3.2. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVERCANTILEVER BEAM WITH SINGLE POINT LOAD AT FREE END

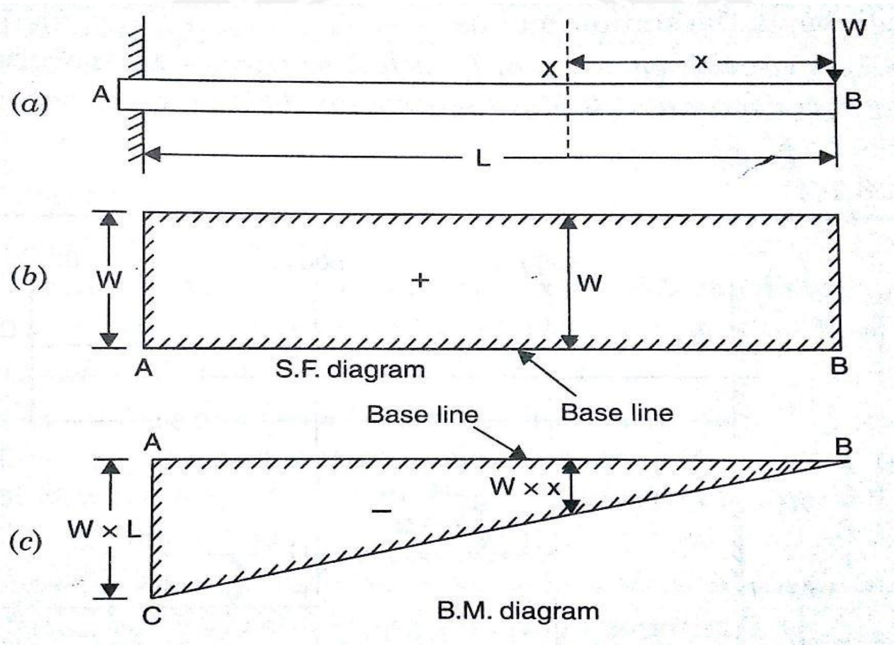
ThefollowingfigureshowsacantileverABoflengthLfixedatAandfreeatBand carrying a point load W at the free end B.

Let F_x=ShearforceatXand M_x=BendingmomentatX

TakeasectionXatadistancexfromthefreeend.Considertherightportionofthe section.

ShearForceCalculation

TheshearforceatthesectionBisequaltotheresultantforceactingontherightportion



atthegivensectionBasWanditactingdownwardisconsideredaspositive.

The shear force at this section X-X is equal to the resultant force acting on the right portion at the given section X-X as W and it acting downward is considered as positive.

The shear force will be constant at all section of the cantilever between A and B asthere is no load between A and B. The shear force diagram is shown in fig.

SF at B= + W (+veduetorightsidedownwardload) SF

at X-X= +W (BecausenoloadbetweenBandX-X) SF

at B= +W (due to same load as above)

Shearforcediagram:

BendingMomentCalculation:

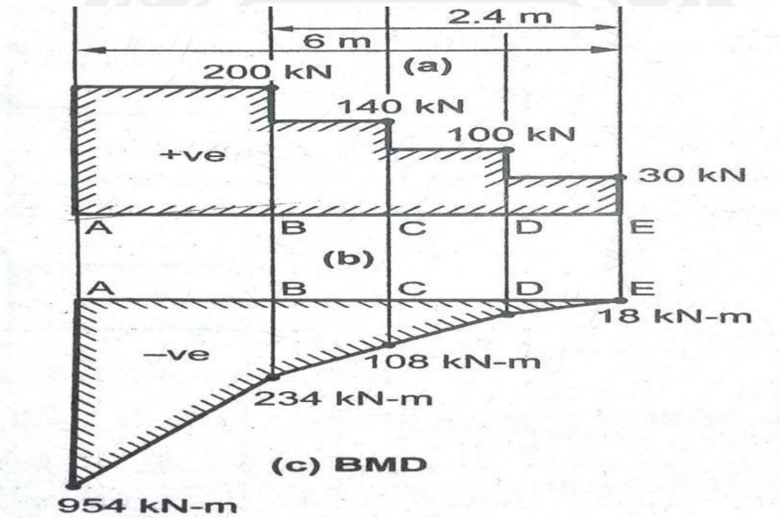
TheBendingMoment atthesectionBisproportionaltothedistanceofthesectionfrom the free end as (W x 0) and it acting clockwise about that section is considered as negative.

