5.5 IMPACT OF BODIES

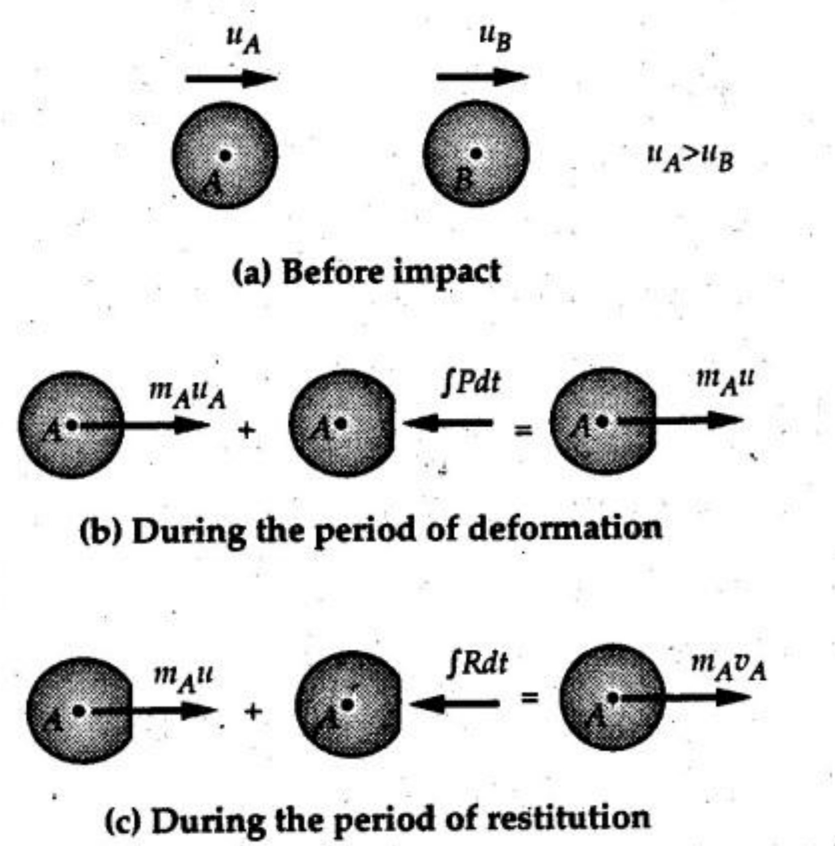
Impact occurs when two bodies collide during a very short time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The line of impact is a line through the mass centers of the colliding particles.

The common normal to the surfaces in contact during the impact is known as line of impact.

• If the velocities of both objects are along the line of impact, the impact is called direct central impact.

If velocity of one or both objects is at an angle with line of impact, it is called oblique impact.

• Consider direct central impact of two objects A and B moving with velocities v_A and v_B ($u_A > u_B$) as shown in Fig.



• During impact, a force P is exerted on A towards left and on B towards right due to which they get deformed.

• After maximum deformation, the objects start regaining their original shape. The time during which the objects regain their original shape is called the time of restitution. In general, the force R exerted by B on A during restitution is different from the force P exerted during deformation.

• By impulse-momentum principle during period of deformation,

$$m_A u_A - \int P\,dt = m_A u$$

and during period of restitution

$$m_A u - \int R\,dt = m_A v_A$$

• The ratio of magnitude of impulse during the period of restitution to the magnitude of impulse during the period of deformation is called the coefficient of restitution denoted by *e*.

$$e = \frac{\int R\,dt}{\int P\,dt}$$

From equations (9.3.19) and (9.3.20),

$$e = \frac{u - v_A}{u_A - u}$$

Similarly for *B*,

$$e = \frac{v_B - u}{u - u_B}$$

From equation (9.3.21),

$$eu_A - eu = u - v_A$$

$$eu_A + v_A = eu + u$$

From equation (9.3.22),

$$eu - eu_B = v_B - u$$

$$eu + u = eu_B + v_B$$

$$eu_A + v_A = eu_B + v_B$$

$$e(u_A - u_B) = v_B - v_A$$

$$\boxed{e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)}$$

$$\therefore \quad e = -\left(\frac{v_{A/B}}{u_{A/B}}\right)$$

From (9.3.26), we can define the coefficient of restitution as the negative ratio of relative velocity after collision to the relative velocity before collision.

