GRAPH

Network graph is simply called as graph. It consists of a set of nodes connected by branches. In graphs, a node is a common point of two or more branches. Sometimes, only a single branch may connect to the node. A branch is a line segment that connects two nodes.

Any electric circuit or network can be converted into its equivalent graph by replacing the passive elements and voltage sources with short circuits and the current sources with open circuits. That means, the line segments in the graph represent the branches corresponding to either passive elements or voltage sources of electric circuit

Terms and definitions

The description of networks in terms of their geometry is referred to as network topology. The adequacy of a set of equations for analyzing a network is more easily determined topologically than algebraically.

Graph (or linear graph): A network graph is a network in which all nodes and loops are retained but its branches are represented by lines. The voltage sources are replaced by short circuits and current sources are replaced by open circuits. (Sources without internal impedances or admittances can also be treated in the same way because they can be shifted to other branches by E-shift and/or I-shift operations.)

Branch: A line segment replacing one or more network elements that are connected in series or parallel. Node: Interconnection of two or more branches. It is a terminal of a branch. Usually interconnections of three or more branches are nodes.

Path: A set of branches that may be traversed in an order without passing through the same node more than once.

Loop: Any closed contour selected in a graph. Mesh: A loop which does not contain any other loop within it.

Planar graph: A graph which may be drawn on a plane surface in such a way that no branch passes over any other branch.

Non-planar graph: Any graph which is not planar.

Oriented graph: When a direction to each branch of a graph is assigned, the resulting graph is called an oriented graph or a directed graph.

Connected graph: A graph is connected if and only if there is a path between every pair of nodes.

Sub graph: Any subset of branches of the graph.

Tree: A connected sub-graph containing all nodes of a graph but no closed path. i.e. it is a set of branches of graph which contains no loop but connects every node to every other node not necessarily directly. A number of different trees can be drawn for a given graph.

Link: A branch of the graph which does not belong to the particular tree under consideration. The links form a sub-graph not necessarily connected and is called the co-tree.

Tree compliment: Totality of links i.e. Co-tree.

Independent loop: The addition of each link to a tree, one at a time, results one closed path called an independent loop. Such a loop contains only one link and other tree branches. Obviously, the number of such independent loops equals the number of links.

Tie set: A set of branches contained in a loop such that each loop contains one link and the remainder are tree branches.

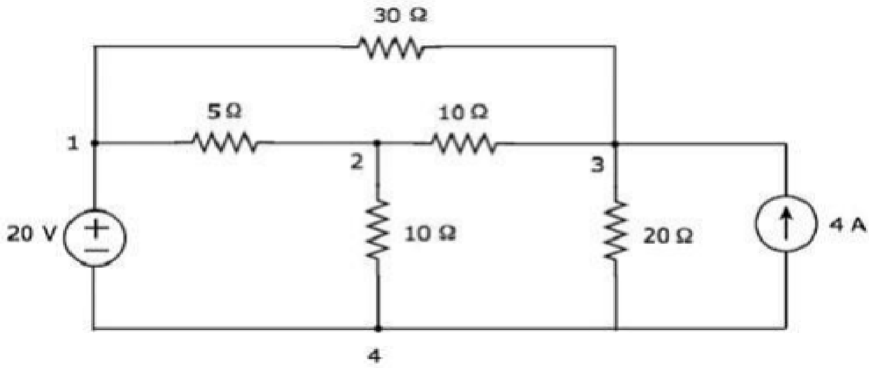
Tree branch voltages: The branch voltages may be separated in to tree branch voltages and link voltages. The tree branches connect all the nodes. Therefore if the tree branch voltages are forced to be zero, then all the node potentials become coincident and hence all branch voltages are forced to be zero. As the act of setting only the tree branch voltages to zero forces all voltages in the network to be zero, it must be possible to express all the link voltages uniquely in terms of tree branch voltages. Thus tree branch form an independent set of equations.

Cut set:

A set of elements of the graph that dissociates it into two main portions of a network such that replacing any one element will destroy this property. It is a set of branches that if removed divides a connected graph in to two connected sub-graphs. Each cut set contains one tree branch and the remaining being links.