The Bending Moment at this section X-X is proportional to the distance of the section from the free end as *x* as (W x *x*) and it acting clockwise about that section is considered as negative.

The Bending Moment at this section fixed end is proportional to the distance of the section from the free end as L as (W x L) and it acting clockwise about that section is considered as negative.The Bending Moment diagram is shown in fig.

Problem 2.2.1: A cantilever 6m long carries load of 30, 70, 40 and 60 kN at a distance of 0,0.6, 1.5 and 2m respectively from the free end. Draw the SF and BM diagram for the cantilever.

GivenData:showninfigure. **To find:** SFD and BMD **Solution:**



Shear Force Calculation:

(Sum of vertical forces)

SFatB=+30kN

SFatC=+30+70=+100kN

SFatD=+30+70 +40=+140kN

SFatE=+ 30+70 + 40+60=+ 200kN

ShearForceDiagram:

Verticaldownwardpointloadaredrawnasupwardverticalline No load are drawn as horizontal line.

BendingmomentCalculation:[Sumof(VerticalforcexActingdistance)] BM at B = **-**(30 x 0) kN = 0 kNm

BMatC= -(30x 0.6) –(70x0)=-18kNm

BMatD= -(30x1.5) –(70x0.9)–(40x0) =-108kNm

BMatE=-(30x2.4)–(70x1.8)–(40x0.9)–(60x0)=-234kNm

BMatA= -(30x6)–(70x5.4)– (40x4.5)–(60x3.6)=-954kNm

BendingmomentDiagram:

Verticaldownwardpointloadaredrawnasinclinedline.AllBMareinnegativeside.

Result: TheSFDandBMDaredrawnasshowninfig.

3.2.2. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVER BEAM WITH UNIFORMLY DISTRIBUTED LOAD

Consider a cantilever beam of length *L* fixed at A and carrying a uniformly distributed load of *w* per unit length over the entire length of the beam.

Take the section X at a distance of ' *x'* from the free end B. Here we have consider the right portion of the beam section.

LetF_x=ShearforceatX and M_x=BendingmomentatX

ShearForceCalculation:

The shear force at the section X will be equal to the resultant force acting on the right portion up to the section.

Theresultantforceontherightportion=loadxdistanceofrightportion= *wxx*

Theresultantforceactingontherightportionactingdownwardisconsideredpositive.

∴ ShearforceatX,$_x$=+$w.x$

Theaboveequationshowsthattheshearforcefollowsastraightline law.

SFatB,when$x$= 0hence=+(loadxdistance)=wx0=+0

SFatA,when$x$= Lhence=+(loadxdistance)=wxL=+$w.L$

ShearForceDiagram:

WhenanUDLactingonthebeamisindicatedinShearforcediagramasan***inclined line***. The shear force diagram shown in fig.

BendingmomentCalculation:

The UDL over a section ofbeam is converted into point load acting at the C.G ofthesection.

Thebendingmomentwillbenegativeasfortherightportionofthesection,the moment of the load at *x* is clockwise.

ThebendingmomentatthesectionXisgivenby

M_x =-(totalloadonrightportion)x(distanceofC.Gofrightportionfrom X)

$= -(w.x) \times (\frac{x}{2}) = -w\frac{x^2}{2}$

From the above eqn. it is clear that B.M. at any section is proportional to the square of the distance from the free end. This follows a parabolic law.

BMatB, when$x$=0hence= $-\frac{w}{2} \times 0 = 0$

BMatA,when$x$=Lhence=$-\frac{w}{2} \times L^2 = -w.\frac{L^2}{2}$

BendingmomentDiagram:

WhenanUDLactingonthebeamisindicatedinBendingMomentdiagramasan ***paraboliccurvedline***.TheBendingMomentdiagramshowninfig.

Problem2.2.3: Acantileveroflength2mcarriesaUDLof3kN/m.DrawtheSFandBM diagram.