For perfectly elastic collision, where kinetic energy and momentum are both conserved, $e = 1$. For plastic collisions, where both objects move together with same velocity after collision, $e = 0$. For other collisions, $0 < e < 1$. Note that momentum is conserved in all types of collisions.

General Procedure for Solving Problems

1) Use conservation of momentum equation.

ii) Use coefficient of restitution equation.

iii) Solve the two equations to solve for the two unknown velocities after impact.

iv) If energy imparted to an object during impact is converted to another form or dissipated in friction, use conservation of energy or work-energy principle for further analysis.

v) For collision of an object with a rigid surface like the ground, the velocity of rigid surface before and after impact is zero.

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)$$

Putting $u_B = v_B = 0$,

$$e = -\frac{v_A}{u_A}$$

$$\therefore \quad v_A = -eu_A$$

i.e., velocity after impact is '*e*' times the velocity before impact in opposite direction.

Solved Examples

1.*A ball of mass 20 kg moving with a velocity of 5 m/s strikes directly another ball of mass 10 kg moving in the opposite direction with a velocity of 10 m/s. If the coefficient of restitution is equal to 5/6, then determine the velocity of each ball after impact.*

Solution:

m_1 = 20 kg, u_1 = 5 m/s

m_2 = 10 kg, u_2 = -10 m/s

e = 5/6

By conservation of momentum,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$(20)(5)+(10)(-10) = 20 v_1 + 10 v_2$$

$$\therefore \quad 2 v_1 + v_2 = 0 \quad \text{... (1)}$$

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

$$\frac{5}{6} = -\left(\frac{v_1 - v_2}{5+10}\right)$$

$$-v_1 + v_2 = \frac{5}{6}\times 15$$

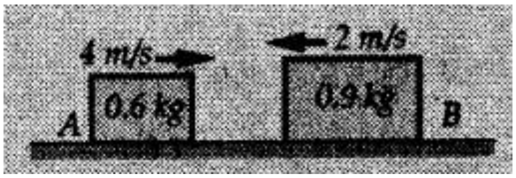
$$\therefore \quad -v_1 + v_2 = 12.5 \quad \text{... (2)}$$

From (1) and (2),

$\therefore$ $\boxed{v_1 = 4.167 \text{ m/s}}$ **Opposite to its initial velocity**

$\therefore$ $\boxed{v_2 = 8.33 \text{ m/s}}$ **also opposite to initial velocity.**

2.*Two blocks approaching each other along smooth surface, along same line are as shown. After impact, velocity of B is observed to be 2.5 m/s to the right. Determine coefficient of restitution.*



Solution :

By conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

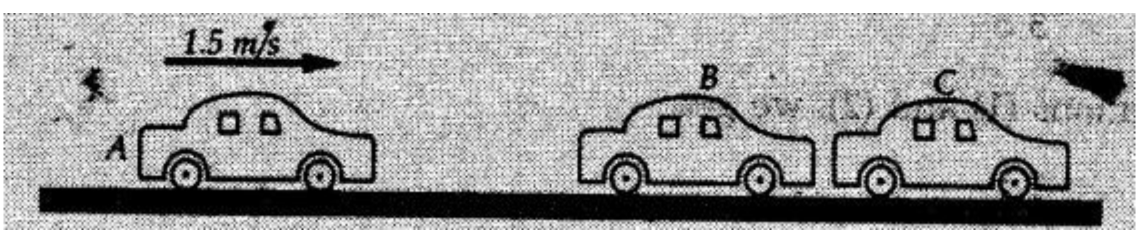
$$\therefore \quad (0.6)(4) + (0.9)(-2) = (0.6) v_A + (0.9)(2.5)$$

$$\therefore \quad v_A = -2.75 \text{ m/s}$$

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right) = -\left(\frac{-2.75 - 2.5}{4 - (-2)}\right)$$

$$\therefore \quad \boxed{e = 0.875}$$

3.Two identical cars B and C are at rest on a loading dock with their brakes released. Car A of same model is pushed by workers, hits car B with velocity 1.5 m/s, causing series of collisions between three cars. Assuming e = 0.50 for impact between A and B, while e = 1.0 for impact between B and C, find velocity of each car after all collisions have taken place.