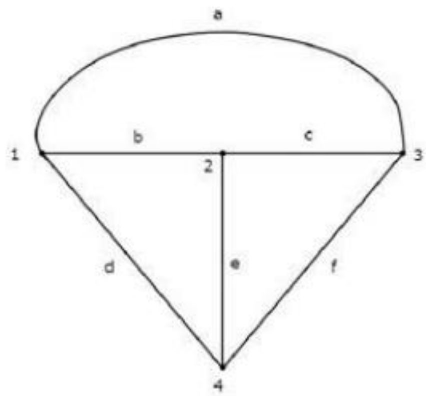
Example

Let us consider the following **electric circuit**.



In the above circuit, there are **four principal nodes** and those are labelled with 1, 2, 3, and 4. There are **seven branches** in the above circuit, among which one branch contains a 20 V voltage source, another branch contains a 4 A current source and the remaining five branches contain resistors having resistances of 30 Ω, 5 Ω, 10 Ω, 10 Ω and 20 Ω respectively.

An equivalent **graph** corresponding to the above electric circuit is shown in the following figure.



In the above graph, there are **four nodes** and those are labelled with 1, 2, 3 & 4 respectively. These are same as that of principal nodes in the electric circuit. There are **six branches** in the above graph and those are labelled with a, b, c, d, e & f respectively.

In this case, we got **one branch less** in the graph because the 4A current source is made as open circuit, while converting the electric circuit into its equivalent graph.

From this Example, we can conclude the following points −

- The **number of nodes** present in a graph will be equal to the number of principal nodes present in an electric circuit.
- The **number of branches** present in a graph will be less than or equal to the number of branches present in an electric circuit.

Types of Graphs

Following are the types of graphs −

- Connected Graph
- Unconnected Graph
- Directed Graph
- Undirected Graph

Connected Graph

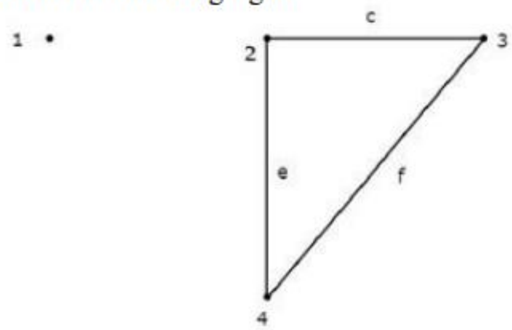
If there exists at least one branch between any of the two nodes of a graph, then it is called as a **connected graph**. That means, each node in the connected graph will be having one or more branches that are connected to it. So, no node will present as isolated or separated.

The graph shown in the previous Example is a **connected graph**. Here, all the nodes are connected by three branches.

Unconnected Graph

If there exists at least one node in the graph that remains unconnected by even single branch, then it is called as an **unconnected graph**. So, there will be one or more isolated nodes in an unconnected graph.

Consider the graph shown in the following figure.

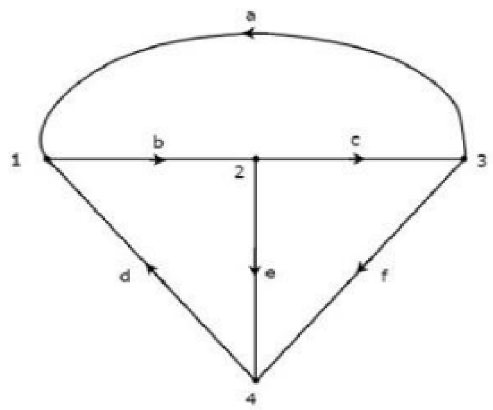


In this graph, the nodes 2, 3, and 4 are connected by two branches each. But, not even a single branch has been connected to the **node 1**. So, the node 1 becomes an **isolated node**. Hence, the above graph is an **unconnected graph**.

Directed Graph

If all the branches of a graph are represented with arrows, then that graph is called as a **directed graph**. These arrows indicate the direction of current flow in each branch. Hence, this graph is also called as **oriented graph**.

Consider the graph shown in the following figure.



In the above graph, the direction of current flow is represented with an arrow in each branch. Hence, it is a **directed graph**.

Undirected Graph

If the branches of a graph are not represented with arrows, then that graph is called as an **undirected graph**. Since, there are no directions of current flow; this graph is also called as an **unoriented graph**.

The graph that was shown in the first Example of this chapter is an unoriented graph, because there are no arrows on the branches of that graph.