GivenData:showninfigure. **To find:** SFD and BMD **Solution:**

ShearForceCalculation:(Sumofverticalforces)

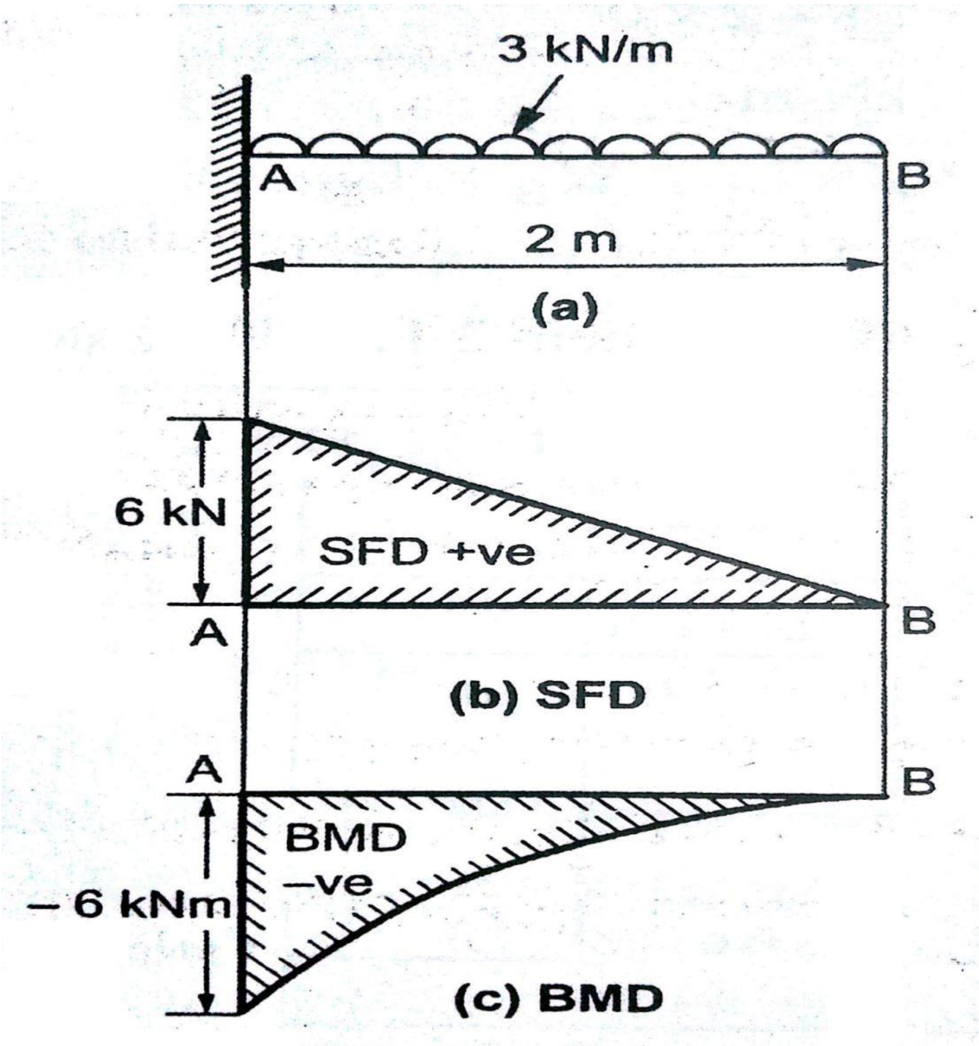
ForUDL,itwillbeconvertedintopointloadas(Pointload=UDLxloadacting distance) and the converted point load acting at its middle means divided by 2

SFatB=+0kN

SFatA=+ 0+(3x 2) =+6kN

ShearForceDiagram:

VerticaldownwardUDLaredrawnasinclinedlinebasedonsign No load are drawn as horizontal line.



BendingmomentCalculation:[Sumof(VerticalforcexDistanceofloadactingfromrequired section)]

ForUDL,itwillconvertintopointloadandthatPLactatitsmiddle BM at B =

-(0 x 0) kN = 0 kNm

BMatC=-(0x2)–[(3x2)x $\frac{2}{2}$]=-6kNm

BendingmomentDiagram:

VerticaldownwardUDLaredrawnasparaboliccurvedlinebasedontheirsign.