Solution:

Let mass of each car be m. For collision between A and B,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(m)(1.5) + 0 = m v_A + m v_B$$

$$\therefore \quad v_A + v_B = 1.5 \qquad \dots (1)$$

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)$$

$$0.5 = -\left(\frac{v_A - v_B}{1.5 - 0}\right)$$

$$-v_A + v_B = 0.75 \qquad \dots (2)$$

From equations (1) and (2),

$v_A = 0.375$ m/s →

$v_B = 1.125$ m/s →

For collision between B and C,.

$$m_B u_B + m_C u_C = m_B v_B + m_C v_C$$

$$m(1.125) + 0 = m v_B + m v_C$$

$$v_B + v_C = 1.125 \qquad \dots (3)$$

$$e = -\left(\frac{v_B - v_C}{u_B - u_C}\right)$$

$$1 = -\left(\frac{v_B - v_C}{1.125 - 0}\right)$$

$$\therefore \quad -v_B + v_C = 1.125 \qquad \dots (4)$$

From equations (3) and (4)

$v_B = 0$

$v_C = 1.125$ m/s →

As *A* is moving towards right at 0.375 m/s and B is stationary, there will be another collision between A and B.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$m(0.375) + 0 = m v_A + m v_B$$

$$v_A + v_B = 0.375 \quad \ldots (5)$$

$$e = -\left(\frac{v_A - v_B}{u_A - u_B}\right)$$

$$0.5 = -\left(\frac{v_A - v_B}{0.375 - 0}\right)$$

$$-v_A + v_B = 0.1875 \quad \ldots (6)$$

From equations (5) and (6),

$v_A = 0.09375$ m/s →

$v_B = 0.28125$ m/s →

Now there is no further collision as $v_A < v_B < v_C$ and all the three are moving towards right.

$$\therefore \quad \boxed{v_A = 0.09375 \text{ m/s} \rightarrow}$$

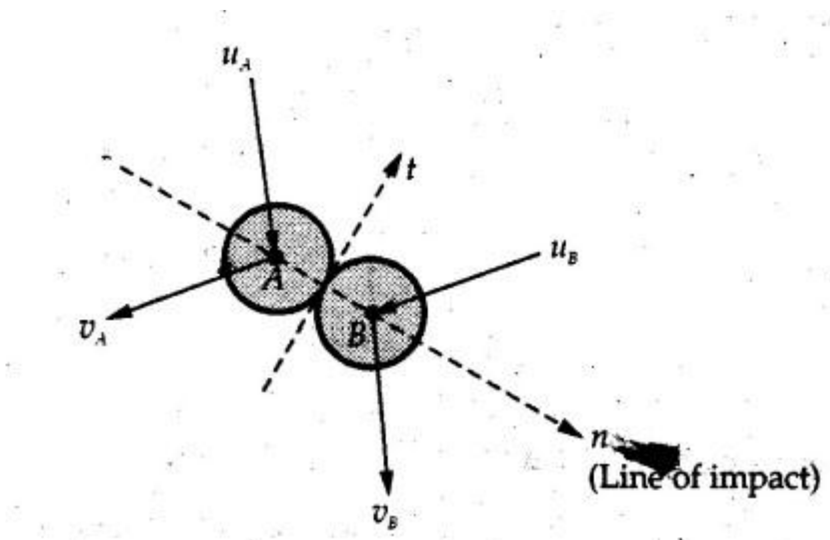
$$\boxed{v_B = 0.28125 \text{ m/s} \rightarrow}$$

$$\boxed{v_C = 1.125 \text{ m/s} \rightarrow}$$

Oblique Impact

• If the velocities of the two colliding bodies are not along the line of impact, the impact is said to be oblique.

• Consider oblique impact of two bodies A and B as shown in Fig.



• μ_A and μ_B are velocities of A and B respectively before impact and v_A and v_B after impact.

• During impact, the internal forces are directed only along the line of impact.

