-p_2} = P_1 - P_2 = 0.10 - 0.05 = 0.05$$

$$\sigma_p = \sigma_{p_1-p_2} = \sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$$

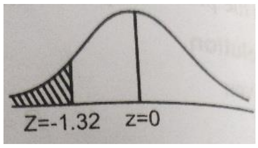
$$\sigma_p = \sigma_{p_1-p_2} = \sqrt{\frac{0.10 * 0.90}{250} + \frac{0.05 * 0.95}{300}}$$

$$\sigma_p = 0.0228$$

The required probability is $p(p_1 - p_2 \leq 0.02)$

$$= p\left(\frac{(p_1 - p_2) - (P_1 - P_2)}{\sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}} \leq \frac{0.02 * 0.05}{0.0228}\right)$$

$$= p(Z \leq -1.32)$$



$$= 0.5 - p(0 \leq Z \leq 1.32)$$

$$= 0.5 - 0.4066 = 0.0934$$

4. A manufacturer of pens has determined from experience that 4% of the pens produced are defective. If a random sample of 400 pens is examined, what is the probability of the proportion of defects between 0.025 and 0.048.

   Solution:

$$\mu_p = P = 0.04,$$

$$Q = 1 - P = 1 - 0.04 = 0.96,$$

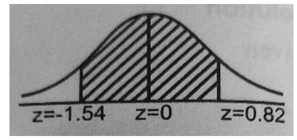
$$n = 400$$

$$and\ \sigma_p = \sqrt{\frac{PQ}{n}} = \sqrt{\frac{0.04 * 0.96}{400}} = 0.0097$$

The required probability is $p(0.025 \leq Z \leq 0.048)$

$$= p\left(\frac{0.025 - 0.04}{0.0097} \le \frac{P - p}{\sqrt{\frac{PQ}{n}}} \le \frac{0.048 - 0.04}{0.0097}\right)$$

$$= p(-1.54 \le Z \le 0.82)$$



$$= 0.4357 + 0.2939 = 0.73$$

5. A research troop stated that 16% of the firms of the particular type A increased their market research budget in the 5 year proceedings the studying for type B firms, the figure was 9%.

   (i) What are the mean and S.D of the sampling distribution of the difference between sample proportion based on independent random samples. 100 firms from each type

   (ii). What proportion of the sample difference would be between 0.05 and 0.10

   Solution

$$P_1 = 0.16, P_2 = 0.09, n_1 = 100, n_2 = 100$$

$$Q_1 = 0.84, Q_2 = 0.91$$

$$\mu_{p_1-p_2} = P_1 - P_2 = 0.16 - 0.09 = 0.07$$

$$\sigma_p = \sigma_{p_1-p_2} = \sqrt{\frac{P_1Q_1}{n_1} + \frac{P_2Q_2}{n_2}}$$

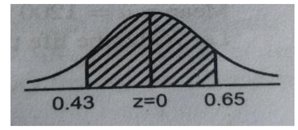
$$\sigma_p = \sigma_{p_1-p_2} = \sqrt{\frac{0.16 * 0.84}{100} + \frac{0.09 * 0.91}{100}}$$

$$\sigma_p = 0.0465$$

The required probability is $p(0.05 \le p \le 0.10)$

$$= p\left(\frac{0.05 - 0.07}{0.0465} \le Z \le \frac{0.10 - 0.07}{0.0465}\right)$$

$$= p(-0.43 \le Z \le 0.65)$$



$$= 0.1664 + 0.2422 = 0.4086$$