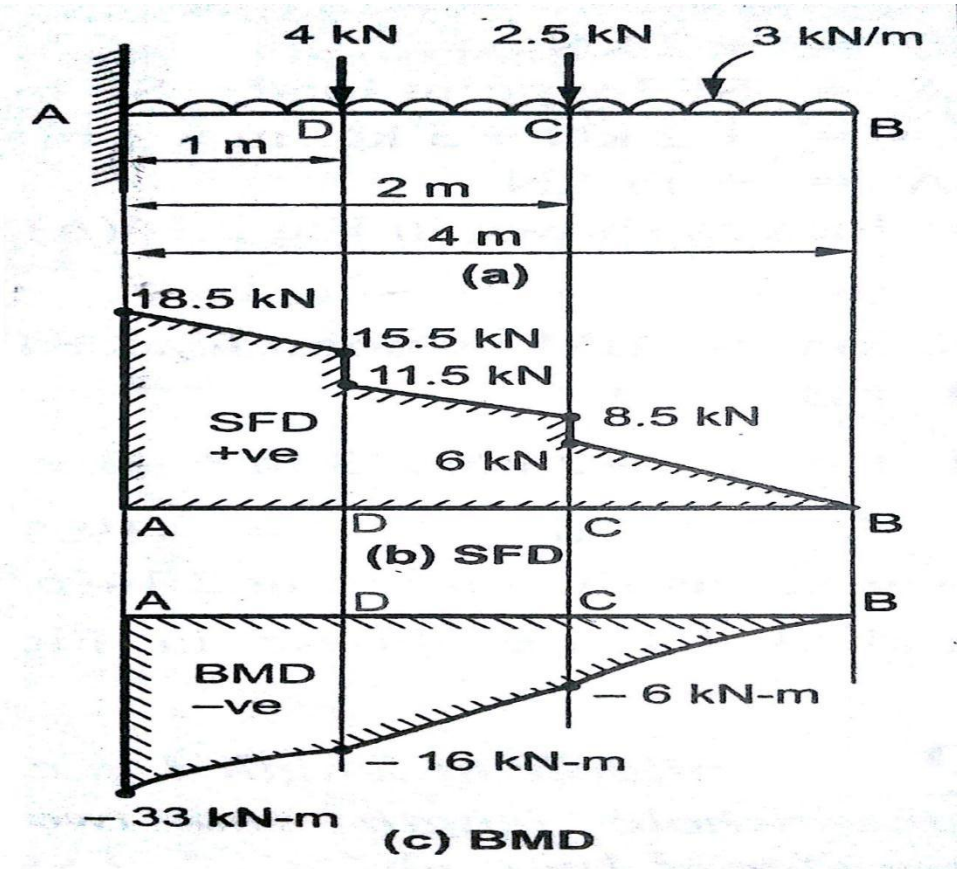
Result: TheSFDandBMDaredrawnasshowninfig.

Problem 2.3: A cantilever of length 4m carries a UDL of 3 kN/m run over the whole length and two point loads of 4kN and 2.5 kN are placed 1m and 2m respectivelyfrom the fixed end. Draw the SF and BM diagram.

GivenData:showninfigure.

Tofind:SFDandBMD

Solution:



Shear Force Calculation:(Sumofverticalforces)

ForUDL,itwillbeconvertedintopointloadas(Pointload=UDLxloadacting distance) and the converted point load acting at its middle means divided by 2

SFatB=+0kN

SFatC=+0+2.5 +(3 x2)=+8.5 kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force

SFatD=+3+2+(3 x1.5)=+9.5kN(WithoutconsiderPL)

SF at D = + 3 + 2 + 2 + (3 x 1.5) = + 11.5 kN (WithconsiderPL)

SF at A = +0 + 2.5 + 4 + (3 x 4) = + 18.5 kN

ShearForceDiagram:

Verticaldownwardpointloadaredrawnasverticallinebasedonsign Vertical downward UDL are drawn as inclined line based on sign

BendingmomentCalculation:[Sumof(VerticalforcexDistanceofloadactingfromrequired section)]

ForUDL,itwillconvertintopointloadandthatPLactatitsmiddle BM at B = -(0 x 0) = 0 kNm