• Hence the components of the two velocities will change only along the line of impact. The components of velocities along the common tangent remain unchanged.

$$\therefore \quad (u_A)_t = (v_A)_t \qquad \text{... (9.3.27)}$$

$$\text{and} \quad (u_B)_t = (v_B)_t \qquad \text{... (9.3.28)}$$

• Momentum is conserved along the line of impact.

$$m_A(u_A)_n + m_B(u_B)_n = m_A(v_A)_n + m_B(v_B)_n \qquad \text{... (9.3.29)}$$

• The relative velocities along line of impact are related to the coefficient of restitution.

$$\therefore \quad e = -\left[\frac{(v_A)_n - (v_B)_n}{(u_A)_n - (u_B)_n}\right] \qquad \text{... (9.3.30)}$$

• Equations (9.3.29) and (9.3.30) can be solved simultaneously to find $(v_A)_n$ and $(v_B)_n$. Knowing $(v_A)_t$ and $(v_B)_t$ from equations (9.3.27) and (9.3.28), the velocities of A and B can be calculated after impact.

Solved Examples

1.A smooth spherical ball A of mass 15 kg is moving from left to right with a velocity of 5 m/s in a horizontal plane. Another identical ball B travelling in a perpendicular direction with a velocity of 15 m/s collides with A in such a way that the line of impact is in the direction of motion of ball A. Assuming e = 0.6, determine the velocity of balls A and B after impact.

Solution:

The collision is shown in Fig.