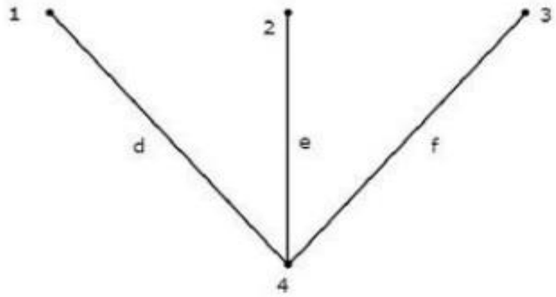
Subgraph and its Types

A part of the graph is called as a **subgraph**. We get subgraphs by removing some nodes and/or branches of a given graph. So, the number of branches and/or nodes of a subgraph will be less than that of the original graph. Hence, we can conclude that a subgraph is a subset of a graph.

Tree

Tree is a connected subgraph of a given graph, which contains all the nodes of a graph. But, there should not be any loop in that subgraph. The branches of a tree are called as **twigs**.

Consider the following **connected subgraph** of the graph, which is shown in the Example of the beginning of this chapter.

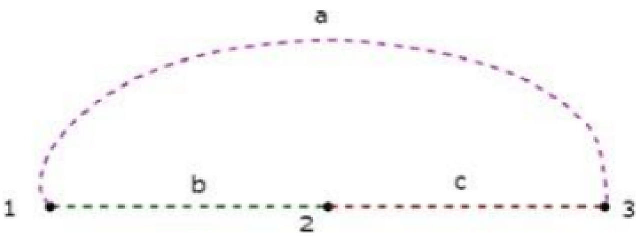


This connected subgraph contains all the four nodes of the given graph and there is no loop. Hence, it is a **Tree**. This Tree has only three branches out of six branches of given graph. Because, if we consider even single branch of the remaining branches of the graph, then there will be a loop in the above connected subgraph. Then, the resultant connected subgraph will not be a Tree.

From the above Tree, we can conclude that the **number of branches** that are present in a Tree should be equal to ***n* - 1** where 'n' is the number of nodes of the given graph.

Co-Tree

Co-Tree is a subgraph, which is formed with the branches that are removed while forming a Tree. Hence, it is called as **Complement** of a Tree. For every Tree, there will be a corresponding Co-Tree and its branches are called as **links** or chords. In general, the links are represented with dotted lines.
The **Co-Tree** corresponding to the above Tree is shown in the following figure.



This Co-Tree has only three nodes instead of four nodes of the given graph, because Node 4 is isolated from the above Co-Tree. Therefore, the Co-Tree need not be a connected subgraph. This Co-Tree has three branches and they form a loop.

The **number of branches** that are present in a co-tree will be equal to the difference between the number of branches of a given graph and the number of twigs. Mathematically, it can be written as

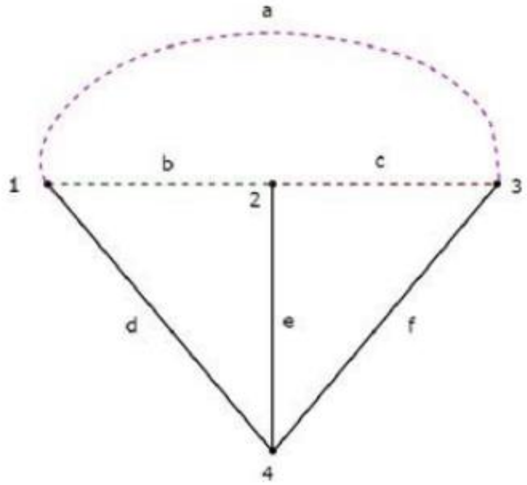
$$l = b - (n - 1)$$
$$l = b - n + 1$$

Where,

- ***l*** is the number of links.
- ***b*** is the number of branches present in a given graph.
- ***n*** is the number of nodes present in a given graph.

The Tree branches d, e & f are represented with solid lines. The Co-Tree branches a, b & c are represented with dashed lines.

If we combine a Tree and its corresponding Co-Tree, then we will get the **original graph** as shown below.



Network Topology Matrices

Network Topology Matrices are useful for solving any electric circuit or network problem by using their equivalent graphs.

Matrices Associated with Network Graphs

Following are the three matrices that are used in Graph theory.

- Incidence Matrix
- Fundamental Loop Matrix
- Fundamental Cut set Matrix