BMatC=-(0x2)-(2.5x0)-[(3x2)x$\frac{2}{2}$]=-6kNm

BMatD= -(0x3)-(2.5x1)-(4x 0)-[(3x3)x $\frac{3}{2}$]=-16kNm

BMatA= -(0x4)–(2.5x2)–(4x1)–[(3x4)x$\frac{4}{2}$]=-33kNm

BendingmomentDiagram:

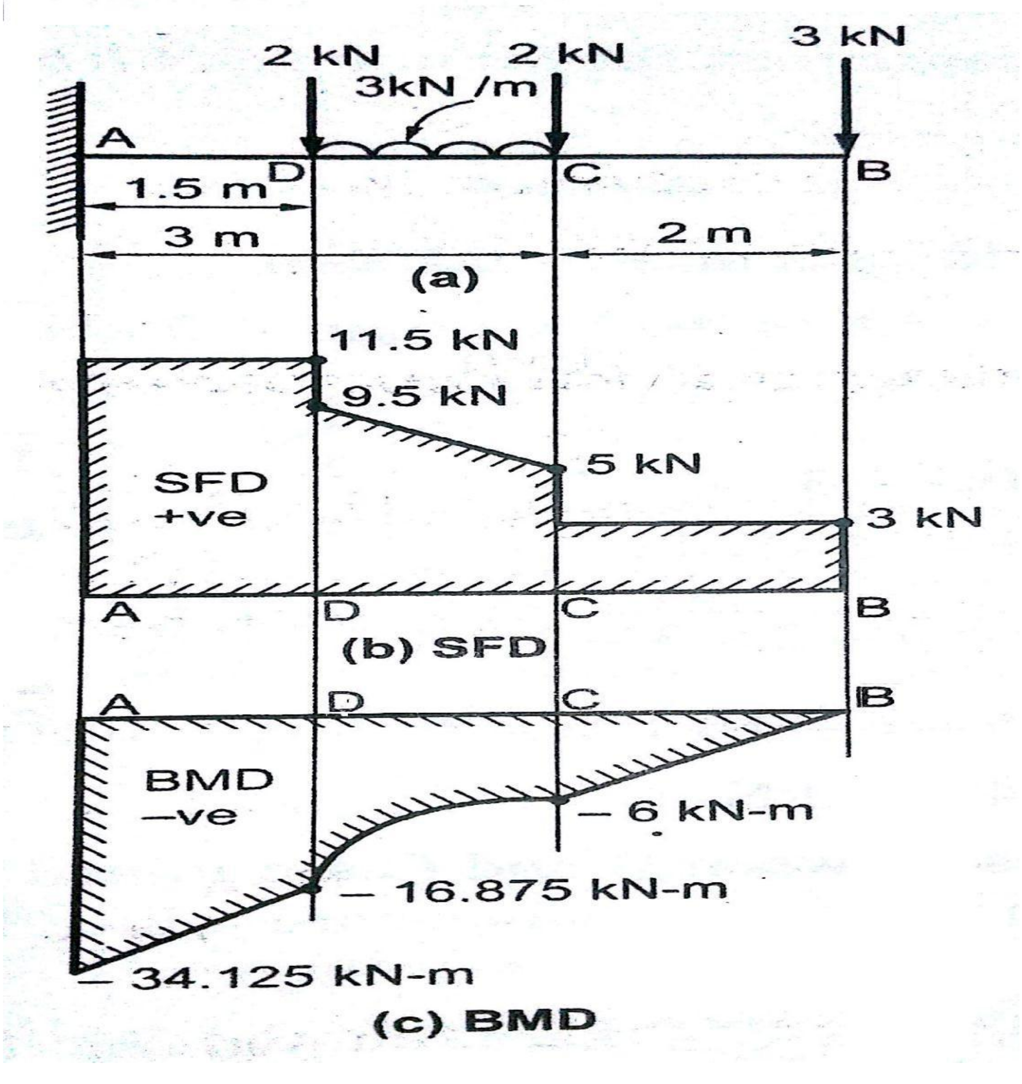
VerticaldownwardUDLaredrawnasparaboliccurvedlinebasedontheirsign.

Result:

TheSFDandBMDaredrawnasshowninfig.