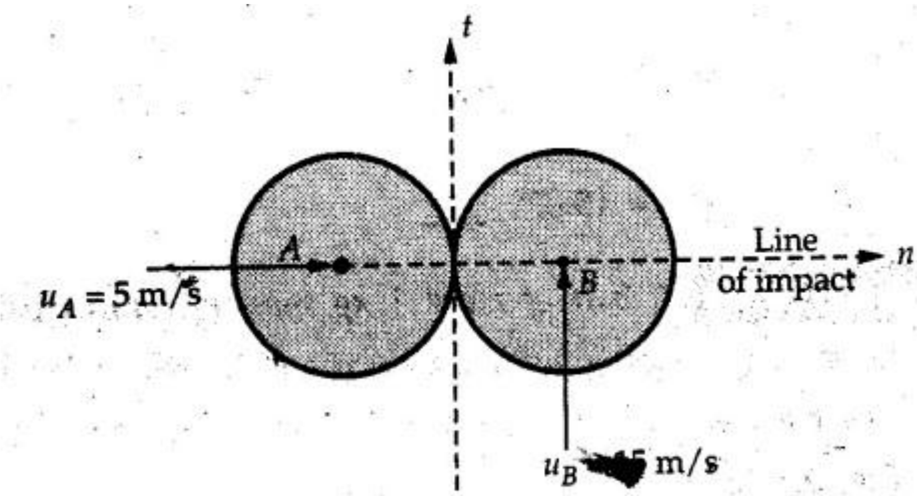


Fig. 9.3.35

$$(u_A)_t = 0$$

$$(u_A)_n = 5 \text{ m/s} \rightarrow$$

$$(u_B)_t = 15 \text{ m/s} \uparrow$$

$$(u_B)_n = 0$$

As the velocity components remain unchanged along the *t*-axis,

$(v_A)_t = 0$

and

$(v_B)_t = 15$ m/s

By conservation of momentum along *n*-axis,

$$m_A(u_A)_n + m_B(u_B)_n = m_A(v_A)_n + m_B(v_B)_n$$

$$\therefore \quad 15 \times 5 + 0 = 15(v_A)_n + 15(v_B)_n$$

$$\therefore (v_A)_n + (v_B)_n = 5 \qquad \text{... (1)}$$

$$e = -\left[\frac{(v_A)_n - (v_B)_n}{(u_A)_n - (u_B)_n}\right]$$

$$\therefore \quad 0.6 = -\left[\frac{(v_A)_n - (v_B)_n}{5 - 0}\right]$$

$$-(v_A)_n + (v_B)_n = 3 \qquad \text{... (2)}$$

Adding equations (1) and (2),

$2(v_B)_m = 8$

□ $(v_B)_m = 4$ m/s

Substituting in equation (1),

$$(v_A)_n + 4 = 5$$

$$\therefore \quad (v_A)_n = 1 \text{ m/s}$$

$$v_A = \sqrt{(v_A)_n^2 + (v_A)_t^2} = \sqrt{1^2 + 0}$$

$$\therefore \quad \boxed{v_A = 1 \text{ m/s} \rightarrow}$$

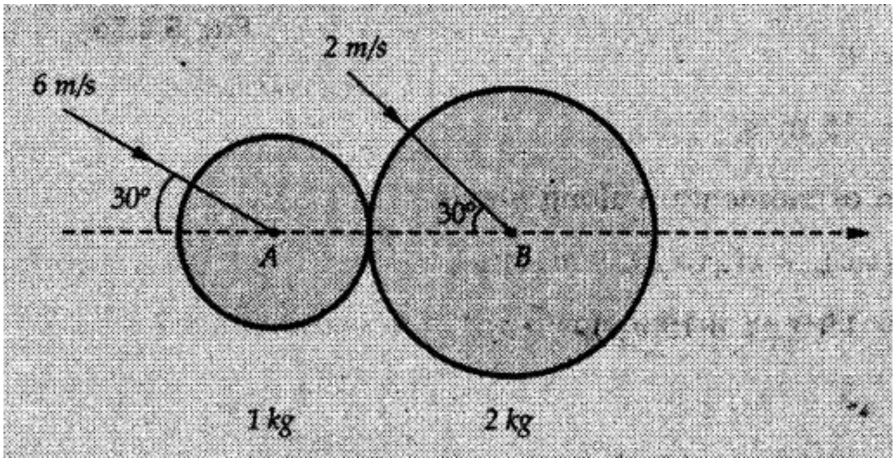
$$v_B = \sqrt{(v_B)_n^2 + (v_B)_t^2} = \sqrt{4^2 + 15^2}$$

$$\therefore \quad \boxed{v_B = 15.524 \text{ m/s}}$$

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_t}{(v_B)_n}\right] = \tan^{-1}\left(\frac{15}{4}\right)$$

$$\therefore \quad \boxed{\theta_B = 15.07^\circ}$$

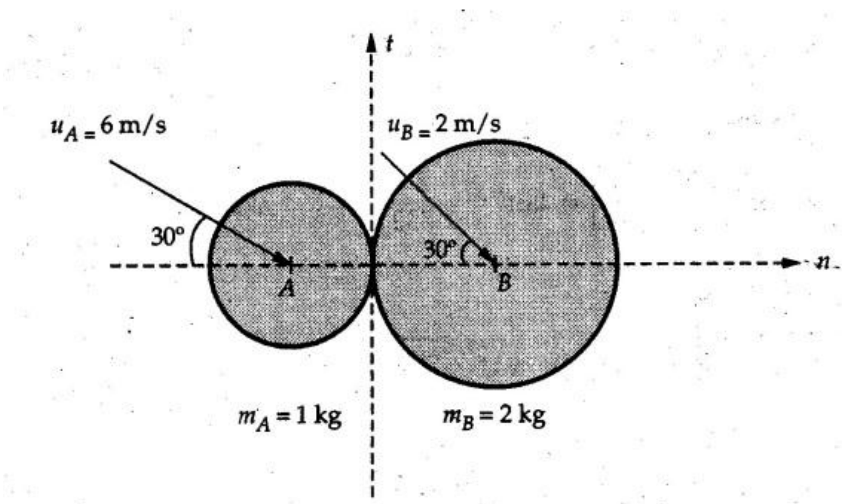
2.A ball of mass 1 kg moving with a velocity of 6 m/s strikes another ball of mass 2 kg moving with a velocity of 2 m/s at the instant of impact the velocities of the two balls are parallel and inclined at 30° to the line joining their centers as shown in the Fig.



If the coefficient of restitution is 0.5, find the velocity and the direction of the two balls after impact. Also calculate the loss in kinetic energy due to impact and the percentage of loss.

Solution:

The *t*-axis along the common tangent and the n-axis perpendicular to it are shown in Fig.



Resolving the initial velocities of A and B along *t* and *n*-axis.

$$(u_A)_n = 6\cos 30, \quad (u_A)_t = -6\sin 30$$

$$(u_B)_n = 2\cos 30, \quad (u_B)_t = -2\sin 30$$

The *t*-component of velocity remains unchanged after impact.

$$(v_A)_t = -6\sin 30 = -3 \text{ m/s}$$

and $$(v_B)_t = -2\sin 30 = -1 \text{ m/s}$$

Momentum is conserved along the *n*-axis.

$$m_A(u_A)_n + m_B(u_B)_n = m_A(v_A)_n + m_B(v_B)_n$$

$$1\times 6\cos 30 + 2\times 2\cos 30 = 1\times (v_A)_n + 2(v_B)_n$$

$$\therefore \quad (v_A)_n + 2(v_B)_n = 8.66 \qquad \dots (1)$$

The coefficient of restitution is given by

$$e = -\left[\frac{(v_B)_n - (v_A)_n}{(u_B)_n - (u_A)_n}\right]$$

$$0.5 = -\left[\frac{(v_B)_n - (v_A)_n}{2\cos 30 - 6\cos 30}\right]$$

$$(v_A)_n - (v_B)_n = -1.732 \qquad \dots (2)$$

From equations (1) and (2),

$(v_A)_n = 1.732$ m/s

$(v_B)_n = 3.464$ m/s

$$v_A = \sqrt{(v_A)_t^2 + (v_A)_n^2} = \sqrt{3^2 + 1.732^2}$$

$$\therefore \quad \boxed{v_A = 3.464 \text{ m/s}}$$

$$\theta_A = \tan^{-1}\left[\frac{(v_A)_t}{(v_A)_n}\right] = \tan^{-1}\left(\frac{3}{1.732}\right)$$

$$\therefore \quad \boxed{\theta_A = 60^\circ}$$

$$v_B = \sqrt{(v_B)_t^2 + (v_B)_n^2} = \sqrt{1^2 + 3.464^2}$$

$$\therefore \quad \boxed{v_B = 3.605 \text{ m/s}}$$

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_t}{(v_B)_n}\right] = \tan^{-1}\left(\frac{1}{3.464}\right)$$

$$\therefore \quad \boxed{\theta_B = 16.1^\circ}$$

$$\text{Initial } K.E. = \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B u_B^2 = \frac{1}{2} \times 1 \times 6^2 + \frac{1}{2} \times 2 \times 2^2$$

$$\therefore \quad \text{Initial } K.E. = 22 \text{ J}$$

$$\text{Final } K.E. = \frac{1}{2} m\, v_A^2 + \frac{1}{2} m_A\, v_B^2 = \frac{1}{2} \times 1 \times 3.464^2 + \frac{1}{2} \times 2 \times 3.605^2$$

$$\therefore \quad \text{Final } K.E. = 19 \text{ J}$$

$$\% \text{ Loss of } K.E. = \left[\frac{\text{Initial } K.E - \text{Final } K.E}{\text{Initial } K.E}\right] \times 100 = \left(\frac{22-19}{22}\right) \times 100$$

$$\therefore \quad \boxed{\% \text{ Loss of K.E.} = 13.64 \%}$$

3.A ball of mass 2 kg, moving with a velocity of 3 m/s, impinges on a ball of mass 4 kg moving with a velocity of 1 m/s. The velocities of the two balls are parallel and inclined at 30° to the line of joining their centres at the instant of impact.

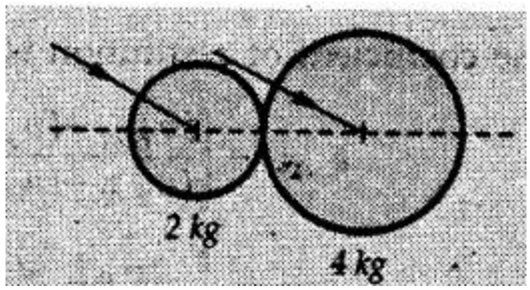
If the coefficient of restitution is 0.5, find

i) Direction, in which the 4 kg ball will move after impact ;

ii) Velocity of the 4 kg ball after impact ;