Problem 2.4: A cantilever of length 5m carries a UDL of 3 kN/m run over the length of 1.5m start from 1.5m from fixed end and two point load 2kN acting at 1.5m and 3m respectively from fixed end and another point load 3kN acting at the free end. Draw the SF and BM diagram.

GivenData:showninfigure. **To find:** SFD and BMD **Solution:**



ShearForceCalculation:(Sumofverticalforces)

ForUDL,itwillbeconvertedintopointloadas(Pointload=UDLxloadacting distance) and the converted point load acting at its middle means divided by 2

SFatB=+3kN

SFatC=+3+2= +5kN

When point load and UDL acting at a particular point, first we have not consider Point Load and next consider point load to find shear force

SFatD=+3+2+(3x1.5)=+9.5kN(WithoutconsiderPL)

SF at D = + 3 + 2 + 2 + (3 x 1.5) = + 11.5 kN (WithconsiderPL)

SF at A = + 3 + 2 + 2 + (3 x1.5) = + 11.5 kN

ShearForceDiagram:

Verticaldownwardpointloadaredrawnasverticallinebasedonsign.

Vertical downward UDL are drawn as inclined line based on sign.

Noloadaredrawnashorizontalline.

BendingmomentCalculation:[Sumof(VerticalforcexDistanceofloadactingfromrequired section)]

ForUDL,itwillconvertintopointloadandthatPLactatitsmiddle BM at B = - (3 x 0) = 0 kNm

BMatC= -(3x2) -(2.x0)=-6kNm

BMatD=-(3x3.5)-(2x1.5)-(2x0)-[(3x1.5)x$\frac{1.5}{2}$]=-16.875kNm

BMatA=-(3x5)–(2x3)–(2x1.5)–[(3x 1.5)x($\frac{1.5}{2}$+1.5)]=-34.125kNm

BendingmomentDiagram:

VerticaldownwardPLaredrawnasinclinedlinebasedontheirsign.Vertical downward UDL are drawn as parabolic curved line based on their sign.

Result:

TheSFDandBMDaredrawnasshowninfig.

3.2.3. SHEAR FORCE AND BENDING MOMENT DIAGRAM FOR A CANTILEVER BEAM WITH UNIFORMLY VARYING LOAD

Consideracantileverbeamoflength LfixedatAandcarryingauniformlyvarying load from zero at the free and to *w* per unit length at the fixed end of beam.

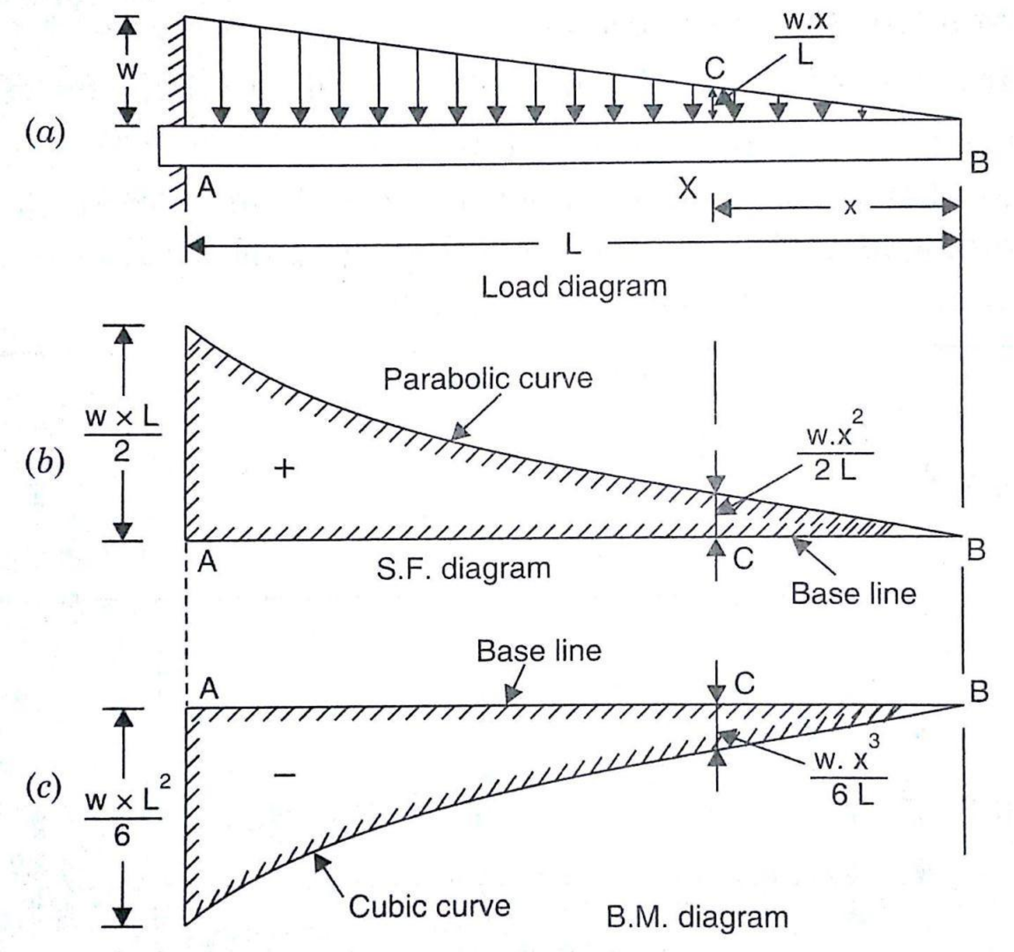
Take the section X at a distance of ' x' from the free end B. Here we have consider the right portion of the beam section.

Let F_x=ShearforceatXandM_x=BendingmomentatX

ShearForceCalculation:

Let us firstfind the rate of loading at the section X. The rate of loading is zero at B and is 'w' per meter run at A. This means that rate of loading for a length L is w per unit length.

Hencerateofloadingforalengthof$x$=$\frac{w}{L}$x$x$perunitlength.Whichisequaltothe loadacting atX=CX=$\frac{w}{L}$xx



Theresultantforceactingontherightportionactingdownward isconsideredpositive. The shear force at the section X at a distance x from free end is given by

F_x =+(Totalloadonthecantileverforalength $x$fromthefreeendB)

=+(AreaoftriangleBCX)

=+($\frac{1}{2}$xXBx XC)

$=+\frac{1}{2}\text{x}x\ (\frac{w}{L}\ \text{x}x)=+w.\ \frac{x^2}{2L}$ $(\therefore XB=x, C=\frac{w}{L}\ \text{x}x)$

Fromtheaboveeqn.showsthattheSFvariesaccordingtotheparaboliclaw.

SFatB, when$x$=0hence=+ $w.\frac{0^2}{2L}=0$

SFatA,when$x$= Lhence=+$w.\frac{L^2}{2L}$ =+$\frac{w.L}{2}$

ShearForceDiagram:

When an UVL acting on the beam is indicated in Shear force diagram as an ***Parabolic curved line***. The shear force diagram shown in fig.

BendingmomentCalculation:

The UVL over a section ofbeam is converted into point load acting at the C.G ofthesection.

Thebendingmomentwillbenegativeasfortherightportionofthesection,the moment of the load at x is clockwise about the section.

ThebendingmomentatthesectionXisgivenby

M_x =-(totalloadonrightportion)x(DistanceoftheloadfromX)

=-(AreaoftriangleBCX)x(DistanceofC.Goftrianglefrom X)

$=-(w.\frac{x^2}{2L})\mathrm{x}(\frac{x}{3})=-\frac{wx^3}{6L}$

Fromtheabove eqn. itisclearthatB.M.atanysection isproportionaltothecube of the distance from the free end. This follows a cubic law.

BMat B,when$x$ =0 hence =-$\frac{wx0}{6L}$=0

BMatA,when x =L hence=-$\frac{wxL^3}{6L}$ =-$w.\frac{L^2}{6}$

BendingmomentDiagram:

When an UDL actingonthebeamisindicated inBendingMomentdiagramas an ***cubic curved line***. The Bending Moment diagram shown in fig.

Problem2.5: Acantileveroflength4mcarriesagradually increasingload,zeroatthefree end to 2kN/m at the fixed end. Draw the SF and BM diagrams for the cantilever.