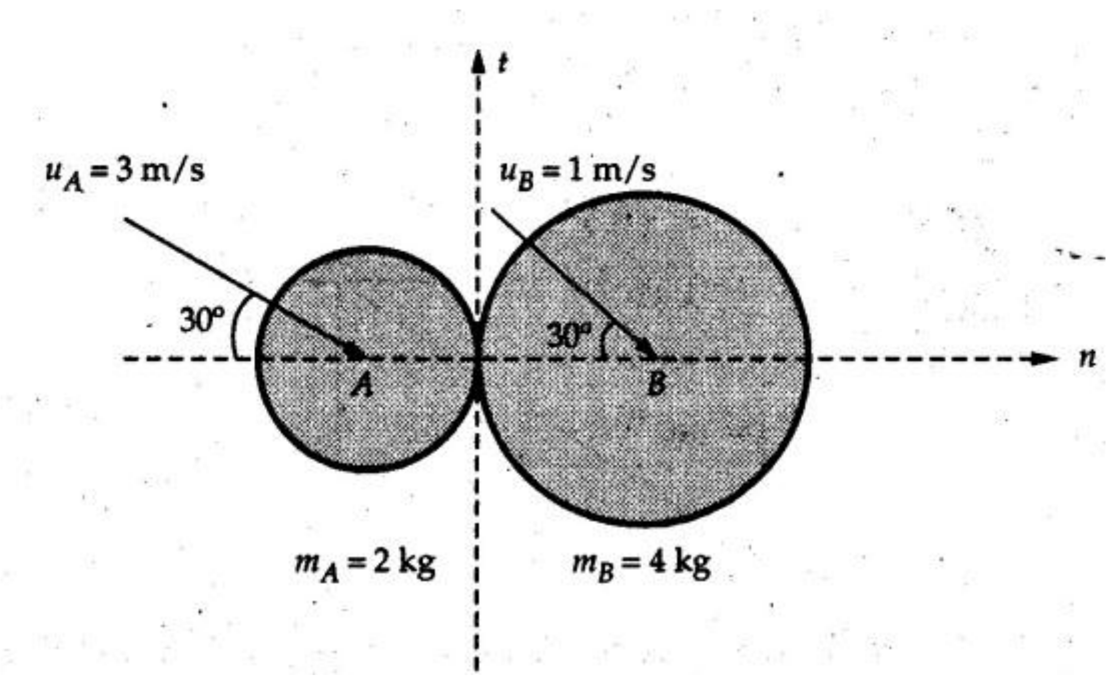
iii) Direction, in which the 2 kg ball will move after impact ;

iv) Velocity of the 2 kg ball after impact.



Solution:

The *t*-axis along the common tangent and the n-axis perpendicular to it are shown in Fig. 9.3.37 (a).



Resolving the initial velocities of A and B along *t* and *n*-axes,

$$(u_A)_n = 3 \cos 30, \quad (u_A)_t = -3 \sin 30$$

$$(u_B)_n = 1 \cos 30, \quad (u_B)_t = -1 \sin 30$$

The *t*-component of velocity remains unchanged after impact.

$$(v_A)_t = -3\sin 30 = -1.5 \text{ m/s}$$

and $$(v_B)_t = -1\sin 30 = -0.5 \text{ m/s}$$

Momentum is conserved along the *n*-axis.

$$m_A(u_A)_n + m_B(u_B)_n = m_A(v_A)_n + m_B(v_B)_n$$

$$2\times 3\cos 30 + 4\times 1\cos 30 = 2\times(v_A)_n + 4(v_B)_n$$

$$\therefore 2(v_A)_n + 4(v_B)_n = 8.66 \qquad \dots (1)$$

The coefficient of restitution is given by

$$e = -\left[\frac{(v_B)_n - (v_A)_n}{(u_B)_n - (u_A)_n}\right]$$

$$0.5 = -\left[\frac{(v_B)_n - (v_A)_n}{1\cos 30 - 3\cos 30}\right]$$

$$(v_A)_n - (v_B)_n = -0.866 \qquad \dots (2)$$

From equations (1) and (2),

$$(v_A)_n = 0.866 \text{ m/s}$$

$$(v_B)_n = 1.732 \text{ m/s}$$

$$v_A = \sqrt{(v_A)_t^2 + (v_A)_n^2} = \sqrt{1.5^2 + 0.866^2}$$

$$\therefore \quad \boxed{v_A = 1.732 \text{ m/s}}$$

$$\theta_A = \tan^{-1}\left[\frac{(v_A)_t}{(v_A)_n}\right] = \tan^{-1}\left(\frac{1.5}{0.866}\right)$$

$$\therefore \quad \boxed{\theta_A = 60^\circ}$$

$$v_B = \sqrt{(v_B)_t^2 + (v_B)_n^2} = \sqrt{0.5^2 + 1.732^2}$$

$$\therefore \quad \boxed{v_B = 1.803 \text{ m/s}}$$

$$\theta_B = \tan^{-1}\left[\frac{(v_B)_t}{(v_B)_n}\right] = \tan^{-1}\left(\frac{0.5}{1.732}\right)$$

$$\therefore \quad \boxed{\theta_B = 16.1^\circ}$$