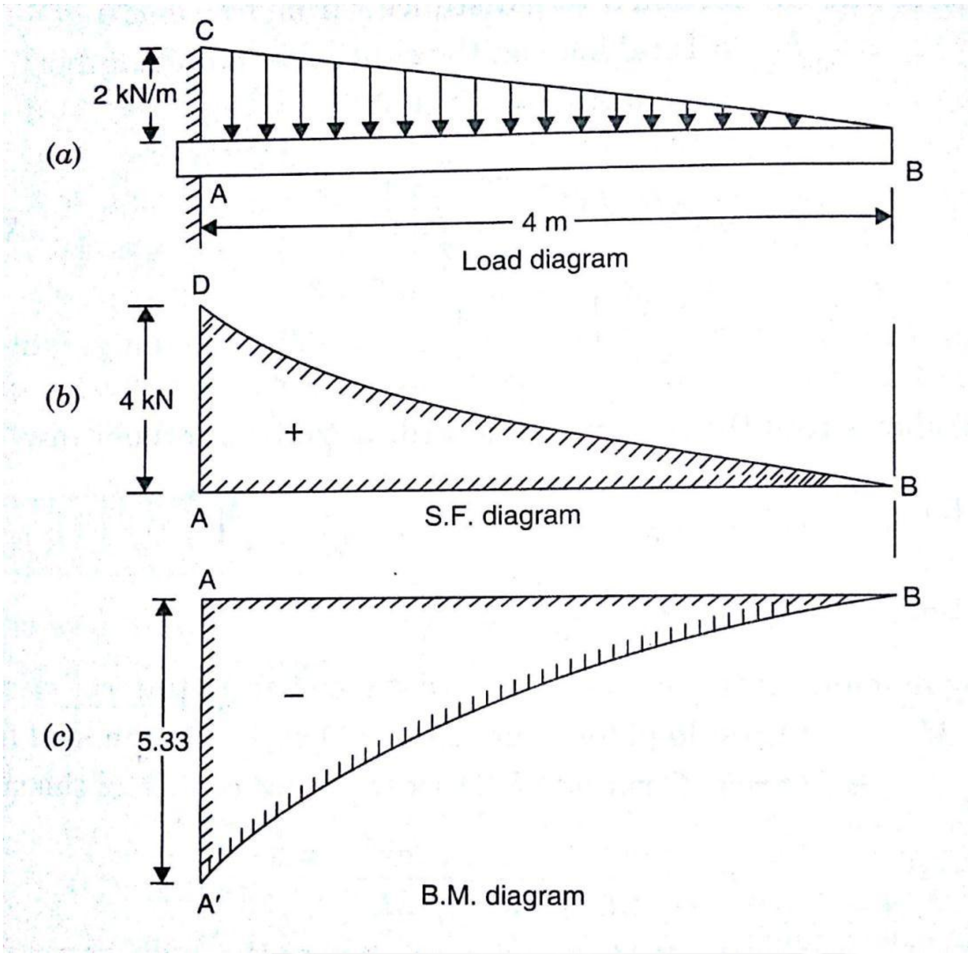
GivenData:showninfigure. **To find:** SFD and BMD **Solution:**

ShearForceCalculation:(Sumofverticalforces)

ForUVL,itwillbeconvertedintopointloadas(Pointload=Areaoftriangle=$\frac{1}{2}$x UVLxloadactingdistance)andtheconvertedpointloadactingdistanceatits $\frac{l}{3}$ fromthe higherload end.

SFatB=+0kN

SFatA=+0+($\frac{1}{2}$x2x4)=+4kN



ShearForceDiagram:

DownwardUVLaredrawnasparaboliccurvedlinebasedonsign.

BendingmomentCalculation:[Sumof(VerticalforcexDistanceofloadactingfromrequired section)]

ForUVL,itwillconvertintopointloadandthatPLactatits $\frac{l}{3}$ fromthehigherloadend.

BMatB= -(0x0) =0 kNm

BMatA=-(0x4)–[($\frac{1}{2}$x2x4)x$\frac{4}{3}$]=-5.33kNm

BendingmomentDiagram:

VerticaldownwardUVLaredrawnascubiccurvedlinebasedontheirsign.

Result: TheSFDandBMDaredrawnasshowninfig.